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ARTICLE

Supra-Approximation Spaces Using Combined Edges Systems

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Abstract

The primary in this paper's notion, the i-space using incident edges system (resp. n-space using non-incidental edges system), is what this study is responsible for generating and investigating. Additionally, we used c-interior to define the c-lower approximations in generalized rough set theory (resp. i-interior and n-interior) Additionally, the c-upper approximations are defined using c-closure (as opposed to i-closure and n-closure), and some of its characteristics are studied.

2010 Mathematics Subject classification: 05C20, 04A05, 54A05

Keywords: Graph, Topology, Supra-approximation space, c-lower approximation and c-upper approximation

1. Introduction and preliminaries

For two reasons, discrete mathematics mainly relies on graph theory, an interesting and significant area of mathematics. In theory, the graph is mathematically appealing. They can be used to depict topographic space, harmonic objects, and a wide variety of other mathematical graphs despite being simple relation graphs. The second reason is that when many concepts are empirically represented by graphs, they will be incredibly helpful in practice. The concepts of topological graph theory [1,2,3,4,5,8,9] are a subfield of mathematics that have numerous applications in both theoretical and practical contexts. We predict that topological graph structure will play a key role in bridging the topology and applications divide. For all graph theory slang and notation, we refer to Harary [6] and all terminology and notation in topology, we refer to Moller [7]. Some basic concepts of graph theory [10] are presented. A undirected graph or graph is pair $\Omega=(\mho(\Omega),\mathcal{E}(\Omega))$ where $\mho(\Omega)$ is a non-empty set whose elements are called points or vertices (called vertex set) and $\mathcal{E}(\Omega)$ is the set of unordered pairs of elements of $\mho(\Omega)$ (called edge set). An edge of a graph that joins a vertex to itself is called a loop. If two edges of a graph are joined by an vertex then these edges are called the edges ϱ incident with the edges ϱ_1. the set of ϱ is $\{\varrho_1\in\mathcal{E}(\Omega):\varrho_1$ incident with $\varrho\}$ and the edges ϱ non incident with the edges ϱ_1. the set of ϱ is $\{\varrho_1\in\mathcal{E}(\Omega):\varrho_1$ nonincident with $\varrho\}$. A graph is symmetric if $(\vartheta_1,\vartheta_2)\in\mathcal{E}(\Omega)$ implies $(\vartheta_2,\vartheta_1)\in\mathcal{E}(\Omega)$, antisymmetric if $(\vartheta_1,\vartheta_2)\in\mathcal{E}(\Omega)$ and $(\vartheta_2,\vartheta_1)\in\mathcal{E}(\Omega)$ implies $\vartheta_2=\vartheta_1$. A sub graph of a graph Ω is a graph each of whose vertices belong to $\mho(\Omega)$ and each of whose edges belong to $\mathcal{E}(\Omega)$. An empty graph if the vertices set and edge set is empty. A degree of a vertex ϑ in a graph Ω is the number of edges of Ω incident with ϑ. A star graph of order n (denoted by S_n) is a graph that all edges are incident to each other. Let $\Omega=(\mho(\Omega),\mathcal{E}(\Omega))$ be und. g. and a edge $\varrho\in\mathcal{E}(\Omega)$. The incident edges set of ϱ is denoted by $I\mathcal{E}(\varrho)$ and defined by (see Tables 1–6, Figs. 1–3).

$I\mathcal{E}(\varrho)=\{\varrho_1\in\mathcal{E}(\Omega):\varrho_1$incident with $\varrho\}$ and The non-incident edges set of ϱ is denoted by $NI\mathcal{E}(\varrho)$ and defined by $NI\mathcal{E}(\varrho)=\{\varrho_1\in\mathcal{E}(\Omega):\varrho_1$ nonincident with $\varrho\}$. an und. g., $\Omega=(\mho(\Omega),\mathcal{E}(\Omega))$ the incident edges system (resp. non incident edges system) of a edge $\varrho\in\mathcal{E}(\Omega)$ is denoted by $I\mathcal{E}S(\varrho)$ (resp. $NI\mathcal{E}S(\varrho)$) and defined by: $I\mathcal{E}S(\varrho)=\{I\mathcal{E}(\varrho)\}$ (resp. $NI\mathcal{E}S(\varrho)=\{NI\mathcal{E}(\varrho)\}$). The Combined edges System of a edge $\varrho\in\mathcal{E}(\Omega)$ is denoted by $C\mathcal{E}S(\varrho)$ and defined by $C\mathcal{E}S(\varrho)=\{I\mathcal{E}S(\varrho),NI\mathcal{E}S(\varrho)\}$. Let $\Omega=(\mho(\Omega),\mathcal{E}(\Omega))$ be an und. g. and suppose that $þ_c:\mathcal{E}(\Omega)\rightarrow P(P(\mathcal{E}(\Omega)))$ is a mapping which assigns for each ϱ in $\mathcal{E}(\Omega)$ its

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Table 1. $L_i(\mathcal{E}(ƕ))$, $L_n(\mathcal{E}(ƕ))$ and $L_c(\mathcal{E}(ƕ))$ for all $ƕ \subseteq \Omega$.

$\mathcal{E}(ƕ)$	$L_i(\mathcal{E}(ƕ))$	$L_n(\mathcal{E}(ƕ))$	$L_c(\mathcal{E}(ƕ))$
$\{ϙ_1\}$	ϕ	ϕ	ϕ
$\{ϙ_2\}$	ϕ	ϕ	ϕ
$\{ϙ_3\}$	ϕ	ϕ	ϕ
$\{ϙ_4\}$	ϕ	$\{ϙ_4\}$	$\{ϙ_4\}$
$\{ϙ_5\}$	ϕ	ϕ	ϕ
$\{ϙ_1, ϙ_2\}$	ϕ	ϕ	ϕ
$\{ϙ_1, ϙ_3\}$	ϕ	ϕ	ϕ
$\{ϙ_1, ϙ_4\}$	ϕ	$\{ϙ_4\}$	$\{ϙ_4\}$
$\{ϙ_1, ϙ_5\}$	ϕ	ϕ	ϕ
$\{ϙ_2, ϙ_3\}$	ϕ	ϕ	ϕ
$\{ϙ_2, ϙ_4\}$	ϕ	$\{ϙ_4\}$	$\{ϙ_4\}$
$\{ϙ_2, ϙ_5\}$	ϕ	ϕ	ϕ
$\{ϙ_3, ϙ_4\}$	ϕ	$\{ϙ_4\}$	$\{ϙ_4\}$
$\{ϙ_3, ϙ_5\}$	ϕ	ϕ	ϕ
$\{ϙ_4, ϙ_5\}$	ϕ	$\{ϙ_4\}$	$\{ϙ_4\}$
$\{ϙ_1, ϙ_2, ϙ_3\}$	ϕ	ϕ	ϕ
$\{ϙ_1, ϙ_2, ϙ_4\}$	ϕ	$\{ϙ_4\}$	$\{ϙ_4\}$
$\{ϙ_1, ϙ_2, ϙ_5\}$	ϕ	ϕ	ϕ
$\{ϙ_2, ϙ_3, ϙ_4\}$	ϕ	$\{ϙ_4\}$	$\{ϙ_4\}$
$\{ϙ_2, ϙ_3, ϙ_5\}$	ϕ	ϕ	ϕ
$\{ϙ_3, ϙ_4, ϙ_1\}$	ϕ	$\{ϙ_4\}$	$\{ϙ_4\}$
$\{ϙ_3, ϙ_4, ϙ_5\}$	ϕ	$\{ϙ_4\}$	$\{ϙ_4\}$
$\{ϙ_4, ϙ_5, ϙ_1\}$	ϕ	$\{ϙ_4\}$	$\{ϙ_4\}$
$\{ϙ_4, ϙ_5, ϙ_2\}$	ϕ	$\{ϙ_4\}$	$\{ϙ_4\}$
$\{ϙ_1, ϙ_3, ϙ_5\}$	ϕ	ϕ	ϕ
$\{ϙ_1, ϙ_2, ϙ_3, ϙ_4\}$	ϕ	$\{ϙ_4\}$	$\{ϙ_1, ϙ_2, ϙ_3, ϙ_4\}$
$\{ϙ_1, ϙ_2, ϙ_3, ϙ_5\}$	ϕ	$\{ϙ_1, ϙ_2, ϙ_3, ϙ_5\}$	$\{ϙ_1, ϙ_2, ϙ_3, ϙ_5\}$
$\{ϙ_2, ϙ_3, ϙ_4, ϙ_5\}$	ϕ	$\{ϙ_4\}$	$\{ϙ_2, ϙ_3, ϙ_4, ϙ_5\}$
$\{ϙ_1, ϙ_3, ϙ_4, ϙ_5\}$	ϕ	$\{ϙ_4\}$	$\{ϙ_1, ϙ_3, ϙ_4, ϙ_5\}$
$\{ϙ_1, ϙ_2, ϙ_4, ϙ_5\}$	ϕ	$\{ϙ_4\}$	$\{ϙ_1, ϙ_2, ϙ_4, ϙ_5\}$
$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$
ϕ	ϕ	ϕ	ϕ

Table 2. $U_i(\mathcal{E}(ƕ))$, $U_n(\mathcal{E}(ƕ))$ and $U_c(\mathcal{E}(ƕ))$ for all $ƕ \subseteq \Omega$.

$\mathcal{E}(ƕ)$	$U_i(\mathcal{E}(ƕ))$	$U_n(\mathcal{E}(ƕ))$	$U_c(\mathcal{E}(ƕ))$
$\{ϙ_1\}$	$\mathcal{E}(\Omega)$	$\{ϙ_1, ϙ_2, ϙ_3, ϙ_5\}$	$\{ϙ_1\}$
$\{ϙ_2\}$	$\mathcal{E}(\Omega)$	$\{ϙ_1, ϙ_2, ϙ_3, ϙ_5\}$	$\{ϙ_2\}$
$\{ϙ_3\}$	$\mathcal{E}(\Omega)$	$\{ϙ_1, ϙ_2, ϙ_3, ϙ_5\}$	$\{ϙ_3\}$
$\{ϙ_4\}$	$\mathcal{E}(\Omega)$	$\{ϙ_4\}$	$\{ϙ_4\}$
$\{ϙ_5\}$	$\mathcal{E}(\Omega)$	$\{ϙ_1, ϙ_2, ϙ_3, ϙ_5\}$	$\{ϙ_5\}$
$\{ϙ_1, ϙ_2\}$	$\mathcal{E}(\Omega)$	$\{ϙ_1, ϙ_2, ϙ_3, ϙ_5\}$	$\{ϙ_1, ϙ_2, ϙ_3, ϙ_5\}$
$\{ϙ_1, ϙ_3\}$	$\mathcal{E}(\Omega)$	$\{ϙ_1, ϙ_2, ϙ_3, ϙ_5\}$	$\{ϙ_1, ϙ_2, ϙ_3, ϙ_5\}$
$\{ϙ_1, ϙ_4\}$	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$
$\{ϙ_1, ϙ_5\}$	$\mathcal{E}(\Omega)$	$\{ϙ_1, ϙ_2, ϙ_3, ϙ_5\}$	$\{ϙ_1, ϙ_2, ϙ_3, ϙ_5\}$
$\{ϙ_2, ϙ_3\}$	$\mathcal{E}(\Omega)$	$\{ϙ_1, ϙ_2, ϙ_3, ϙ_5\}$	$\{ϙ_1, ϙ_2, ϙ_3, ϙ_5\}$
$\{ϙ_2, ϙ_4\}$	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$
$\{ϙ_2, ϙ_5\}$	$\mathcal{E}(\Omega)$	$\{ϙ_1, ϙ_2, ϙ_3, ϙ_5\}$	$\{ϙ_1, ϙ_2, ϙ_3, ϙ_5\}$
$\{ϙ_3, ϙ_4\}$	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$
$\{ϙ_3, ϙ_5\}$	$\mathcal{E}(\Omega)$	$\{ϙ_1, ϙ_2, ϙ_3, ϙ_5\}$	$\{ϙ_1, ϙ_2, ϙ_3, ϙ_5\}$
$\{ϙ_4, ϙ_5\}$	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$
$\{ϙ_1, ϙ_2, ϙ_3\}$	$\mathcal{E}(\Omega)$	$\{ϙ_1, ϙ_2, ϙ_3, ϙ_5\}$	$\{ϙ_1, ϙ_2, ϙ_3, ϙ_5\}$
$\{ϙ_1, ϙ_2, ϙ_4\}$	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$
$\{ϙ_1, ϙ_2, ϙ_5\}$	$\mathcal{E}(\Omega)$	$\{ϙ_1, ϙ_2, ϙ_3, ϙ_5\}$	$\{ϙ_1, ϙ_2, ϙ_3, ϙ_5\}$
$\{ϙ_2, ϙ_3, ϙ_4\}$	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$
$\{ϙ_2, ϙ_3, ϙ_5\}$	$\mathcal{E}(\Omega)$	$\{ϙ_1, ϙ_2, ϙ_3, ϙ_5\}$	$\{ϙ_1, ϙ_2, ϙ_3, ϙ_5\}$
$\{ϙ_3, ϙ_4, ϙ_1\}$	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$
$\{ϙ_3, ϙ_4, ϙ_5\}$	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$
$\{ϙ_4, ϙ_5, ϙ_1\}$	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)\}$	$\mathcal{E}(\Omega)$
$\{ϙ_4, ϙ_5, ϙ_2\}$	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$
$\{ϙ_1, ϙ_3, ϙ_5\}$	$\mathcal{E}(\Omega)$	$\{ϙ_1, ϙ_2, ϙ_3, ϙ_5\}$	$\{ϙ_1, ϙ_2, ϙ_3, ϙ_5\}$
$\{ϙ_1, ϙ_2, ϙ_3, ϙ_4\}$	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$
$\{ϙ_1, ϙ_2, ϙ_3, ϙ_5\}$	$\mathcal{E}(\Omega)$	$\{ϙ_1, ϙ_2, ϙ_3, ϙ_5\}$	$\{ϙ_1, ϙ_2, ϙ_3, ϙ_5\}$
$\{ϙ_2, ϙ_3, ϙ_4, ϙ_5\}$	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$
$\{ϙ_1, ϙ_3, ϙ_4, ϙ_5\}$	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$
$\{ϙ_1, ϙ_2, ϙ_4, ϙ_5\}$	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$
$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$
ϕ	ϕ	ϕ	ϕ

Combined edges System in $P(P(\mathcal{E}(\Omega)))$. The pair $(\Omega, þ_c)$ is called the C-space.

2. c-Lower and c-upper approximations

We introduce the topological spaces i-space and n-space in this section. The unguarded. i-interior, n-interior, i-closure, and n-closure were all defined. Finally, using i-interior (resp. n-interior and c-interior), we define the c-lower and c-upper approximations in generalized rough set theory and look into some of its aspects.

Definition 2.1. Let $\Omega = (\mho(\Omega), \mathcal{E}(\Omega))$ be an und. g. and suppose that $þ_i : \mathcal{E}(\Omega) \to P(P(\mathcal{E}(\Omega)))$ (resp. $þ_n : \mathcal{E}(\Omega) \to P(P(\mathcal{E}(\Omega))))$ is a mapping which assigns for each ϙ in $\mathcal{E}(\Omega)$ it's incident (resp. non incident) edges system in $P(P(\mathcal{E}(\Omega)))$. The pair $(\Omega, þ_i)$ (resp. $(\Omega, þ_n)$) is called an i-space (resp. n-space).

Definition 2.2. Let $(\Omega, þ_i)$ be an i-space and $(\Omega, þ_n)$ be an n-space and let $ƕ \subseteq \Omega$. Then

(a) The i-derived and n-derived of an und. g. ƕ are defined respectively by:

$$[\mathcal{E}(ƕ)]_i^{'} = \{ϙ \in \mathcal{E}(\Omega); I\mathcal{E}(ϙ), I\mathcal{E}(ϙ) \cap (\mathcal{E}(ƕ) - \{ϙ\}) \neq \varnothing\}$$

$$[\mathcal{E}(ƕ)]_n^{'} = \{ϙ \in \mathcal{E}(\Omega); NI\mathcal{E}(ϙ), NI\mathcal{E}(ϙ) \cap (\mathcal{E}(ƕ) - \{ϙ\}) \neq \varnothing\}$$

(b) The classes of i-closed and n-closed of an und. g. in i-space and n-space are defined respectively by:

$$\mathfrak{T}_{þ_i} = \left\{\mathcal{E}(ƕ) \subseteq \mathcal{E}(\Omega);\ [\mathcal{E}(ƕ)]_i^{'} \subseteq \mathcal{E}(ƕ)\right\}$$

$$\mathfrak{T}_{þ_n} = \left\{\mathcal{E}(ƕ) \subseteq \mathcal{E}(\Omega);\ [\mathcal{E}(ƕ)]_n^{'} \subseteq \mathcal{E}(ƕ)\right\}$$

(c) The classes of i-open and n-open of an und. g. in i-space and n-space are defined respectively:

$Ɣ_{þ_i} = \{\mathcal{E}(Ю) \subseteq \mathcal{E}(\Omega); \mathcal{E}(Ю) = \mathcal{E}(\Omega) - \mathcal{E}(ƕ)$ such that $\mathcal{E}(ƕ) \in \mathfrak{T}_{þ_i}\}$,
$Ɣ_{þ_n} = \{\mathcal{E}(Ю) \subseteq \mathcal{E}(\Omega); \mathcal{E}(Ю) = \mathcal{E}(\Omega) - \mathcal{E}(ƕ)$ such that $\mathcal{E}(ƕ) \in \mathfrak{T}_{þ_n}\}$,

Table 3. $B_i(\mathcal{E}(ƕ))$, $B_n(\mathcal{E}(ƕ))$ and $B_c(\mathcal{E}(ƕ))$ for all ƕ⊆Ω.

$\mathcal{E}(ƕ)$	$B_i(\mathcal{E}(ƕ))$	$B_n(\mathcal{E}(ƕ))$	$B_c(\mathcal{E}(ƕ))$
$\{\varrho_1\}$	$\mathcal{E}(\Omega)$	$\{\varrho_1,\varrho_2,\varrho_3,\varrho_5\}$	$\{\varrho_1\}$
$\{\varrho_2\}$	$\mathcal{E}(\Omega)$	$\{\varrho_1,\varrho_2,\varrho_3,\varrho_5\}$	$\{\varrho_2\}$
$\{\varrho_3\}$	$\mathcal{E}(\Omega)$	$\{\varrho_1,\varrho_2,\varrho_3,\varrho_5\}$	$\{\varrho_3\}$
$\{\varrho_4\}$	$\mathcal{E}(\Omega)$	ϕ	ϕ
$\{\varrho_5\}$	$\mathcal{E}(\Omega)$	$\{\varrho_1,\varrho_2,\varrho_3,\varrho_5\}$	$\{\varrho_5\}$
$\{\varrho_1,\varrho_2\}$	$\mathcal{E}(\Omega)$	$\{\varrho_1,\varrho_2,\varrho_3,\varrho_5\}$	$\{\varrho_1,\varrho_2,\varrho_3,\varrho_5\}$
$\{\varrho_1,\varrho_3\}$	$\mathcal{E}(\Omega)$	$\{\varrho_1,\varrho_2,\varrho_3,\varrho_5\}$	$\{\varrho_1,\varrho_2,\varrho_3,\varrho_5\}$
$\{\varrho_1,\varrho_4\}$	$\mathcal{E}(\Omega)$	$\{\varrho_1,\varrho_2,\varrho_3,\varrho_5\}$	$\{\varrho_1,\varrho_2,\varrho_3,\varrho_5\}$
$\{\varrho_1,\varrho_5\}$	$\mathcal{E}(\Omega)$	$\{\varrho_1,\varrho_2,\varrho_3,\varrho_5\}$	$\{\varrho_1,\varrho_2,\varrho_3,\varrho_5\}$
$\{\varrho_2,\varrho_3\}$	$\mathcal{E}(\Omega)$	$\{\varrho_1,\varrho_2,\varrho_3,\varrho_5\}$	$\{\varrho_1,\varrho_2,\varrho_3,\varrho_5\}$
$\{\varrho_2,\varrho_4\}$	$\mathcal{E}(\Omega)$	$\{\varrho_1,\varrho_2,\varrho_3,\varrho_5\}$	$\{\varrho_1,\varrho_2,\varrho_3,\varrho_5\}$
$\{\varrho_2,\varrho_5\}$	$\mathcal{E}(\Omega)$	$\{\varrho_1,\varrho_2,\varrho_3,\varrho_5\}$	$\{\varrho_1,\varrho_2,\varrho_3,\varrho_5\}$
$\{\varrho_3,\varrho_4\}$	$\mathcal{E}(\Omega)$	$\{\varrho_1,\varrho_2,\varrho_3,\varrho_5\}$	$\{\varrho_1,\varrho_2,\varrho_3,\varrho_5\}$
$\{\varrho_3,\varrho_5\}$	$\mathcal{E}(\Omega)$	$\{\varrho_1,\varrho_2,\varrho_3,\varrho_5\}$	$\{\varrho_1,\varrho_2,\varrho_3,\varrho_5\}$
$\{\varrho_4,\varrho_5\}$	$\mathcal{E}(\Omega)$	$\{\varrho_1,\varrho_2,\varrho_3,\varrho_5\}$	$\{\varrho_1,\varrho_2,\varrho_3,\varrho_5\}$
$\{\varrho_1,\varrho_2,\varrho_3\}$	$\mathcal{E}(\Omega)$	$\{\varrho_1,\varrho_2,\varrho_3,\varrho_5\}$	$\{\varrho_1,\varrho_2,\varrho_3,\varrho_5\}$
$\{\varrho_1,\varrho_2,\varrho_4\}$	$\mathcal{E}(\Omega)$	$\{\varrho_1,\varrho_2,\varrho_3,\varrho_5\}$	$\{\varrho_1,\varrho_2,\varrho_3,\varrho_5\}$
$\{\varrho_1,\varrho_2,\varrho_5\}$	$\mathcal{E}(\Omega)$	$\{\varrho_1,\varrho_2,\varrho_3,\varrho_5\}$	$\{\varrho_1,\varrho_2,\varrho_3,\varrho_5\}$
$\{\varrho_2,\varrho_3,\varrho_4\}$	$\mathcal{E}(\Omega)$	$\{\varrho_1,\varrho_2,\varrho_3,\varrho_5\}$	$\{\varrho_1,\varrho_2,\varrho_3,\varrho_5\}$
$\{\varrho_2,\varrho_3,\varrho_5\}$	$\mathcal{E}(\Omega)$	$\{\varrho_1,\varrho_2,\varrho_3,\varrho_5\}$	$\{\varrho_1,\varrho_2,\varrho_3,\varrho_5\}$
$\{\varrho_3,\varrho_4,\varrho_1\}$	$\mathcal{E}(\Omega)$	$\{\varrho_1,\varrho_2,\varrho_3,\varrho_5\}$	$\{\varrho_1,\varrho_2,\varrho_3,\varrho_5\}$
$\{\varrho_3,\varrho_4,\varrho_5\}$	$\mathcal{E}(\Omega)$	$\{\varrho_1,\varrho_2,\varrho_3,\varrho_5\}$	$\{\varrho_1,\varrho_2,\varrho_3,\varrho_5\}$
$\{\varrho_4,\varrho_5,\varrho_1\}$	$\mathcal{E}(\Omega)$	$\{\varrho_1,\varrho_2,\varrho_3,\varrho_5\}$	$\{\varrho_1,\varrho_2,\varrho_3,\varrho_5\}$
$\{\varrho_4,\varrho_5,\varrho_2\}$	$\mathcal{E}(\Omega)$	$\{\varrho_1,\varrho_2,\varrho_3,\varrho_5\}$	$\{\varrho_1,\varrho_2,\varrho_3,\varrho_5\}$
$\{\varrho_1,\varrho_3,\varrho_5\}$	$\mathcal{E}(\Omega)$	$\{\varrho_1,\varrho_2,\varrho_3,\varrho_5\}$	$\{\varrho_1,\varrho_2,\varrho_3,\varrho_5\}$
$\{\varrho_1,\varrho_2,\varrho_3,\varrho_4\}$	$\mathcal{E}(\Omega)$	$\{\varrho_1,\varrho_2,\varrho_3,\varrho_5\}$	$\{\varrho_5\}$
$\{\varrho_1,\varrho_2,\varrho_3,\varrho_5\}$	$\mathcal{E}(\Omega)$	ϕ	ϕ
$\{\varrho_2,\varrho_3,\varrho_4,\varrho_5\}$	$\mathcal{E}(\Omega)$	$\{\varrho_1,\varrho_2,\varrho_3,\varrho_5\}$	$\{\varrho_1\}$
$\{\varrho_1,\varrho_3,\varrho_4,\varrho_5\}$	$\mathcal{E}(\Omega)$	$\{\varrho_1,\varrho_2,\varrho_3,\varrho_5\}$	$\{\varrho_2\}$
$\{\varrho_1,\varrho_2,\varrho_4,\varrho_5\}$	$\mathcal{E}(\Omega)$	$\{\varrho_1,\varrho_2,\varrho_3,\varrho_5\}$	$\{\varrho_3\}$
$\mathcal{E}(\Omega)$	ϕ	ϕ	ϕ
ϕ	ϕ	ϕ	ϕ

Table 4. $\zeta_i(\mathcal{E}(ƕ))$, $\zeta_n(\mathcal{E}(ƕ))$ and $\zeta_c(\mathcal{E}(ƕ))$ for all ƕ⊆Ω.

$\mathcal{E}(ƕ)$	$\zeta_i(\mathcal{E}(ƕ))$	$\zeta_n(\mathcal{E}(ƕ))$	$\zeta_c(\mathcal{E}(ƕ))$
$\{\varrho_1\}$	0	1/5	4/5
$\{\varrho_2\}$	0	1/5	4/5
$\{\varrho_3\}$	0	1/5	4/5
$\{\varrho_4\}$	0	1	1
$\{\varrho_5\}$	0	1/5	4/5
$\{\varrho_1,\varrho_2\}$	0	1/5	1/5
$\{\varrho_1,\varrho_3\}$	0	1/5	1/5
$\{\varrho_1,\varrho_4\}$	0	1/5	1/5
$\{\varrho_1,\varrho_5\}$	0	1/5	1/5
$\{\varrho_2,\varrho_3\}$	0	1/5	1/5
$\{\varrho_2,\varrho_4\}$	0	1/5	1/5
$\{\varrho_2,\varrho_5\}$	0	1/5	1/5
$\{\varrho_3,\varrho_4\}$	0	1/5	1/5
$\{\varrho_3,\varrho_5\}$	0	1/5	1/5
$\{\varrho_4,\varrho_5\}$	0	1/5	1/5
$\{\varrho_1,\varrho_2,\varrho_3\}$	0	1/5	1/5
$\{\varrho_1,\varrho_2,\varrho_4\}$	0	1/5	1/5
$\{\varrho_1,\varrho_2,\varrho_5\}$	0	1/5	1/5
$\{\varrho_2,\varrho_3,\varrho_4\}$	0	1/5	1/5
$\{\varrho_2,\varrho_3,\varrho_5\}$	0	1/5	1/5
$\{\varrho_3,\varrho_4,\varrho_1\}$	0	1/5	1/5
$\{\varrho_3,\varrho_4,\varrho_5\}$	0	1/5	1/5
$\{\varrho_4,\varrho_5,\varrho_1\}$	0	1/5	1/5
$\{\varrho_4,\varrho_5,\varrho_2\}$	0	1/5	1/5
$\{\varrho_1,\varrho_3,\varrho_5\}$	0	1/5	1/5
$\{\varrho_1,\varrho_2,\varrho_3,\varrho_4\}$	0	1/5	4/5
$\{\varrho_1,\varrho_2,\varrho_3,\varrho_5\}$	0	1	1
$\{\varrho_2,\varrho_3,\varrho_4,\varrho_5\}$	0	1/5	4/5
$\{\varrho_1,\varrho_3,\varrho_4,\varrho_5\}$	0	1/5	4/5
$\{\varrho_1,\varrho_2,\varrho_4,\varrho_5\}$	0	1/5	4/5
$\mathcal{E}(\Omega)$	1	1	1
ϕ	1	1	1

(d) The i-interior and n-interior of an und. g. ƕ are defined respectively by:

$$\mathrm{Int_i}(\mathcal{E}(ƕ)) = \cup\{\mathcal{E}(Ю) \in Ɣ_{þ_i}; \mathcal{E}(Ю) \subseteq \mathcal{E}(ƕ)\},$$

$$\mathrm{Int_n}(\mathcal{E}(ƕ)) = \cup\{\mathcal{E}(Ю) \in Ɣ_{þ_n}; \mathcal{E}(Ю) \subseteq \mathcal{E}(ƕ)\},$$

(e) The i-closure and n-closure of an und. g. ƕ are defined respectively by:

$$\mathrm{Cl_i}(\mathcal{E}(ƕ)) = \cap\{\mathcal{E}(ƙ) \in \mathfrak{T}_{þ_i}; \mathcal{E}(ƕ) \subseteq \mathcal{E}(ƙ)\},$$

$$\mathrm{Cl_n}(\mathcal{E}(ƕ)) = \cap\{\mathcal{E}(ƙ) \in \mathfrak{T}_{þ_n}; \mathcal{E}(ƕ) \subseteq \mathcal{E}(ƙ)\},$$

(f) The i-boundary and n-boundary of an und. g. ƕ are defined respectively by:

$$[\mathcal{E}(ƕ)]_i^b = \mathrm{Cl_i}(\mathcal{E}(ƕ)) - \mathrm{Int_i}(\mathcal{E}(ƕ)),$$

$$[\mathcal{E}(ƕ)]_n^b = \mathrm{Cl_n}(\mathcal{E}(ƕ)) - \mathrm{Int_n}(\mathcal{E}(ƕ)).$$

Theorem 2.3. In $(\Omega, þ_i)$ (resp. $(\Omega, þ_n)$) is an i-space (resp. n-space) and ƕ⊆Ω, then ƕ is an i-open (resp. n-open) if and only if it contains the incident edges (resp. non incident edges) of each of its edges.

Proof: Let $(\Omega, þ_i)$ be an i-space and ƕ be an i-open und. g. contained in Ω and $\varrho \in \mathcal{E}(ƕ)$. Suppose that $I\mathcal{E}(\varrho)$ is incident edges with ϱ, and $I\mathcal{E}(\varrho) \not\subseteq \mathcal{E}(ƕ)$, thus $I\mathcal{E}(\varrho) \bigcap [\mathcal{E}(\Omega) - \mathcal{E}(ƕ)] \neq \varnothing$ which implies

Table 5. The i-derived and n-derived of an und. g. ƕ .

$\mathcal{E}(ƕ)$	$[\mathcal{E}(ƕ)]_i^{'}$	$[\mathcal{E}(ƕ)]_n^{'}$
$\{\varrho_1\}$	$\{\varrho_2,\varrho_3,\varrho_4\}$	$\varnothing$
$\{\varrho_2\}$	$\{\varrho_1,\varrho_3,\varrho_4\}$	$\varnothing$
$\{\varrho_3\}$	$\{\varrho_1,\varrho_2,\varrho_4\}$	$\varnothing$
$\{\varrho_4\}$	$\{\varrho_1,\varrho_2,\varrho_3\}$	$\varnothing$
$\{\varrho_1,\varrho_2\}$	$\mathcal{E}(\Omega)$	$\varnothing$
$\{\varrho_1,\varrho_3\}$	$\mathcal{E}(\Omega)$	$\varnothing$
$\{\varrho_1,\varrho_4\}$	$\mathcal{E}(\Omega)$	$\varnothing$
$\{\varrho_2,\varrho_3\}$	$\mathcal{E}(\Omega)$	$\varnothing$
$\{\varrho_2,\varrho_4\}$	$\mathcal{E}(\Omega)$	$\varnothing$
$\{\varrho_3,\varrho_4\}$	$\mathcal{E}(\Omega)$	$\varnothing$
$\{\varrho_1,\varrho_2,\varrho_3\}$	$\mathcal{E}(\Omega)$	$\varnothing$
$\{\varrho_1,\varrho_2,\varrho_4\}$	$\mathcal{E}(\Omega)$	$\varnothing$
$\{\varrho_1,\varrho_3,\varrho_4\}$	$\mathcal{E}(\Omega)$	$\varnothing$
$\{\varrho_2,\varrho_3,\varrho_4\}$	$\mathcal{E}(\Omega)$	$\varnothing$
$\mathcal{E}(\Omega)$	$\mathcal{E}(\Omega)$	$\varnothing$
$\varnothing$	$\varnothing$	$\varnothing$

Table 6. The i-derived and n-derived of an und. g. ƕ.

$\mathcal{E}$(ƕ)	$[ƕ]_i^{'}$	$[ƕ]_n^{'}$
$\{\varrho_1\}$	$\varnothing$	$\{\varrho_2,\varrho_3,\varrho_4\}$
$\{\varrho_2\}$	$\varnothing$	$\{\varrho_1,\varrho_3,\varrho_4\}$
$\{\varrho_3\}$	$\varnothing$	$\{\varrho_1,\varrho_2,\varrho_4\}$
$\{\varrho_4\}$	$\varnothing$	$\{\varrho_1,\varrho_2,\varrho_3\}$
$\{\varrho_1,\varrho_2\}$	$\varnothing$	$\mathcal{E}(\Omega)$
$\{\varrho_1,\varrho_3\}$	$\varnothing$	$\mathcal{E}(\Omega)$
$\{\varrho_1,\varrho_4\}$	$\varnothing$	$\mathcal{E}(\Omega)$
$\{\varrho_2,\varrho_3\}$	$\varnothing$	$\mathcal{E}(\Omega)$
$\{\varrho_2,\varrho_4\}$	$\varnothing$	$\mathcal{E}(\Omega)$
$\{\varrho_3,\varrho_4\}$	$\varnothing$	$\mathcal{E}(\Omega)$
$\{\varrho_1,\varrho_2,\varrho_3\}$	$\varnothing$	$\mathcal{E}(\Omega)$
$\{\varrho_1,\varrho_2,\varrho_4\}$	$\varnothing$	$\mathcal{E}(\Omega)$
$\{\varrho_1,\varrho_3,\varrho_4\}$	$\varnothing$	$\mathcal{E}(\Omega)$
$\{\varrho_2,\varrho_3,\varrho_4\}$	$\varnothing$	$\mathcal{E}(\Omega)$
$\mathcal{E}(\Omega)$	$\varnothing$	$\mathcal{E}(\Omega)$
$\varnothing$	$\varnothing$	$\varnothing$

$\varrho\in[\mathcal{E}(\Omega)-\mathcal{E}(ƕ)]_i^{'}$. But $\Omega-ƕ$ is i-closed since ƕ is i-open and so $[\mathcal{E}(\Omega)-\mathcal{E}(ƕ)]_i^{'}\subseteq[\mathcal{E}(\Omega)-\mathcal{E}(ƕ)]$ and hence $\varrho\in[\mathcal{E}(\Omega)-\mathcal{E}(ƕ)]$. Therefore $\varrho\notin\mathcal{E}(ƕ)$ which contradicts with $\varrho\in\mathcal{E}(ƕ)$ and consequently if $ƕ\subseteq\Omega$ is i-open and $\varrho\in\mathcal{E}(ƕ)$, then the incident edges with ϱ which is contained in $\mathcal{E}(ƕ)$. Conversely, Let ƕ contains the incident edges with each of its edges, i.e. for all $\varrho\in\mathcal{E}(ƕ)$ then $I\mathcal{E}(\varrho)\subseteq\mathcal{E}(ƕ)$. Let $\varrho_1\in[\mathcal{E}(\Omega)-\mathcal{E}(ƕ)]_i^{'}$ then $\varrho_1\notin\mathcal{E}(ƕ)$. If $\varrho_1\in\mathcal{E}(ƕ)$ there would be incident edges with ϱ_1, $I\mathcal{E}(\varrho_1)$, such that $I\mathcal{E}(\varrho_1)\subseteq\mathcal{E}(ƕ)$ and this would imply that $I\mathcal{E}(\varrho_1)\bigcap[\mathcal{E}(\Omega)-\mathcal{E}(ƕ)]=\varnothing$, thus $\varrho_1\notin[\mathcal{E}(\Omega)-\mathcal{E}(ƕ)]_i^{'}$ which is impossible. Accordingly, $\varrho_1\in[\mathcal{E}(\Omega)-\mathcal{E}(ƕ)]$ and so $[\mathcal{E}(\Omega)-\mathcal{E}(ƕ)]_i^{'}\subseteq[\mathcal{E}(\Omega)-\mathcal{E}(ƕ)]$ which implies $\Omega-ƕ$ is i-closed and hence ƕ is i-open. Similarly, we can prove that ƕ is n-open if and only if it contains the non incident edges with each of its edges.

Definition 2.4. Let $\mathfrak{Z}=(\mho(\Omega),\mathcal{E}(\Omega))$ be a generalized approximation space and $ƕ\subseteq\Omega$. Then is called incident composed (resp. non incident composed) if ƕ contains the incident edges (resp. non incident edges) with each of its edges i.e. for each $\varrho\in\mathcal{E}(ƕ)$, $I\mathcal{E}(\varrho)\subseteq\mathcal{E}(ƕ)$ (resp. for each $\varrho\in\mathcal{E}(ƕ)$, $NI\mathcal{E}(\varrho)\subseteq\mathcal{E}(ƕ)$).

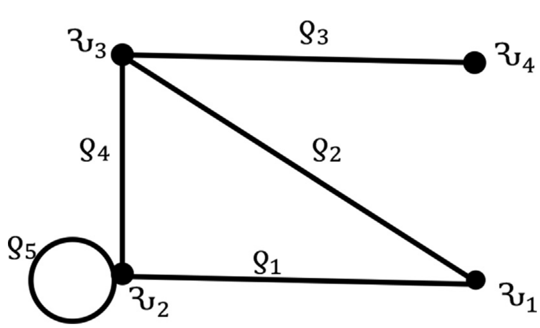

Fig. 1. und. g. Ω given in Example (3.2).

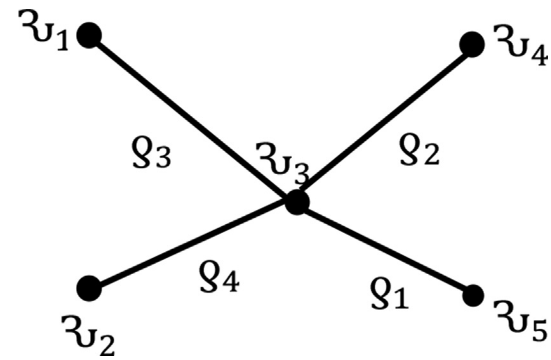

Fig. 2. und. g. Ω given in Example (3.14).

Definition 2.5. *Let* $\mathfrak{Z}=(\mho(\Omega),\mathcal{E}(\Omega))$ be a generalized approximation space, then the class of all incident composed (resp. non incident composed) und. g. are denoted by $Ʈ_i$ (*resp.* $Ʈ_n$) and defined by:
$Ʈ_i=\{ƕ\subseteq\Omega;\text{ for each }\varrho\in\mathcal{E}(ƕ), I\mathcal{E}(\varrho)\subseteq\mathcal{E}(ƕ)\}$
(resp. $Ʈ_n=\{ƕ\subseteq\Omega;\text{ for each}\varrho\in\mathcal{E}(ƕ), NI\mathcal{E}(\varrho)\subseteq\mathcal{E}(ƕ)\}$)

Proposition 2.6. Let $\mathfrak{Z}=(\mho(\Omega),\mathcal{E}(\Omega))$ be a generalized approximation space, then $Ʈ_i$ (resp. $Ʈ_n$) forms a topology on Ω.
Proof:

1. $\mathcal{E}(\Omega),\varnothing\in Ʈ_i$ (resp. $Ʈ_n$)
2. Let $\mathcal{E}(ƕ),\mathcal{E}(ƙ)\in Ʈ_i$ (resp. $Ʈ_n$) and let for each $\varrho\in\mathcal{E}(ƕ)$ and $\varrho\in\mathcal{E}(ƙ)$, which implies that $I\mathcal{E}(\varrho)\subseteq\mathcal{E}(ƕ)$ and $I\mathcal{E}(\varrho)\subseteq\mathcal{E}(ƙ), I\mathcal{E}(\varrho)\subseteq\mathcal{E}(ƕ)\bigcap\mathcal{E}(ƙ)$ $\Rightarrow\mathcal{E}(ƕ)\bigcap\mathcal{E}(ƙ)\in Ʈ_i$ (resp. $Ʈ_n$)
3. Let $\mathcal{E}(ƕ_i)\in Ʈ_i$ (resp. $Ʈ_n$) $\forall i\in I$. Then $\varrho\in\bigcup_i\mathcal{E}(ƕ_i)$ imply that $\exists i_o\in I$ such that $\varrho\in\mathcal{E}(ƕ_{i_o})\subseteq\bigcup_i\mathcal{E}(ƕ_i)$, hence $I\mathcal{E}(\varrho)\subseteq\mathcal{E}(ƕ_{i_o})\subseteq\bigcup_i\mathcal{E}(ƕ_i)$, that is $\bigcup_i\mathcal{E}(ƕ_i)\in Ʈ_i$ (resp. $Ʈ_n$) $\forall i\in I$.

Theorem 2.7. Let $\mathfrak{Z}=(\mho(\Omega),\mathcal{E}(\Omega))$ be a generalized approximation space, then $\mathfrak{T}_{þ_i}=Ɣ_{þ_i}$ *and* $\mathfrak{T}_{þ_n}=Ɣ_{þ_n}$.
Proof: Let $ƕ\in Ɣ_{þ_i}$ be a proper then ƕ is i-open, to prove that $\Omega-ƕ$ is i-open. We will prove that by contradiction, Let $\varrho\in\Omega-ƕ$ and $I\mathcal{E}(\varrho)\not\subseteq\Omega-ƕ$ then

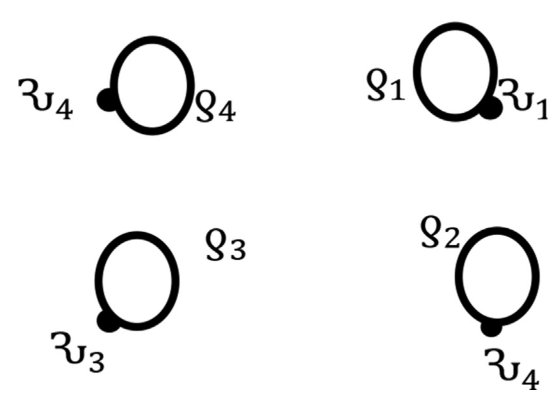

Fig. 3. und. g. Ω given in Example (3.16).

there exist at lest $\varrho_1 \in I\mathcal{E}(\varrho)$ and $\varrho_1 \notin \Omega - \hbar$ and hence $\varrho_1 \in \hbar$, since $\varrho_1 \in I\mathcal{E}(\varrho)$ then $\varrho \in I\mathcal{E}(\varrho_1)$ and because $\varrho_1 \in \hbar$ and $\hbar$ is i-open then by theorem(2.3), we get $I\mathcal{E}(\varrho_1) \subseteq \hbar$ and hence $\varrho \in \hbar$, this contradiction with $\varrho \in \Omega - \hbar$, there for each $\varrho \in \Omega - \hbar$ then $I\mathcal{E}(\varrho) \subseteq \Omega - \hbar$ and by theorem(2.3), we get $\Omega - \hbar$ is i-open and hence we get $\hbar$ is i-closed and $\hbar \in \mathfrak{T}_{\flat_i}$, thus $\gamma_{\flat_i} \subseteq \mathfrak{T}_{\flat_i}$. Now, let $\Bbbk \in \mathfrak{T}_{\flat_i}$ then $\Bbbk$ is i-closed and hence $\Omega - \Bbbk$ is i-open and by the above we get that $\Omega - (\Omega - \Bbbk) = \Bbbk$ is i-open and $\Bbbk \in \gamma_{\flat_i}$, thus $\mathfrak{T}_{\flat_i} \subseteq \gamma_{\flat_i}$ and since $\mathfrak{T}_{\flat_i}, \gamma_{\flat_i}$ are both contains $\emptyset$,$\mathcal{E}(\Omega)$, and hence $\mathfrak{T}_{\flat_i} = \gamma_{\flat_i}$. And by the same way we can prove that $\mathfrak{T}_{\flat_n} = \gamma_{\flat_n}$.

Proposition 2.8. Let $\mathfrak{Z} = (\mho(\Omega), \mathcal{E}(\Omega))$ be a generalized approximation space, then $\top_i = \mathfrak{T}_{\flat_i} = \gamma_{\flat_i}$ and $\top_n = \mathfrak{T}_{\flat_n} = \gamma_{\flat_n}$.
Proof: the proof of $\top_i = \gamma_{\flat_i}$ and $\top_n = \gamma_{\flat_n}$ is immediately follows from definition(2.4), definition(2.5) and theorem(2.3). and by theorem(2.7), we get that $\top_i = \mathfrak{T}_{\flat_i} = \gamma_{\flat_i}$ and $\top_n = \mathfrak{T}_{\flat_n} = \gamma_{\flat_n}$.

Remark 2.9. An immediate consequence of proposition(2.8) and proposition(2.6) we have $\gamma_{\flat_i}$ and $\gamma_{\flat_n}$ form topologies on Ω.

Definition 2.10. Let $\mathfrak{Z} = (\mho(\Omega), \mathcal{E}(\Omega))$ be a generalized approximation space and $\gamma_{\flat_i}$, $\gamma_{\flat_n}$ *and* $\gamma_{\flat_c}$ be the supra topologies induced by $\mathfrak{Z}$ *and let* $\hbar \subseteq \Omega$. *Then*

a) The i-lower and i-upper approximations of $\hbar$ are defined respectively by:

$$L_i(\mathcal{E}(\hbar)) = Int_i(\mathcal{E}(\hbar)),$$

$$U_i(\mathcal{E}(\hbar)) = Cl_i(\mathcal{E}(\hbar)),$$

b) The n-lower and n-upper approximations of $\hbar$ are defined respectively by:

$$L_n(\mathcal{E}(\hbar)) = Int_n(\mathcal{E}(\hbar)),$$

$$U_n(\mathcal{E}(\hbar)) = Cl_n(\mathcal{E}(\hbar)),$$

c) The c-lower and c-upper approximations of $\hbar$ are defined respectively by:

$$L_c(\mathcal{E}(\hbar)) = Int_c(\mathcal{E}(\hbar)),$$

$$U_c(\mathcal{E}(\hbar)) = Cl_c(\mathcal{E}(\hbar)).$$

Definition 2.11. Let $\mathfrak{Z} = (\mho(\Omega), \mathcal{E}(\Omega))$ be a generalized approximation space and $\gamma_{\flat_i}$, $\gamma_{\flat_n}$ and $\gamma_{\flat_c}$ be the supra topologies induced by $\mathfrak{Z}$ and let $\hbar \subseteq \Omega$. Then

a) The i-boundary, i-positive and i-negative regions of $\hbar$ are defined respectively by:

$$Bd_i(\mathcal{E}(\hbar)) = U_i(\mathcal{E}(\hbar)) - L_i(\mathcal{E}(\hbar)),$$

$$POS_i(\mathcal{E}(\hbar)) = L_i(\mathcal{E}(\hbar)),$$

$$NEG_i(\mathcal{E}(\hbar)) = \mathcal{E}(\Omega) - U_i(\mathcal{E}(\hbar)),$$

b) The n-boundary, n-positive and n-negative regions of $\hbar$ are defined respectively by:

$$Bd_n(\mathcal{E}(\hbar)) = U_n(\mathcal{E}(\hbar)) - L_n(\mathcal{E}(\hbar)),$$

$$POS_n(\mathcal{E}(\hbar)) = L_n(\mathcal{E}(\hbar)),$$

$$NEG_n(\mathcal{E}(\hbar)) = \mathcal{E}(\Omega) - U_n(\mathcal{E}(\hbar)),$$

c) The c-boundary, c-positive and c-negative regions of $\hbar$ are defined respectively by:

$$Bd_c(\mathcal{E}(\hbar)) = U_c(\mathcal{E}(\hbar)) - L_c(\mathcal{E}(\hbar)),$$

$$POS_c(\mathcal{E}(\hbar)) = L_c(\mathcal{E}(\hbar)),$$

$$NEG_c(\mathcal{E}(\hbar)) = \mathcal{E}(\Omega) - U_c(\mathcal{E}(\hbar)).$$

3. Accuracy of the lower, upper and boundary approximation spaces

In this section, we investigate some of properties of the accuracy of lower, upper, and boundary approximations in generalized rough set theory, which is defined by employing i-interior (resp. n-interior and c-interior).

Definition 3.1. Let $\mathfrak{Z} = (\mho(\Omega), \mathcal{E}(\Omega))$ be a generalized approximation space. The accuracy of the approximation of a sub und. g. $\hbar \subseteq \Omega$ using ($\flat_i, \flat_n$ and $\flat_c$) are defined respectively by:

$$\zeta_i(\mathcal{E}(\hbar)) = 1 - \frac{|Bd_i(\mathcal{E}(\hbar))|}{|\mathcal{E}(\Omega)|},$$

$$\zeta_n(\mathcal{E}(\hbar)) = 1 - \frac{|Bd_n(\mathcal{E}(\hbar))|}{|\mathcal{E}(\Omega)|},$$

$$\zeta_c(\mathcal{E}(\hbar)) = 1 - \frac{|Bd_c(\mathcal{E}(\hbar))|}{|\mathcal{E}(\Omega)|}.$$

It is obvious that $0 \le \zeta_i(\mathcal{E}(\hbar)) \le 1$, $0 \le \zeta_n(\mathcal{E}(\hbar)) \le 1$ and $0 \le \zeta_c(\mathcal{E}(\hbar)) \le 1$. Moreover, if $\zeta_i(\mathcal{E}(\hbar)) = 1$ or $\zeta_n(\mathcal{E}(\hbar)) = 1$ or $\zeta_c(\mathcal{E}(\hbar)) = 1$ then $\hbar$

is called ƕ-definable (ƕ-exact) und. g. otherwise, it is called ƕ-rough.

Example 3.2. Let $\Omega = (Ʊ(\Omega), \mathcal{E}(\Omega))$ such that $Ʊ(ƕ) = \{ʋ_1, ʋ_2, ʋ_3, ʋ_4\}$, $\mathcal{E}(ƕ) = \{ϙ_1, ϙ_2, ϙ_3, ϙ_4, ϙ_5\}$.

$þ_i(ϙ_1)=\{\{ϙ_2,ϙ_4,ϙ_5\}\}, þ_i(ϙ_2)=\{\{ϙ_1,ϙ_3,ϙ_4\}\}, þ_i(ϙ_3)=\{ \times \{ϙ_2,ϙ_4\}\}, þ_i(ϙ_4)=\{\{ϙ_1,ϙ_2,ϙ_3,ϙ_5\}\}, þ_i(ϙ_5)=\{ \times \{ϙ_1,ϙ_4,ϙ_5\}\}.$

$þ_n(ϙ_1)=\{\{ϙ_3\}\}, þ_n(ϙ_2)=\{\{ϙ_5\}\}, þ_n(ϙ_3)=\{\{ϙ_1,ϙ_5\} \times \}, þ_n(ϙ_4)=\varnothing, þ_n(ϙ_5)=\{\{ϙ_2,ϙ_3\}\}.$

$þ_c(ϙ_1)=\{\{ϙ_2,ϙ_4,ϙ_5\},\{ϙ_3\}\}, þ_c(ϙ_2)=\{ \times \{ϙ_1,ϙ_3,ϙ_4\},\{ϙ_5\}\}, þ_c(ϙ_3)=\{\{ϙ_2,ϙ_4\},\{ϙ_1,ϙ_5\} \times \}, þ_c(ϙ_4)=\{\{ϙ_1,ϙ_2,ϙ_3,ϙ_5\},\varnothing\}, þ_c(ϙ_5)=\{ \times \{ϙ_1,ϙ_4,ϙ_5\},\{ϙ_2,ϙ_3\}\}.$

$Ɣ_{þ_i}=\{\mathcal{E}(\Omega),\varnothing\}$

$Ɣ_{þ_n}=\{\mathcal{E}(\Omega),\varnothing,\{ϙ_4\},\{ϙ_1,ϙ_2,ϙ_3,ϙ_5\}\},$

$$Ɣ_{þ_c}=\left\{\begin{array}{l}\mathcal{E}(\Omega),\varnothing,\{ϙ_4\},\{ϙ_2,ϙ_3,ϙ_4,ϙ_5\},\{ϙ_1,ϙ_3,ϙ_4,ϙ_5\},\{ϙ_1,ϙ_2,ϙ_4,ϙ_5\},\{ϙ_1,ϙ_2,ϙ_3,ϙ_5\},\\ \{ϙ_1,ϙ_2,ϙ_3,ϙ_4\}\end{array}\right\}$$

We can get the following four tables:

Proposition 3.3. Let $\mathfrak{Z} = (Ʊ(\Omega), \mathcal{E}(\Omega))$ be a generalized approximation space, and $\mathfrak{T}_{þ_i}$, $\mathfrak{T}_{þ_n}$ and $\mathfrak{T}_{þ_c}$ be the classes of i-closed, n-closed and c-closed graphs induced by $\mathfrak{Z}$. Then any i-closed (or n-closed) und. g. is c-closed.

Proof: Let $ƕ\subseteq\Omega$ be an i-closed und. g., then $[\mathcal{E}(ƕ)]_i^{'}\subseteq \mathcal{E}(ƕ)$.
$[\mathcal{E}(ƕ)]_i^{'} = \{ϙ \in\mathcal{E}(\Omega); I\mathcal{E}(ϙ), I\mathcal{E}(ϙ) \cap(\mathcal{E}(ƕ) -\{ϙ\}) \neq \varnothing\}$ and $[\mathcal{E}(ƕ)]_c^{'} = \{ϙ \in\mathcal{E}(\Omega) : C\mathcal{E}S(ϙ) \cap (\mathcal{E}(ƕ) - \{ϙ\}) \neq \phi\} = \{ϙ \in\mathcal{E}(\Omega); I\mathcal{E}(ϙ), I\mathcal{E}(ϙ) \cap(\mathcal{E}(ƕ) - -\{ϙ\}) \neq \varnothing \text{and } NI\mathcal{E}(ϙ) \cap(\mathcal{E}(ƕ) -\{ϙ\}) \neq \varnothing\}$. Consequently, $[\mathcal{E}(ƕ)]_c^{'}\subseteq[\mathcal{E}(ƕ)]_i^{'}$ and so $[\mathcal{E}(ƕ)]_c^{'}\subseteq\mathcal{E}(ƕ)$ which implies ƕ is c-closed. Therefore any i-closed und. g. is c-closed. Similarly, we can prove that any n-closed is c-closed.

Proposition 3.4. Let $\mathfrak{Z} = (Ʊ(\Omega), \mathcal{E}(\Omega))$ be a generalized approximation space and $Ɣ_{þ_i}$, $Ɣ_{þ_n}$ and $Ɣ_{þ_c}$ be the supra topologies induced by $\mathfrak{Z}$. Then any i-open (or n-open) und. g. is c-open.

Proof: Let $ƙ\subseteq\Omega$ be an i-open und. g. and $Ғ = \Omega - ƙ$. So Ғ is i-closed und. g. and by using proposition(3.3), Ғ is c-closed. Hence $ƙ = \Omega - Ғ$ is c-open. Accordingly, any i-open und. g. is c-open. By the same manner we can prove that any n-open und. g. is c-open.

Proposition 3.5. Let $\mathfrak{Z} = (Ʊ(\Omega), \mathcal{E}(\Omega))$ be a generalized approximation space and $ƕ\subseteq\Omega$. Then.
(a) $L_i(\mathcal{E}(ƕ))\bigcup L_n(\mathcal{E}(ƕ))\subseteq L_c(\mathcal{E}(ƕ))$.
(b) $U_c(\mathcal{E}(ƕ))\subseteq U_i(\mathcal{E}(ƕ))\bigcap U_n(\mathcal{E}(ƕ))$.
(c) $B_c(\mathcal{E}(ƕ))\subseteq B_i(\mathcal{E}(ƕ))\bigcap B_n(\mathcal{E}(ƕ))$.
Proof:
(a) Since $L_i(\mathcal{E}(ƕ)) = \cup\{\mathcal{E}(Ю) \in Ɣ_{þ_i}; \mathcal{E}(Ю) \subseteq\mathcal{E}(ƕ)\}$. Hence $L_i(\mathcal{E}(ƕ))\subseteq\mathcal{E}(ƕ)$ and $L_i(\mathcal{E}(ƕ))$ is i-open since the union of any family of i-open und. g. is i-open. Since $L_n(\mathcal{E}(ƕ)) = \cup\{\mathcal{E}(Ю) \in Ɣ_{þ_n}; \mathcal{E}(Ю) \subseteq\mathcal{E}\ (ƕ)\}$. So $L_n(\mathcal{E}(ƕ))\subseteq\mathcal{E}(ƕ)$ and $L_n(\mathcal{E}(ƕ))$ is n-open since the union of any family of n-open und. g. is n-open. Since $L_i(\mathcal{E}(ƕ))$ is i-open, then by proposition(3.4), it is c-open and since $L_n(\mathcal{E}(ƕ))$ is n-open, then by proposition(3.4), it is also c-open. Hence $L_i(\mathcal{E}(ƕ))\bigcup L_n(\mathcal{E}(ƕ))$ is c-open and $L_i(\mathcal{E}(ƕ))\bigcup L_n (\mathcal{E}(ƕ))\subseteq\mathcal{E}(ƕ)$. But, $L_c(\mathcal{E}(ƕ)) = \bigcup\{\mathcal{E}(Ю) \in Ɣ_{þ_c}; \mathcal{E}(Ю) \subseteq\mathcal{E}(ƕ)\}$. Consequently, $L_i(\mathcal{E}(ƕ))\bigcup L_n(\mathcal{E} (ƕ))\subseteq L_c(\mathcal{E}(ƕ))$.
(b) $U_i(\mathcal{E}(ƕ)) = \cap\{\mathcal{E}(ƙ) \in\mathfrak{T}_{þ_i}; \mathcal{E}(ƕ) \subseteq\mathcal{E}(ƙ)\}$. Hence $\mathcal{E}(ƕ)\subseteq U_i(\mathcal{E}(ƕ))$ and $U_i(\mathcal{E}(ƕ))$ is i-closed since the intersection of any family of i-closed und. g. is i-closed. Since $U_n(\mathcal{E}(ƕ)) = \cap\{\mathcal{E}(ƙ) \in\mathfrak{T}_{þ_n}; \mathcal{E}(ƕ) \subseteq\mathcal{E}(ƙ)\}$. thus $\mathcal{E}(ƕ)\subseteq U_n(\mathcal{E}(ƕ))$ and $U_n(\mathcal{E}(ƕ))$ is n-closed since the intersection of any family of n-closed und. g. is n-closed. Since $U_i(\mathcal{E}(ƕ))$ is i-closed then, by proposition(3.3), it is c-closed and since $U_n(\mathcal{E}(ƕ))$ is n-closed then, by proposition(3.3), it is also c-closed. Hence $U_i(\mathcal{E}(ƕ))\bigcap U_n(\mathcal{E}(ƕ))$ is c-closed and $\mathcal{E}(ƕ)\subseteq U_i(\mathcal{E}(ƕ))\bigcap U_n(\mathcal{E}(ƕ))$. But $U_c(\mathcal{E}(ƕ)) = \cap\{\mathcal{E}(ƙ) \in\mathfrak{T}_{þ_c}; \mathcal{E}(ƕ) \subseteq\mathcal{E}(ƙ)\}$.According.
$U_c(\mathcal{E}(ƕ))\subseteq U_i(\mathcal{E}(ƕ))\bigcap U_n(\mathcal{E}(ƕ))$
(c) Let $ϙ\in B_c(\mathcal{E}(ƕ))$, then $ϙ\in(U_c(\mathcal{E}(ƕ)) - L_c(\mathcal{E}(ƕ)))$ and so $ϙ\in U_c(\mathcal{E}(ƕ))\wedge ϙ\notin L_c(\mathcal{E}(ƕ))$. Since $U_c(\mathcal{E}(ƕ))\subseteq U_i(\mathcal{E}(ƕ))\bigcap U_n(\mathcal{E}(ƕ))$ and $L_i(\mathcal{E}(ƕ))\bigcup L_n(\mathcal{E}(ƕ))\subseteq L_c(\mathcal{E}(ƕ))$. Then $ϙ\in(U_i(\mathcal{E}(ƕ))\bigcap U_n(\mathcal{E} (ƕ)))\wedge ϙ\notin(L_i(\mathcal{E}(ƕ))\bigcup L_n(\mathcal{E}(ƕ)))$, this imply $(ϙ \in U_i (\mathcal{E}(ƕ))\wedge ϙ \in U_n(\mathcal{E}(ƕ)))\wedge(ϙ \notin L_i(\mathcal{E}(ƕ))\wedge ϙ \notin L_n (\mathcal{E} (ƕ)))$, this imply $(ϙ \in U_i(\mathcal{E}(ƕ))\wedge ϙ \notin L_i(\mathcal{E}(ƕ)))$

$\wedge(\varrho \in U_n \ (\mathcal{E}(ƕ))\wedge\varrho \notin L_n(\mathcal{E}(ƕ)))$, this imply $\varrho\in (U_i(\mathcal{E}(ƕ)) - L_i(\mathcal{E}(ƕ)))\wedge\varrho\in(U_n(\mathcal{E}(ƕ)) - L_n(\mathcal{E}\ (ƕ)))$, this imply $\varrho\in B_i(\mathcal{E}(ƕ))\wedge\varrho\in B_n(\mathcal{E}(ƕ))$, this imply $\varrho\in (B_i(\mathcal{E}(ƕ))\bigcap B_n(\mathcal{E}(ƕ)))$. Therefore $B_c(\mathcal{E}(ƕ))\subseteq B_i(\mathcal{E}(ƕ))\bigcap B_n(\mathcal{E}(ƕ))$.

Remark 3.6. Let $\mathfrak{Z} = (Ʊ(\Omega), \mathcal{E}(\Omega))$ be a generalized approximation space and $ƕ\subseteq\Omega$. Then the following statements are not necessarily true.
(a) $L_c(\mathcal{E}(ƕ)) = L_i(\mathcal{E}(ƕ))\bigcup L_n(\mathcal{E}(ƕ))$.
(b) $U_c(\mathcal{E}(ƕ)) = U_i(\mathcal{E}(ƕ))\bigcap U_n(\mathcal{E}(ƕ))$.
(c) $B_c(\mathcal{E}(ƕ)) = B_i(\mathcal{E}(ƕ))\bigcap B_n(\mathcal{E}(ƕ))$.
The next example shows pervious remark.

Example 3.7. According to example(3.2),

(a) Let $ƕ = (Ʊ(ƕ), \mathcal{E}(ƕ))$ such that $Ʊ(ƕ) = \{\mathfrak{v}_1, \mathfrak{v}_2, \mathfrak{v}_3, \mathfrak{v}_4\}$ and $\mathcal{E}(ƕ) = \{\varrho_1, \varrho_2, \varrho_3, \varrho_4\}$. Then $L_c(\mathcal{E}(ƕ)) = \{\varrho_1, \varrho_2, \varrho_3, \varrho_4\}$, $L_i(\mathcal{E}(ƕ)) = \varnothing$ and $L_n(\mathcal{E}(ƕ)) = \{\varrho_4\}$, such that $L_i(\mathcal{E}(ƕ))\bigcup L_n(\mathcal{E}(ƕ)) = \{\varrho_4\}$ and so $L_c(\mathcal{E}(ƕ))\neq L_i(\mathcal{E}(ƕ))\bigcup L_n(\mathcal{E}(ƕ))$.
(b) Let $ƕ = (Ʊ(ƕ), \mathcal{E}(ƕ))$ such that $Ʊ(ƕ) = \{\mathfrak{v}_2\}$ and $\mathcal{E}(ƕ) = \{\varrho_5\}$. Then $U_c(\mathcal{E}(ƕ)) = \{\varrho_5\}$, $U_i(\mathcal{E}(ƕ)) = \mathcal{E}(\Omega)$ and $U_n(\mathcal{E}(ƕ)) = \{\varrho_1, \varrho_2, \varrho_3, \varrho_5\}$, such that $U_i(\mathcal{E}(ƕ))\bigcap U_n\ (\mathcal{E}(ƕ)) = \{\varrho_1, \varrho_2, \varrho_3, \varrho_5\}$ and so $U_c(\mathcal{E}(ƕ))\neq U_i(\mathcal{E}(ƕ))\bigcap U_n(\mathcal{E}(ƕ))$.
(c) Let $ƕ = (Ʊ(ƕ), \mathcal{E}(ƕ))$ such that $Ʊ(ƕ) = \{\mathfrak{v}_2\}$ and $\mathcal{E}(ƕ) = \{\varrho_5\}$. Then $B_c(\mathcal{E}(ƕ)) = \{\varrho_5\}$, $B_i(\mathcal{E}(ƕ)) = \mathcal{E}(\Omega)$ and $B_n(\mathcal{E}(ƕ)) = \{\varrho_1, \varrho_2, \varrho_3, \varrho_5\}$, such that $B_i(\mathcal{E}(ƕ))\bigcap B_n(\mathcal{E}(ƕ)) = \{\varrho_1, \varrho_2, \varrho_3, \varrho_5\}$ and so $B_c(\mathcal{E}(ƕ))\neq B_i(\mathcal{E}(ƕ))\bigcap B_n(\mathcal{E}(ƕ))$.

Theorem 3.8. Let $\mathfrak{Z} = (Ʊ(\Omega), \mathcal{E}(\Omega))$ be a generalized approximation space and ϱ is isolated edge then:
(a) $\{\varrho\}$ is i-closed.
(b) If $\{\varrho\}\subsetneq\mathcal{E}(\Omega)$ then $\{\varrho\}$ is not n-closed.
Proof: Let ϱ is isolated edge in a graph Ω.then
(a) Since ϱ is isolated edge then for every $\varrho_1\in \mathcal{E}(\Omega)$ we get $\varrho\notin I\mathcal{E}(\varrho_1)$ and $I\mathcal{E}(\varrho_1)\cap(\{\varrho\} - \{\varrho_1\}) = \varnothing$ and hence for every $\varrho_1\in\mathcal{E}(\Omega)$ then $\varrho_1\notin[\{\varrho\}]_i^{'}$, thus $[\{\varrho\}]_i^{'} = \varnothing\subseteq\{\varrho\}$, there fore $\{\varrho\}$ is i-closed.
(b) Since $\{\varrho\}\subsetneq\mathcal{E}(\Omega)$ then there exist at least a edge $\varrho_1\in\mathcal{E}(\Omega)$ different from ϱ, since ϱ is isolated edge then $\varrho\in NI\mathcal{E}(\varrho_1)$ and $NI\mathcal{E}(\varrho_1)\cap(\{\varrho\} - \{\varrho_1\}) = \{\varrho\}\neq\varnothing$ and hence $\varrho_1\in[\{\varrho\}]_n^{'}$, thus $[\{\varrho\}]_n^{'}\not\subseteq\{\varrho\}$, therefore $\{\varrho\}$ is not n-closed.

Theorem 3.9. Let $\mathfrak{Z} = (Ʊ(\Omega), \mathcal{E}(\Omega))$ be a generalized approximation space. Then if Ω is connected graph then $\mathfrak{T}_{þ_i} = \{\varnothing, \mathcal{E}(\Omega)\}$.
Proof: Let Ω be a connected graph and let $ƕ$ be any a proper sub und. g. in Ω i.e. $\varnothing\subsetneq ƕ\subsetneq\mathcal{E}(\Omega)$. Since Ω is connected then $\Omega - ƕ$ connected with $ƕ$ and hence there exist at least a edge $\varrho\in\Omega - ƕ$ such that ϱ is incident with at least for some a edge $\varrho_1\in ƕ$ and we get $\varrho_1\in I\mathcal{E}(\varrho)$ and hence $I\mathcal{E}(\varrho)\cap(ƕ - \{\varrho\})$ contains at least a edge ϱ_1, then $I\mathcal{E}(\varrho)\cap(ƕ - \{\varrho\})\neq\varnothing$, thus $\varrho\in[ƕ]_i^{'}$ and $\varrho\notin ƕ$ then we get $[ƕ]_i^{'}\not\subseteq ƕ$, there fore $ƕ$ is not i-closed and the only i-closed are $\varnothing, \mathcal{E}(\Omega)$ and hence $\mathfrak{T}_{þ_i} = \{\varnothing, \mathcal{E}(\Omega)\}$.

Theorem 3.10. Let $\mathfrak{Z} = (Ʊ(\Omega), \mathcal{E}(\Omega))$ be a generalized approximation space. Then if Ω is disconnected graph then $\mathfrak{T}_{þ_n} = \{\varnothing, \mathcal{E}(\Omega)\}$.
Proof: Let Ω be a disconnected graph then there exist at least two disjoint component in Ω say $\mathcal{A}$ and $\mathcal{B}$ that is $\mathcal{A}\cap\mathcal{B} = \varnothing$. Let $ƕ$ be any a proper sub und. g. in Ω i.e. $\varnothing\subsetneq ƕ\subsetneq\mathcal{E}(\Omega)$ the

(i) If $ƕ\subseteq\mathcal{A}$ then $ƕ\not\subseteq\mathcal{B}$ and for all $\varrho\in\mathcal{B}$ then $ƕ\subseteq NI\mathcal{E}(\varrho)$ and $NI\mathcal{E}(\varrho)\cap(ƕ - \{\varrho\}) = ƕ\neq\varnothing$ then $\varrho\in[ƕ]_n^{'}$ and $\varrho\notin ƕ$, there fore $[ƕ]_n^{'}\not\subseteq ƕ$ then $ƕ$ is not n-closed
(ii) If $ƕ\subseteq\mathcal{B}$ then $ƕ\not\subseteq\mathcal{A}$ and for all $\varrho\in\mathcal{A}$ then $ƕ\subseteq NI\mathcal{E}(\varrho)$ and $NI\mathcal{E}(\varrho)\cap(ƕ - \{\varrho\}) = ƕ\neq\varnothing$ then $\varrho\in[ƕ]_n^{'}$ and $\varrho\notin ƕ$, there fore $[ƕ]_n^{'}\not\subseteq ƕ$ then $ƕ$ is not n-closed
(iii) If $ƕ\cap\mathcal{A}\neq\varnothing$ and $ƕ\cap\mathcal{B}\neq\varnothing$ then there exist at least $\varrho\in\mathcal{A}$ and $\varrho_1\in\mathcal{B}$ such that $\varrho, \varrho_1\in ƕ$, since $ƕ\subsetneq\mathcal{E}(\Omega)$ then there exist $\varrho_2\in\mathcal{E}(\Omega)$ and $\varrho_2\notin ƕ$ then ϱ_2 belong to one of them component of Ω, if $\varrho_2\in\mathcal{A}$ then $\varrho_1\in NI\mathcal{E}(\varrho_2)$ and $NI\mathcal{E}(\varrho_2)\cap(ƕ - \{\varrho_2\})$ contain at least ϱ_1 and hence $NI\mathcal{E}(\varrho_2)\cap(ƕ - \{\varrho_2\})\neq\varnothing$, then $\varrho_2\in[ƕ]_n^{'}$ and $\varrho_2\notin ƕ$, thus $[ƕ]_n^{'}\not\subseteq ƕ$ and hence $ƕ$ is not n-closed, if $\varrho_2\in\mathcal{B}$ then $\varrho\in NI\mathcal{E}(\varrho_2)$ and $NI\mathcal{E}(\varrho_2)\cap(ƕ - \{\varrho_2\})$ contain at least ϱ and hence $NI\mathcal{E}(\varrho_2)\cap(ƕ - \{\varrho_2\})\neq\varnothing$, then $\varrho_2\in[ƕ]_n^{'}$ and $\varrho_2\notin ƕ$, thus $[ƕ]_n^{'}\not\subseteq ƕ$ and hence $ƕ$ is not n-closed, there for the only n-closed are $\varnothing$, $\mathcal{E}(\Omega)$.and hence $\mathfrak{T}_{þ_n} = \{\varnothing, \mathcal{E}(\Omega)\}$.

Corollary 3.11. Let $\mathfrak{Z} = (Ʊ(\Omega), \mathcal{E}(\Omega))$ be a generalized approximation space. Then for any und. g. Ω we have that either $\mathfrak{T}_{þ_i} = \{\varnothing, \mathcal{E}(\Omega)\}$ or $\mathfrak{T}_{þ_n} = \{\varnothing, \mathcal{E}(\Omega)\}$.
Proof: The proof is immediately follows from Theorem (3.9) and Theorem (3.10) and by (for any und. g. Ω we have that either Ω is connected or Ω is disconnected.

Example 3.12. Let $\Omega = (Ʊ(\Omega), \mathcal{E}(\Omega))$ be und. g. such that $Ʊ(\Omega)$ contain only one vertex then $\mathfrak{T}_{þ_i} = \mathfrak{T}_{þ_n} = \{\varnothing, \mathcal{E}(\Omega)\}$.

Theorem 3.13. Let $\mathfrak{Z} = (Ʊ(\Omega), \mathcal{E}(\Omega))$ be a generalized approximation space. Then if Ω is star graph then.
(a) $\mathfrak{T}_{þ_i} = \{\varnothing, \mathcal{E}(\Omega)\}$.

(b) $\mathfrak{T}_{\text{þ}_n} = P(\mathcal{E}(\Omega))$.
Proof: Let Ω is star graph then,

(a) Since Ω is star graph then Ω is connected graph and hence by theorem (3.9) we get $\mathfrak{T}_{\text{þ}_i} = \{\varnothing, \mathcal{E}(\Omega)\}$.
(b) We will prove that by contradiction. suppose that $\mathfrak{T}_{\text{þ}_n} \subsetneq P(\mathcal{E}(\Omega))$ then there exists at least sub und. g. $\mho \in P(\mathcal{E}(\Omega))$ such that ℧ is not n-closed then $[\mho]_n^{'} \not\subseteq \mho$ then there exist at least $\varrho \in [\mho]_n^{'}$ and $\varrho \notin \mho$, since $\varrho \in [\mho]_n^{'}$ then $NI\mathcal{E}(\varrho) \cap (\mho - \{\varrho\}) \neq \varnothing$ and hence there exists at least a edge $\varrho_1 \in \mho$ different from ϱ such that $\varrho_1 \in NI\mathcal{E}(\varrho)$ and hence ϱ_1 is non incident with ϱ , thus this contradiction with Ω is star graph. There fore ℧ is n-closed. then we have that for every sub und. g. of Ω is n-closed and hence $\mathfrak{T}_{\text{þ}_n} = P(\mathcal{E}(\Omega))$.

The next example illustrates the above theorem.

Example 3.14. Let $\Omega = (\mho(\Omega), \mathcal{E}(\Omega))$ such that $\mho(\Omega) = \{\nu_1, \nu_2, \nu_3, \nu_4, \nu_5\}$ and $\mathcal{E}(\Omega) = \{\varrho_1, \varrho_2, \varrho_3, \varrho_4\}$.
Hence þ_i is defined by

$\text{þ}_i(\varrho_1) = \{\varrho_2, \varrho_3, \varrho_4\}, \text{þ}_i(\varrho_2) = \{\varrho_1, \varrho_3, \varrho_4\}, \text{þ}_i(\varrho_3) = \{\varrho_1, \varrho_2, \varrho_4\}, \text{þ}_i(\varrho_4) = \{\varrho_1, \varrho_2, \varrho_3\}$. Hence þ_n is defined by $\text{þ}_n(\varrho_1) = \varnothing, \text{þ}_n(\varrho_2) = \varnothing, \text{þ}_n(\varrho_3) = \varnothing$, $\text{þ}_n(\varrho_4) = \varnothing$.

Then $\mathfrak{T}_{\text{þ}_i} = \{\varnothing, \mathcal{E}(\Omega)\}$ because $(\forall \varnothing \subsetneq \mho \subsetneq \Omega$ then $[\mho]_i^{'} \not\subseteq \mho)$ and $\mathfrak{T}_{\text{þ}_n} = P(\mathcal{E}(\Omega))$ because $(\forall \mho \subseteq \Omega$ then $[\mho]_n^{'} \subseteq \mho)$.

Theorem 3.15. Let $\mathfrak{Z} = (\mho(\Omega), \mathcal{E}(\Omega))$ be a generalized approximation space. Then if Ω is antisymmetric graph then.
(a) $\mathfrak{T}_{\text{þ}_n} = \{\varnothing, \mathcal{E}(\Omega)\}$.
(b) $\mathfrak{T}_{\text{þ}_i} = P(\mathcal{E}(\Omega))$.
Proof: Let Ω be an antisymmetric graph then,

(a) Since Ω is antisymmetric graph then Ω is disconnected graph and hence by theorem(3.10) we get $\mathfrak{T}_{\text{þ}_n} = \{\varnothing, \mathcal{E}(\Omega)\}$.
(b) We will prove that by contradiction. suppose that $\mathfrak{T}_{\text{þ}_i} \subsetneq P(\mathcal{E}(\Omega))$ then there exists at least sub und. g. $\mho \in P(\mathcal{E}(\Omega))$ such that ℧ is not i-closed then $[\mho]_i^{'} \not\subseteq \mho$ then there exist at least $\varrho \in [\mho]_i^{'}$ and $\varrho \notin \mho$, since $\varrho \in [\mho]_i^{'}$ then $I\mathcal{E}(\varrho) \cap (\mho - \{\varrho\}) \neq \varnothing$ and hence there exists at least a edge $\varrho_1 \in \mho$ different from ϱ such that $\varrho_1 \in I\mathcal{E}(\varrho)$ and hence ϱ_1 is incident with ϱ, thus this contradiction with Ω is antisymmetric graph. Therefore ℧ is i-closed. then we have that for every sub und. g. of Ω is i-closed and hence $\mathfrak{T}_{\text{þ}_i} = P(\mathcal{E}(\Omega))$.

The next example illustrates the above theorem.

Example 3.16. Let $\Omega = (\mho(\Omega), \mathcal{E}(\Omega))$ such that $\mho(\Omega) = \{\nu_1, \nu_2, \nu_3, \nu_4\}$ and $\mathcal{E}(\Omega) = \{\varrho_1, \varrho_2, \varrho_3, \varrho_4\}$. Hence þ_i is defined by $\text{þ}_i(\varrho_1) = \{\varrho_1\}, \text{þ}_i(\varrho_2) = \{\varrho_2\}, \text{þ}_i(\varrho_3) = \{\varrho_3\}, \text{þ}_i(\varrho_4) = \{\varrho_4\}$. Hence þ_n is defined by" $\text{þ}_n(\varrho_1) = \{\varrho_2, \varrho_3, \varrho_4\}, \text{þ}_n(\varrho_2) = \{\varrho_1, \varrho_3, \varrho_4\}, \text{þ}_n(\varrho_3) = \{\varrho_1, \varrho_2, \varrho_4\}, \text{þ}_n(\varrho_4) = \{\varrho_1, \varrho_2, \varrho_3\}$.
Then $\mathfrak{T}_{\text{þ}_n} = \{\varnothing, \mathcal{E}(\Omega)\}$ because $(\forall \varnothing \subsetneq \mho \subsetneq \Omega$ then $[\mho]_n^{'} \not\subseteq \mho)$ and $\mathfrak{T}_{\text{þ}_i} = P(\mathcal{E}(\Omega))$ because $(\forall \mho \subseteq \Omega$ then $[\mho]_i^{'} \subseteq \mho)$.
Note 3.17. Let $(\Omega, \text{þ}_i)$ be an i-space if Ω is connected (resp. disconnected,star, antisymmetric) then $(\Omega, \text{þ}_i)$ is called connected (resp. disconnected, star, antisymmetric) i-space.

Corollary 3.18. Let $\mathfrak{Z} = (\mho(\Omega), \mathcal{E}(\Omega))$ be a generalized approximation space. Then:

(a)The induced topology by þ_i on connected i-space is indiscrete topology.
(b)The induced topology by þ_n on disconnected n-space is indiscrete topology.
(c)The induced topology by þ_i on star i-space is indiscrete topology and the induced topology by þ_n on star n-space is discrete topology.
(d)The induced topology by þ_i on antisymmetric i-space is discrete topology and the induced topology by þ_n on antisymmetric n-space is indiscrete topology,
(e)For any und. g. Ω we have that either T_i is indiscrete topology or T_n is indiscrete topology.

Proof:

(a) The proof is immediately follows from theorem(3.9) and theorem(2.7).
(b) The proof is immediately follows from theorem(3.10) and theorem(2.7).
(c) The proof is immediately follows from theorem(3.13) and theorem(2.7).
(d) The proof is immediately follows from theorem(3.15) and theorem(2.7).
(e) The proof is immediately follows from Corollary (3.11) and proposition(2.8).

References

[1] Al'Dzhabri KS, Hamzha A, Essa YS. On DG-topological spaces associated with directed graphs. J Discrete Math Sci Cryptogr 2020;32(5):1039−46. https://doi.org/10.1080/09720529.2020.1714886.
[2] Al'Dzhabri KS, Hani MF. On certain types of topological spaces associated with digraphs. J Phys Conf 2020;1591:012055.

[3] Al'Dzhabri KS, et al. DG-domination topology in Digraph. J Prime Res Math 2021;17(2):93−100. http://jprm.sms.edu.pk/.
[4] Al'Dzhabri KS. Enumeration of connected components of acyclic digraph. J Discrete Math Sci Cryptogr 2021;24(7): 2047−58. https://doi.org/10.1080/09720529.2021.1965299.
[5] Beineke LW, Wilson RJ. Topics in topological graph theory. Cambridge University Press; 2009.
[6] Harary F. Graph theory. Reading Mass: Addison-Wesley; 1969.
[7] Moller M. General topology. Authors: Notes; 2007.
[8] Shokry M. Approximations structures generated by trees vertices. Int J Eng Sci Technol 2012;4(2):406−18. 10.
[9] Shokry M. Closure concepts generated by directed trees. J Math Comput Sci 2010;21(1):45−51.
[10] Ray SS. Graph theory with algorithms and its applications: in applied sciences and technology. New Delhi: Springer; 2013.