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The Cyclic Decomposition Of cf(Q_14×C_2p)/ R̄(Q_14×C_2p)

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ABSTRACT

In this paper, we propose the cyclic decomposition of the factor group cf((Q_14×C_2p),Z)/R̄((Q_14×C_2p)), and the group cf((Q_14×C_2p),Z) is Z-valued class functions of the direct product group ((Q_14×C_2p)) under the operation of addition, and R((Q_14×C_2p)) is the subgroup of the generalized characters of the group cf((Q_14×C_2p),Z).Then cf((Q_14×C_2p),Z)/R̄(Q_14×C_2p) is an abelian factor group denoted by K((Q_14×C_10)) where (Q_14 is the quaternion group of order28 and C_2p is the cyclic group of order 2p. Also, we find the rational valued characters table of the group ((Q_14×C_2p)) when p, and q prime numbers and s∈Z^+ is given as follows :

≡^* (Q_14×C_2p)= 〖≡〗 ^* (Q_14)⊗≡^* (C_2p) (1)

and find the cyclic decomposition of group (Q_14× 〖C_2p C〗 _10) in this paper and prove that

K(Q_14×C_2p)=⊕_(i=1)^2 [K(C_14)⊗C_4] ⊕_(i=1)^8 K (C_2p) (2)

1.Introduction

"Any two elements' of a finite group G. are said to be Γ- conjugate if they generated conjugate cyclic subgroups in G. This relation is equivalence on G and its classes are called Γ-classes".

"The number of Γ-classes of G is equal to the rank of cf(G.Z) is the intersection of cf(G.Z) with the group R(G) which is a normal subgroup of cf(G.Z), then, cf(G.Z)/R̄(G) is a finite abelian factor group denoted by K(G)".

"Each element in $\overline{R}((Q_{14} \times \mathbf{C}_{2p}))$ can be written as" $v1\theta 1 + v2\theta 2 + \cdots + vr\theta r$,"where r is the number of $\Gamma -$ classes. $v1. v2. \dots . vr \in Z$ and , where χi is an irreducible character of the group G and σ is any element in Galios group Gal $(Q(\chi i)/Q)$".

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"In 1982, Kirdar M. S. [3] studied the $K(C_n)$. In 1994, Abass H. H. [1] studied the $K(D_n)$ and found $\equiv^* (D_n)$. In 1995, Mahmood N. R. [4] studied the factor group $K(Q_{2m})$ and foun $\equiv^* (Q_{2m})$.. In 2008, Mahdi M. S. [5] studied the factor group K(Dnh) where n is an odd number."

"The aim of this paper to find" $\equiv^* (Q_{14} \times \mathbf{C_{2p}})$) "and to determine the cyclic decomposition of the group" $K(Q_{14} \times \mathbf{C_{2p}})$), when p is primes number Preliminaries. We review some definitions and results which is need in this research.

"Let $T_1 : G1 \rightarrow GL(V1)$ and $T_2: G2 \rightarrow GL(V2)$ be two irreducible representations of the groups c G1 and G2 respectively, then $T_1 \otimes T_2$ is irreducible representations of the group $G_1 \times G_2$."

"Let T be a matrix representation of a group G over the field F, the character χ of a matrix representation T is the mapping $\chi: G \rightarrow F$ defined by $\chi(g) = Tr(T(g))$ for all $g \in G$ where $Tr(T(g))$ refers to the trace of the matrix T(g) and $\chi(1)$ is the degree of χ ."

"A finite group G has a finite number of conjugacy classes and a finite number of distinct k-irreducible representations, the group character of a group representation is constant on a conjugacy class $CL \propto$. $(1 \leq \propto \leq k)$, the values of the characters can be written as a table known as the characters table which is denoted by $\equiv(G)$. "

2 -Basic Concepts

2.1 Definition [2]

A rational valued character θ of G is a character whose values are in Z, which is $\theta(g) \in Z$. for all $g \in G$.

2.2 Proposition:[3]

The rational valued characters

$$\theta_i = \sum_{\sigma \in Gal(Q(\chi_i)/Q)} \sigma(\chi_i) \qquad (3)$$

form basis for $\overline{R}(G)$, where χ_i are the irreducible characters of G and their numbers are equal to the number of all distinct Γ- classes of G.

2.3 Proposition: [3]

The rational valued characters table of the cyclic group C_p of the rank 2 where p is a prime number which is denoted by $(\equiv^* (C_p))$, is given as follows:

Γ − **classes**	$[I]$	$[r]$
θ_1	1	1

θ_2	$p-1$	-1

Table 1. The Rational Valued Characters Table of The Cyclic Group C_p

where its rank 2 represents the number of all distinct Γ-classes.

2.4 Definition [4]

For every positive integer m, The generalized Quaternion Group Q_{2m} of order 4m with two generators x and y then For every element in Q2m satisfies
$x^m = y^2, \quad y^1 x^m y^{-1} = x^{-m}$ which implies $x^m = y^4 = 1$Any element $\lambda \in Q_{2m}$,

can be expressed uniquely in the form $Q_{2m} = \{\lambda = x^h y^k \mid 0 \le h \le 2m-1, k = 0,1\}$

2.5 Proposition: [4]

The rational valued characters table of $Q_{2m}$when m is an odd number is given as follows:

	χ^{2r}				χ^{2r+1}				$[y]$
θ1	1	1	……	1	1	1	……	1	1
θ2	$\equiv^*(C_m)$				$\equiv^*(C_m)$				0
:									:
$\theta^{(I/2)-1}$									0
$\theta^{(I/2)}$									0
$\theta^{(I/2)+1}$	1	1	……	1	1	1	……	1	-1
$\theta^{(I/2)+2}$	$\equiv^*(C_m)$				H				0
:									:
θ^{I-1}									0

θ^{I}									0
θ^{I+1}	2	2	......	2	-2	-2	......	-2	0

Table-2 The rational valued characters table of Q_{2m} when m is an odd number

Where 0< r < m-1 , I is the number of Γ- classes of C_{2m}, θ^{j} such that 1< j < I+1 are the rational valued characters of group Q_{2m} and if we denote Cij the elements $\overset{*}{\equiv}$(Cm) and h_{ij} the elements of H as defined by :

$$h_{ij} = \begin{cases} C_{ij} & if \quad i = 1 \\ -C_{ij} & if \quad i = 1 \end{cases}$$

and where I is the number of Γ- classes ofC_{2m} .

2.6 Theorem: [4]

If m is an odd number, then $K(Q_{2m}) = K(C_{2m}) \oplus C_4$.

3. The Main Results

We study the rational valued characters table of $\left(Q_{14} \times \mathbf{C}_{2p}\right)$. and find the cyclic decomposition $K\left(Q_{14} \times \mathbf{C}_{2p}\right)$.

The group of $Q_{14} \times \mathbf{C}_{2p}$) is the direct product of the Quaternion group $Q_{14}$of order 28 (number), and the group $\mathbf{C}_{2p}$. which is cyclic of order 2p the order of$\left(Q_{14} \times \mathbf{C}_{2p}\right)$.is $28p$

3.1 Theorem

The rational valued character of the group $(Q_{14} \times \mathbf{C}_{2p})$, has the following form

$$\equiv^{*}\left(Q_{14} \times \mathbf{C}_{2p}\right) = \equiv^{*}(Q_{14}) \otimes \equiv^{*}(\mathbf{C}_{2p})$$

Proof

For each element $g_{khl} \in \left(Q_{14} \times \mathbf{C}_{2p}\right)$, we have $g_{khl} = g_k g_h g_l$, such that $g_d \in Q_{14}$,$g_h \in C_{2,} = \prec r_2 \succ = \prec -1 \succ$, $g_l \in \mathbf{C}_p = \prec r \succ$.and $r = e^{2\pi/5}$ Then $g_k = x^t y^f, 0 \le t \le 14, f = 0,1$, and each irreducible character of $\left(Q_{14} \times \mathbf{C}_{2p}\right)$ is $\chi_{ijk} = \chi_i \chi'_j \chi''_k$, where χ_i is an irreducible character of Q_{14}, χ'_j is an irreducible character of C_2, and χ''_k is an irreducible character of $\mathbf{C}_p$

by Proposition (2-2) we get

$$\theta_{ijk}=\theta_{ijk}(g_{dhl})=\left[\!\left[\sum_{\sigma\epsilon\, Gal(\mathcal{Q}(\chi_{ijk}(g_{dhl}))/\mathcal{Q})}\sigma(\chi_{ijk}(g_{dhl}))\right]\!\right]$$

$$=\left[\!\left[\sum_{\sigma\epsilon\, Gal(\mathcal{Q}(\chi_i(g_d))/\mathcal{Q})}\sigma\left(\chi_i(g_d)\right)\left[\sum_{\sigma\epsilon\, Gal(\mathcal{Q}(\acute{\chi}_j(g_h))/\mathcal{Q})}\sigma\left(\acute{\chi}_j(g_h)\right)\left\{\sum_{\sigma\epsilon\, Gal(\mathcal{Q}(\chi''_k(g_l))/\mathcal{Q})}\sigma\left(\chi''_k(g_l)\right)\right\}\right]\right]\!\right]$$

- If j = 1, and k = 1, then for all $g_h \in C_2$ and for all $g_l \in \mathbf{C}_p$ such that $\theta'_j(g_h) = \sum \chi'_j(g_h) = 1, \theta''_k(g_l) = \sum \chi''_k(g_l) = 1$. Then

$$\theta_{ijk}(g_{dhl})=\left[\!\left[\sum_{\sigma\epsilon\, Gal(\mathcal{Q}(\chi_{ijk}(g_{dhl}))/\mathcal{Q})}\sigma(\chi_{ijk}(g_{dhl}))\right]\!\right]=\left[\!\left[\sum_{\sigma\epsilon\, Gal(\mathcal{Q}(\chi_i(g_d))/\mathcal{Q})}\sigma\left(\chi_i(g_d)\right)[1\{1\}]\right]\!\right]$$

$$=\theta_i(g_d) * \theta'_j(g_h) * \theta''_k(g_l)$$

- If j = 1, and k = 2,3,..,p, for Avery $g_h \in C_2$ and g_l is the identity of $\mathbf{C}_p$ that $\theta'_j(g_h) = \sum \chi'_1(g_h) = 1, \theta''_k(g_l) = \sum_{k=2}^{p} \chi''_k(g_l) = p-1$ Then

$$\theta_{ijk}(g_{dhl})=\left[\!\left[\sum_{\sigma\epsilon\, Gal(\mathcal{Q}(\chi_{ijk}(g_{dhl}))/\mathcal{Q})}\sigma(\chi_{ijk}(g_{dhl}))\right]\!\right]=\left[\!\left[\sum_{\sigma\epsilon\, Gal(\mathcal{Q}(\chi_i(g_d))/\mathcal{Q})}\sigma\left(\chi_i(g_d)\right)[1\{4\}]\right]\!\right]$$

$$=\theta_i(g_d)(1)(p-1)=\theta_i(g_d) * \theta'_j(g_h) * \theta''_k(g_l)$$

- If j = 1, and k = 2,3,4,5, for Avery $g_h \in C_2$ and g_l is not identity of C_5 that $\theta'_1(g_h) = \sum \chi'_1(g_h) = 1, \theta''_k(g_l) = \sum_{k=2}^{p} \chi''_k(g_l) = (\varepsilon + \varepsilon^2 + \cdots + \varepsilon^{p-1})_= -1$

$$\theta_{ijk}(g_{dhl})=\left[\!\left[\sum_{\sigma\epsilon\, Gal(\mathcal{Q}(\chi_{ijk}(g_{dhl}))/\mathcal{Q})}\sigma(\chi_{ijk}(g_{dhl}))\right]\!\right]=\left[\!\left[\sum_{\sigma\epsilon\, Gal(\mathcal{Q}(\chi_i(g_d))/\mathcal{Q})}\sigma\left(\chi_i(g_d)\right)[1\{-1\}]\right]\!\right]$$

$$\theta_{ijk}=(1)(-1)=\theta_i(g_d)(1)(-1)=\theta_i(g_d) * \theta'_j(g_h) * \theta''_k(g_l)$$

- If $j = 2$ and $k = 1$, g_h is the identity of C_2 and for all $g_l \in \mathbf{C}_p$

$$\theta_i(g_d) = \sum \chi_i(g_d)\ ,\ \ \theta_j'(g_h) = \chi_2'(g_h) = 1\ , \qquad \theta_k''(g_l) = \chi_k''(g_l) = 1$$

$$\theta_{ijk(g_{dhl})} = \left[\!\!\left[\sum_{\sigma \epsilon\, Gal(Q(\chi_{ijk}(g_{dhl}))/Q)} \sigma(\chi_{ijk}(g_{dhl})) \right]\!\!\right] = \left[\!\!\left[\sum_{\sigma \epsilon\, Gal(Q(\chi_i(g_d))/Q)} \sigma\left(\chi_i(g_d)\right)[1\{1\}] \right]\!\!\right]$$

$$= \theta_i(g_d)(1)(1) = \theta_i(g_d) * \theta_j'(g_h) * \theta_k''(g_l)$$

- If $j = 2$ and $k = 2,3,4,5$, g_h is the identity of C_2 and g_l is the identity $\mathbf{C}_p$

$$\theta_i(g_d) = \sum \chi_i(g_d),\ \ \theta_j'(g_h) = \sum_{k=2}^{2} \chi_j'(g_h) = 1\ , \qquad \theta_k''(g_l) = \sum_{k=2}^{p} \chi_k''(g_l) = \sum_{k=2}^{5} 1 = p - 1$$

$$\theta_{ijk(g_{dhl})} = \left[\!\!\left[\sum_{\sigma \epsilon\, Gal(Q(\chi_{ijk}(g_{dhl}))/Q)} \sigma(\chi_{ijk}(g_{dhl})) \right]\!\!\right] = \left[\!\!\left[\sum_{\sigma \epsilon\, Gal(Q(\chi_i(g_d))/Q)} \sigma\left(\chi_i(g_d)\right)[1\{p-1\}] \right]\!\!\right]$$

$$= \theta_i(g_d)(1)(p-1) = \theta_i(g_d) * \theta_j'(g_h) * \theta_k''(g_l)$$

- If $j = 2$and $k = 2,3,\ldots,p$, g_h is the identity of C_2 and g_l is not identity of $\mathbf{C}_p$

$$\theta_i(g_d) = \sum \chi_i(g_d),\ \ \theta_j'(g_h) = \sum_{k=2}^{2} \chi_j'(g_h) = 1\ ,$$

$$\theta_k''(g_l) = \sum_{k=2}^{p} \chi_k''(g_l) = (\varepsilon + \varepsilon^2 + \cdots + \varepsilon^{p-1}) = -1$$

$$\theta_{ijk(g_{dhl})} = \left[\!\!\left[\sum_{\sigma \epsilon\, Gal(Q(\chi_{ijk}(g_{dhl}))/Q)} \sigma(\chi_{ijk}(g_{dhl})) \right]\!\!\right] = \left[\!\!\left[\sum_{\sigma \epsilon\, Gal(Q(\chi_i(g_d))/Q)} \sigma\left(\chi_i(g_d)\right)[1\{-1\}] \right]\!\!\right]$$

$$= \theta_i(g_d)(1)(-1) = \theta_i(g_d) * \theta_j'(g_h) * \theta_k''(g_l)$$

- If $j = 2$and $k = 1$, g_h is the not identity of C_2 and for all $g_l \in \mathbf{C}_p$

$$\theta_i(g_d) = \sum \chi_i(g_d),\ \ \theta_j'(g_h) = \sum_{k=2}^{2} \chi_j'(g_h) = -1\ , \qquad \theta_k''(g_l) = \sum_{k=2}^{p} \chi_k''(g_l) = 1$$

$$\theta_{ijk(g_{dhl})} = \left[\!\!\left[\sum_{\sigma\epsilon\, Gal(\mathcal{Q}(\chi_{ijk}(g_{dhl}))/\mathcal{Q})} \sigma(\chi_{ijk}(g_{dhl})) \right]\!\!\right] = \left[\!\!\left[\sum_{\sigma\epsilon\, Gal(\mathcal{Q}(\chi_i(g_d))/\mathcal{Q})} \sigma\left(\chi_i(g_d)\right)[1\{1\}] \right]\!\!\right]$$

$$= \theta_i(g_d)(-1)(1) = \theta_i(g_d) * \theta_j^{'}(g_h) * \theta_k^{''}(g_l)$$

- If $j = 2$ and $k = 2,3,\dots,p$, g_h is not identity of C_2 and g_l is the identity of $\mathbf{C}_p$

$$\theta_i(g_d) = \sum \chi_i(g_d), \quad \theta_j'(g_h) = \chi_2^{'}(g_h) = -1, \qquad \theta_k''(g_l) = \sum_{k=2}^{p} \chi_k''(g_l) = \sum_{k=2}^{p} 1 = 4$$

$$\theta_{ijk(g_{dhl})} = \left[\!\!\left[\sum_{\sigma\epsilon\, Gal(\mathcal{Q}(\chi_{ijk}(g_{dhl}))/\mathcal{Q})} \sigma(\chi_{ijk}(g_{dhl})) \right]\!\!\right] = \left[\!\!\left[\sum_{\sigma\epsilon\, Gal(\mathcal{Q}(\chi_i(g_d))/\mathcal{Q})} \sigma\left(\chi_i(g_d)\right)[-1\{4\}] \right]\!\!\right]$$

$$= (-1)(4)\, \theta_i(g_d) = \theta_i(g_d) * \theta_j^{'}(g_h) * \theta_k^{''}(g_l)$$

- If $j = 2$and $k = 2,3,\dots,p$, g_h is not identity of C_2 and g_l is not identity of $\mathbf{C}_p$

$$\theta_i(g_d) = \sum \chi_i(g_d), \quad \theta_j'(g_h) = \chi_2^{'}(g_h) = -1,$$

$$\theta_k''(g_l) = \sum_{k=2}^{p} \chi_k''(g_l) = (\varepsilon + \varepsilon^2 + \cdots + \varepsilon^{p-1}) = -1$$

$$\theta_{ijk(g_{dhl})} = \left[\!\!\left[\sum_{\sigma\epsilon\, Gal(\mathcal{Q}(\chi_{ijk}(g_{dhl}))/\mathcal{Q})} \sigma(\chi_{ijk}(g_{dhl})) \right]\!\!\right] = \left[\!\!\left[\sum_{\sigma\epsilon\, Gal(\mathcal{Q}(\chi_i(g_d))/\mathcal{Q})} \sigma\left(\chi_i(g_d)\right)[-1\{-1\}] \right]\!\!\right]$$

$$\theta_i(g_d)(-1)(-1) = \theta_i(g_d) * \theta_j'(g_h) * \theta_k''(g_l)$$

We have $\theta_{ijk}(g_{dhl}) = \theta_i(g_d) * \theta_j^{'}(g_h) * \theta_k^{''}(g_l)$ for all i, j, k and for all g_d, g_h, g_l where $\theta_{ijk}(g_{dhl}), \theta_i(g_d), \theta_j^{'}(g_h) and\ \theta_k^{''}(g_l)$ are the rational valued character of the group $(Q_{14} \times C_{2p}), (Q_{14}), (C_2)$and, $(\mathbf{C}_p)$ respectively and e is the identity of group

$$\theta_{ijk}(g_{dhl}) = \begin{cases} \theta_i(g_d), \text{if } j = 1, \wedge k = 1, \wedge \forall g_h \in C_2 \wedge \forall g_l \in \mathbf{C}_p \\ (4)\chi_i(g_d), \text{if } j = 1, \wedge k = 2,3,4,5, \wedge \forall g_h \in C_2 \wedge g_l = e \\ -\theta_i(g_d), \text{if } j = 1, \wedge k = 2,3,4,5, \wedge \forall g_h \in C_2 \wedge g_l \neq e \\ \theta_i(g_d), \text{if } j = 2, \wedge k = 1, g_h = e \wedge \forall g_l \in \mathbf{C}_p \\ (4)\chi_i(g_d), \text{if } j = 2, \wedge k = 2,3,4,5, \wedge g_h = e \wedge g_l = e \\ -\chi_i(g_d), \text{if } j = 2, \wedge k = 2,3,4,5, \wedge g_h = e \wedge g_l \neq e \\ -\chi_i(g_d), \text{if } j = 2, \wedge k = 1, g_h \neq e \wedge \forall g_l \in \mathbf{C}_p \\ (-4)\chi_i(g_d), \text{if } j = 2, \wedge k = 2,3,4,5, \neq e \wedge g_l = e \\ \chi_i(g_d), \text{if } j = 2, \wedge k = 2,3,4,5, g_h \neq e \wedge g_l \neq e \end{cases}$$

then

$$\equiv^{*} \left(Q_{14} \times \mathbf{C}_{2p} \right) = \equiv^{*} (Q_{14}) \otimes \equiv^{*} (\mathbf{C}_{2p}) \quad \square$$

Then, the rational characters table of ($Q_{14} \times C_{2p}$) is given in the following table

Γ-classes	[I , I]	[I , r_2]	[I , r]	[I , r r_2]	[x^2, I]	[x^2, r_2]	[x^2, r]	[x^2, r r_2]	[x^q, I]	[x^q, r_2]	[x^q , r]	[x^q , r r_2]	[x , I]	[x , r_2]	[x , r]	[x , r r_2]	[y , I]	[y , r_2]	[y , r]	[y , r r_2]
| cl_α |	1	1	p-1	p-1	6	6	6(p-1)	6(p-1)	1	1	p-1	p-1	6	6	6(p-1)	6(p-1)	14	14	14(p-1)	14(p-1)
θ_{11}	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
θ_{13}	1	1-	1	1-	1	1-	1	1-	1	1-	1	1-	1	1-	1	1-	1	1-	1	1-
θ_{13}	p-1	p-1	-1	-1	p-1	p-1	-1	-1	p-1	p-1	-1	-1	p-1	p-1	-1	-1	p-1	p-1	-1	-1
θ_{14}	p-1	1-p	-1	1	p-1	1-p	-1	1	p-1	1-p	-1	1	p-1	1-p	-1	1	p-1	1-p	-1	1
θ_{21}	6	6	6	-6	-1	-1	-1	-1	6	6	6	6	-1	-1	-1	-1	0	0	0	0
θ_{22}	6	-6	6	-6	-1	1	-1	1	6	-6	6	-6	-1	1	-1	1	0	0	0	0
θ_{23}	6(p-1)	6(p-1)	-6	-6	1-p	1-p	1	1	6(p-1)	6(p-1)	-6	-6	1-p	1-p	1	1	0	0	0	0
θ_{24}	6(p-1)	6(1	-6	6	1-p	p-1	1	-1	6(p-1)	6(1-p)	-6	6	1-p	p-1	1	-1	0	0	0	0
θ_{31}	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-1	-1	-1	-1
θ_{32}	1	1-	1	1-	1	1-	1	1-	1	1-	1	1-	1	1-	1	1-	-1	1	-1	1
θ_{33}	p-1	p-1	-1	-1	p-1	p-1	-1	-1	p-1	p-1	-1	-1	p-1	p-1	-1	-1	1-p	1-p	1	1
θ_{34}	p-1	1-p	-1	1	p-1	1-p	-1	1	p-1	1-p	-1	1	p-1	1-p	-1	1	1-p	p-1	1	-1
θ_{41}	6	6	6	6	-1	-1	-1	-1	-6	-6	-6	-6	1	1	1	1	0	0	0	0
θ_{42}	6	-6	6	-6	-1	1	-1	1	-6	6	-6	6	1	-1	1	-1	0	0	0	0
θ_{43}	6(p-1)	6(p-1)	-6	-6	1-p	1-p	1	1	6(1-p)	6(1-p)	6	6	p-1	p-1	-1	-1	0	0	0	0
θ_{44}	6(p-1)	6(1-p)	-6	6	1-p	p-1	1	-1	6(1-p)	6(p-1)	6	-6	p-1	1-p	-1	1	0	0	0	0
θ_{51}	2	2	2	2	2	2	2	2	-2	-2	-2	-2	-2	-2	-2	-2	0	0	0	0
θ_{52}	2	-2	2	-2	2	-2	2	-2	-2	2	-2	2	-2	2	-2	2	0	0	0	0
θ_{53}	2(p-1)	2(p-1)	-2	-2	2(p-1)	2(p-1)	-2	-2	2(1-p)	2(1-p)	2	2	2(1-p)	2(1-p)	2	2	0	0	0	0
θ_{54}	2(p-1)	2(1-p)	-2	-2	2(p-1)	2(1-p)	-2	2	2(1-p)2(1-p)	2(p-1)	2	-2	-8	2(p-1)	2	-2	0	0	0	0

Table-3 the rational characters table of ($Q_{14} \times C_{2p}$)

3.2 Theorem

the cyclic decomposition of the group ($Q_{14} \times C_{2p}$)is given by

$$K(Q_{14} \times C_{10}) = \oplus_{i=1}^{4} K(C_{14}) \oplus_{i=1}^{4} C_4 \oplus_{i=1}^{10} C_2 \oplus_{i=1}^{10} C_p$$

$K\left(Q_{2m} \times C_{2p}\right) = \oplus_{i=1}^{4} K(Q_{14}) \oplus_{i=1}^{5} K(\boldsymbol{C}_{2p})$

Proof: Let A and B be matrices defining as follows:

$$A=\begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \quad , B = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} A & A & A & A \\ 0 & A & 0 & A \\ 0 & 0 & A & A \\ 0 & 0 & 0 & A \end{bmatrix} \equiv^* \left(Q_{2m} \times C_{2p}\right) \begin{bmatrix} B & 0 & 0 & 0 \\ B & B & 0 & 0 \\ (p-1)B & 0 & B & 0 \\ (p-1)B & (p-1)B & B & B \end{bmatrix} = \begin{bmatrix} E_3 & 0 & 0 & 0 \\ 0 & E_2 & 0 & 0 \\ 0 & 0 & E_1 & 0 \\ 0 & 0 & 0 & E_0 \end{bmatrix}$$

Such that $E_{0,}E_1 , E_2, E_3$ of degree A and B the invariant factors of the matrix$\equiv^* \left(Q_{14} \times C_{10}\right)$ it's the same invariant factors of the matrices $E_{0,}E_1 , E_2, E_3$, so as will the proof

$$E_0 = \begin{bmatrix} -4 & -4 & 0 & 0 & 0 \\ 2 & -12 & 0 & 0 & 0 \\ 0 & 0 & -2 & -2 & 0 \\ 0 & 0 & 1 & -6 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

such a $(E_3 = -2pE_0), (E_2 = -pE_0), (E_1 = 2E_0)$ And we will defined two matrices Lp and Wp , such that :

$$L_q = \begin{bmatrix} 1 & 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, W_q = \left[\begin{bmatrix} 6 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 6 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}\right]$$

Such that; $L_q . E_0 . W_q = diag(-28,2,-14,1,-1)$

$L_q . E_1 . W_q = diag(-56,4,-28,2,-2)$

$L_q . E_2 . W_q = diag(28p,-2p,14p,-p,p)$

$L_q . E_3 . W_q = diag(56p,4p,140,-2p,2p)$

Then: $K(Q_{14} \times C_{10}) = \oplus_{i=1}^{4} K(C_{14}) \oplus_{i=1}^{4} C_4 \oplus_{i=1}^{10} C_p \oplus_{i=1}^{10} C_2$

$K(Q_{14} \times C_{10}) = \oplus_{i=1}^{4} K(Q_{14}) \oplus_{i=1}^{5} K(\mathbf{C}_{2p})$

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