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The Cyclic Decomposition Of cf(Q_4q×C_5)/ R̅(Q_4q×C_5)

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ABSTRACT

In this paper, we propose the cyclic decomposition of the factor group cf(Q_4q×C_5),Z)/R̄(Q_4q×C_5), and the group cf(Q_4q×C_5) is Z-valued class functions of the direct product group (Q_4q×C_5) under the operation of addition, and R(Q_4q×C_5)is the subgroup of the generalized characters of the group cf((Q_4q×C_5),Z).Then cf(Q_4q×C_5)/(Q_4q×C_5))) is an abelian factor group denoted by K(Q_4q×C_5) where (Q_4q is the quaternion group of order8q and C_5 is the cyclic group of order 5. Also, we find the rational valued characters table of the group (Q_4q×C_5) when q is prime numbers is given as follows :

≡^* (Q_4q×C_5)= 〖≡〗^* (Q_4q)⊗≡^* (C_5) (1)

and find the cyclic decomposition of group ((Q_4q×C_5)) in this paper and prove that

K(Q_4q×C_5)=⊕_(i=1)^2 [K(Q_4q)] ⊕_(i=1)^8 K (C_5) (2)

1.Introduction

"Any two elements' of a finite group G. are said to be $\Gamma -$ conjugate if they generated conjugate cyclic subgroups in G. This relation is equivalence on G and its classes are called Γ-classes".

"The number of $\Gamma -$ classes of G is equal to the rank of $\mathrm{cf}(G.Z)$ is the intersection of $\mathrm{cf}(G.Z)$ with the group $R(G)$ which is a normal subgroup of $\mathrm{cf}(G.Z)$, then, $\mathrm{cf}(G.Z)/\overline{R}(G)$ is a finite abelian factor group denoted by $K(G)$".

"Each element in $\overline{R}\left(Q_{4q} \times C_5\right)$ can be written as" $v1\theta1 + v2\theta2 + \cdots + vr\theta r$,"where r is the number of $\Gamma -$ classes. $v1.v2.\ldots.vr \in Z$ and , where χi is an irreducible character of the group G and σ is any element in Galios group Gal $(Q(\chi i)/Q)$".

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"In 1982, Kirdar M. S. [3] studied the $K(C_n)$. In 1994, Abass H. H. [1] studied the $K(D_n)$ and found $\equiv^*(D_n)$. In 1995, Mahmood N. R. [4] studied the factor group $K(Q_{2m})$ and foun $\equiv^*(Q_{2m})$.. In 2008, Mahdi M. S. [5] studied the factor group K(Dnh) where n is an odd number."

"The aim of this paper to find" $\equiv^*\left(Q_{4q} \times C_5\right)$ "and to determine the cyclic decomposition of the group" $K\left(Q_{4q} \times C_5\right)$, when q is prime numbers

A finite group G has a finite number of conjugacy classes and a finite number of distinct k-irreducible representations, the group character of a group representation is constant on a conjugacy class $CL \propto .(1 \leq \propto \leq k)$, the values of the characters can be written as a table known as the characters table which is denoted by $\equiv(G)$." Let $T_1: G_1 \rightarrow GL(n.K)$ and $T_2: G_2 \rightarrow GL(m.K)$ are two matrix representations of the groups G_1 and G_2 . χ_1 and χ_2 be two characters of T_1 and T_2 respectively, then the character of $T_1 \otimes T_2$ is $\chi_1\chi_2$.

2 -Basic Concepts

2.1 Definition [2]

A rational valued character θ of G is a character whose values are in Z, which is $\theta(g) \in Z$. for all $g \in G$.

2.2 Proposition:[3]

The rational valued characters

$$\theta_i = \sum_{\sigma \in Gal(Q(\chi_i)/Q)} \sigma(\chi_i) \qquad (3)$$

form basis for $\overline{R}(G)$, where χ_i are the irreducible characters of G and their numbers are equal to the number of all distinct Γ- classes of G.

2.3 Proposition: [3]

The rational valued characters table of the cyclic group C_p of the rank 2 where p is a prime number which is denoted by ($\equiv^*(C_p)$), is given as follows:

$\Gamma -$ **classes**	$[I]$	$[r]$
θ_1	1	1
θ_2	$p-1$	-1

Table 1. The Rational Valued Characters Table of The Cyclic Group C_p

where its rank 2 represents the number of all distinct Γ-classes.

2.4 Definition [4]

For every positive integer m, The generalized Quaternion Group Q_{2m} of order 4m with two generators x and y then For every element in Q2m satisfies $x^m = y^2$,
$y^1 x^m \ y^{-1} = x^{-m}$ which implies $x^m = y^4 = 1$Any element λ∈Q2m ,

can be expressed uniquely in the form $\mathrm{Q2m} = \{\lambda = x^h y^k \mid 0 \leq \mathrm{h} \leq 2\mathrm{m} - 1, \mathrm{k} = 0,1\}$

2.5 Lemma [4]

The rational valued characters table of Q_{2m} when m is an even number is given in table(2.2)
$\equiv^* (Q_{2m}) =$

Γ-classes of C_{2m}				
	x2r	x2r+1	[y]	[xky]
θ1	1 1 ... 1 1	1 1 ... 1 1	1	1
θ2	1 1 ... 1 1	-1 -1 ... -1 -1	-1	1
			0	0
:			0	0
:	$\equiv^*(C_{2m})$		:	:
			:	:
θl-1			0	0
Θl			0	0
θl+1	1 ... 1	1 ... 1	-1	-1
θl+2	1 ... 1	-1 ... -1	1	-1

Table-2 The rational valued characters table of Q_{2m} when m is an even number

Where 0 ≤ r ≤ m-1, l is the number of Γ-classes of C_{2m}, θj such that 1 ≤ j ≤ l+2 are the rational valued characters of the group Q_{2m}

3. The Main Results

We study the rational valued characters table of $(Q_{4q} \times C_5)$. and find the cyclic decomposition of $K\big((Q_{4q} \times C_5) \big)$. The group of $(Q_{4q} \times C_5)$ is the direct product of the Quaternion group $Q_{4q}$of order 8q (number), and the group C_5. which is cyclic of order 5 the order of$\big((Q_{4q} \times C_5) \big)$.is $= 8q$

3.1 Theorem:

The rational valued character table of the group $(Q_{4q} \times C_5)$ has the following form

$$\equiv^* \left(Q_{4q} \times C_5\right) = \equiv^* \left(Q_{4q}\right) \otimes \equiv (C_5)$$

Proof

For each element $g_{dh} \in \left(Q_{4q} \times C_5\right)$, we have $g_{dh} = g_d g_h$ such that $g_d \in Q_{4q}$,$g_h \in C_5 = \prec r \succ$ Then $g_d = x^t y^k, 0 \leq t \leq 4q, k = 0,1,$ and each irreducible character of $\left(Q_{4q} \times C_p\right)$ is $\chi_{ij} = \chi_i \chi_j'$, where χ_i is an irreducible character of Q_{4q}, χ_j' is an irreducible character of C_5,

by proposition(2.2)

$$\theta_{ij=}\theta_{ij(g_{dh})=}\left[\!\left[\sum_{\sigma\epsilon\, Gal(\mathcal{Q}(\chi_{ij}(g_{khl}))/\mathcal{Q})} \sigma(\chi_{ij}(g_{dh}))\right]\!\right]$$

$$= \left[\!\left[\sum_{\sigma\epsilon\, Gal(\mathcal{Q}(\chi_i(g_d))/\mathcal{Q})} \sigma\left(\chi_i(g_d)\right)\left[\sum_{\sigma\epsilon\, Gal(\mathcal{Q}(\acute{\chi}_j(g_h))/\mathcal{Q})} \sigma\left(\acute{\chi}_j(g_h)\right)\right]\right]\!\right]$$

- If j = 1, then for all $g_h \in C_5$such that $\theta'_j(g_h) = \sum \chi_j'(g_h) = 1$. Then

$$\theta_{ij(g_{dh})=}\left[\!\left[\sum_{\sigma\epsilon\, Gal(\mathcal{Q}(\chi_{ij}(g_{kh}))/\mathcal{Q})} \sigma(\chi_{ij}(g_{dh}))\right]\!\right] = \left[\!\left[\sum_{\sigma\epsilon\, Gal(\mathcal{Q}(\chi_i(g_d))/\mathcal{Q})} \sigma\left(\chi_i(g_d)\right)[1]\right]\!\right]$$

$$= \theta_i(g_d) * \theta'_j(g_h)$$

- If j = 2 ,3,4,5, and g_h is the identity ofC_5 that

$$\theta'_2(g_h) = \sum_{i=2}^{p} \chi_2'(g_h) = \sum_{k=2}^{5} 1 = \underbrace{1 + 1 + \cdots + 1) =}_{(4)\ \text{times}} 4 \text{ Then}$$

$$\theta_{ij(g_{dh})=}\left[\!\left[\sum_{\sigma\epsilon\, Gal(\mathcal{Q}(\chi_{ij}(\mathrm{dh}))/\mathcal{Q})} \sigma(\chi_{ij}(g_{dh}))\right]\!\right] = \left[\!\left[\sum_{\sigma\epsilon\, Gal(\mathcal{Q}(\chi_i(g_d))/\mathcal{Q})} \sigma\left(\chi_i(g_d)\right)[4]\right]\!\right]$$

$$= \theta_i(g_d)(4) = \theta_i(g_d) * \theta'_j(g_h)$$

- If j = 2,3,4,5, and g_h is not identity of C_5 that $\theta'_2(g_h) = \sum_{i=2}^{5} \chi_2'(g_h) = (\varepsilon + \varepsilon^2 + \varepsilon^2 + \cdots + \varepsilon^4)'_{=} - 1$

Then

$$\theta_{ij(g_{dh})}= \left[\!\!\left[\sum_{\sigma\epsilon\, Gal(Q(\chi_{ij}(dh))/Q)} \sigma(\chi_{ij}(g_{dh})) \right]\!\!\right] = \left[\!\!\left[\sum_{\sigma\epsilon\, Gal(Q(\chi_i(g_d))/Q)} \sigma\left(\chi_i(g_d)\right)[-1] \right]\!\!\right]$$

$$= \theta_i(g_d)(-1) = \theta_i(g_d) * \theta_i(g_d)$$

Where $\theta_{ij}(g_{dh})$, $\theta_i(g_d)$, *and* $\theta_i(g_d)$are the rational valued characters of the groups $(Q_{4q} \times C_5)$, (Q_{4q})*and* (C_5) respectively.

Then we get $(\theta_{ij}(g_{dh}) = \theta_i(g_d) * \theta'_j(g_h)$ for all i,j, $g_{dh} \in (Q_{4q} \times C_5), g_d \in Q_{4q} g_h \in C_5$

$\equiv^* (Q_{4q} \times C_5) =\equiv^* (Q_{4q}) \otimes \equiv^* (C_5)$ □

To calculate the rational valued characters table of $(Q_{4q} \times C_5)$

$$\theta_{11} = \psi_{11} \ , \theta_{12} = \sum_{i=2}^{5} \psi_{1i} , \theta_{21} = \psi_{41} \ , \theta_{22} = \sum_{i=2}^{6} \psi_{4i} ,$$

$$\theta_{71} = \psi_{21} \ , \theta_{72} = \sum_{i=2}^{5} \psi_{2i} , \theta_{81} = \psi_{31} \ , \theta_{82} = \sum_{i=2}^{5} \psi_{3i} ,$$

$$\theta_{51} = \chi_{51} \ , \theta_{52} = \sum_{i=2}^{5} \chi_{5i},$$

The elements of Gal (λ 1i) / Q are :

$\{ \sigma_{1i} \ , \sigma_{3i} , \dots \ , \sigma_{2q-1\,i} \ \}$

$\sigma_{1i}(\chi_{1i}) = \chi_{1i} \ , \sigma_{3i}(\chi_{1i}) = \chi_{3i}, \dots, \sigma_{2q-1\,i}(\chi_{1\,i}) = \chi_{2q-1\,i}$

and i = 1 , 2 , 3,4,5

$\theta_{6i} = \sigma_{1i}(\chi_{11}) + \sigma_{3i}(\chi_{1i}) + \cdots + \sigma_{2q-1\,i}(\chi_{1\,i}), \quad \theta_{61} = \chi_{11} + \chi_{3i} + \cdots + \chi_{2q-1\,1}, = \theta_{61} = \chi_{11} + \chi_{3i} + \cdots + \chi_{2q-1\,1}, \quad \theta_{62} = \sum_{i=2}^{5} \sigma_{1i}(\chi_{11}) + \sigma_{3i}(\chi_{1i}) + \cdots + \sigma_{2q-1\,i}(\chi_{1\,i})$

$\theta_{4\,i} = \sigma_{2i}(\chi_{1i}) + \sigma_{6i}(\chi_{1i}) + \cdots + \sigma_{2q-2\,i}(\chi_{1\,i}), \quad + \cdots + \chi_{2q-2\,i}, \quad = \quad , , \theta_{41} = \chi_{2i} + \chi_{6i} + \cdots + \chi_{2q-2\,i} \ , \quad \theta_{4\,2} = \sum_{i=2}^{5} \sigma_{2i}(\chi_{1i}) + \sigma_{6i}(\chi_{1i}) + \cdots + \sigma_{2q-4\,i}(\chi_{1\,i})$

$$\theta_{3\,i} = \sigma_{4i}(\chi_{1i}) + \sigma_{8i}(\chi_{1i}) + \cdots + \sigma_{2q-4\,i}(\chi_{1\,i}), \quad = \theta_{31} = \chi_{4i} + \chi_{8i} + \cdots + \chi_{2q-4\,i}, \quad , \quad \theta_{32} = \sum_{i=2}^{5} \sigma_{4i}(\chi_{1i}) + \sigma_{8i}(\chi_{1i}) + \cdots + \sigma_{2q-4\,i}(\chi_{1\,i})$$

We can write the rational valued characters table of $\equiv^* \left(Q_{4q} \times C_5\right)$ as follows:

	(I,I)	(I,r)	$X^2, I)$	$X^2, r)$	X^4, I	$X^4, r)$	X^{2q}, I	X^{2q}, r	$X, I)$	X, r	$X^3, I)$	X^3, r	y,I	I,r	xy ,I	xy, r
Γ-classes|	1	4	q-1	4q-4	q-1	4q-4	1	4	q-1	4q-4	q-1	4q-4	2q	8q	2q	8q
θ_{11}	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
θ_{12}	4	-1	4	-1	4	-1	4	-1	4	-1	4	-1	4	-1	4	-1
θ_{21}	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1
θ_{22}	4	-1	4	-1	4	-1	4	-1	-4	1	-4	-1	-4	1	4	-1
θ_{31}	q-1	q-1	-1	-1	-1	-1	q-1	q-1	-1	-1	q-1	q-1	0	0	0	0
θ_{32}	4q-4	1-q	-4	1	-4	1	4q-4	1-q	-4	1	4-4q	1-q	0	0	0	0
θ_{41}	q-1	q-1	-1	-1	-1	-1	q-1	q-1	1	1	1-q	1-q	0	0	0	0
θ_{42}	4q-4	1-q	-4	1	-4	1	4q-4	1-q	4	-1	4-4q	q-1	0	0	0	0
θ_{51}	2	2	-2	-2	2	2	-2	-2	0	0	0	0	0	0	0	0
θ_{52}	8	-2	-8	2	8	- 2	-8	2	0	0	0	0	0	0	0	0
θ_{61}	2q-2	2q-2	2	2	-2	-2	2-2q	2-2q	0	0	0	0	0	0	0	0
θ_{62}	8q-8	2-2q	2q-2	-2	2-2q	2	8-8q	2q-2	0	0	0	0	0	0	0	0
θ_{71}	1	1	1	1	1	1	1	1	1	1	1	1	-1	-1	-1	-1
θ_{72}	4	-1	4	-1	4	-1	4	-1	4	-1	4	-1	-4	1	-4	1
Θ_{81}	1	1	1	1	1	1	1	1	-1	-1	-1	-1	1	1	-1	-1
Θ_{82}	4	-1	4	-1	4	-1	4	-1	-4	1	-4	1	4	-1	-4	1

Table-3 The rational valued characters table of the group $\left(Q_{4q} \times C_5\right)$

3.2 Theorem:

If $m = 2q$, q is prime numbers and $q \neq 5$, then the cyclic decomposition of $K(Q_{4q} \times C_5)$ is :

$$K(Q_{4q.} \times C_5) = \oplus_{i=1}^{2} K(Q_{4q.}) \oplus_{i=1}^{8} C_5$$

Proof:

Let A and B be matrices defined as follows

$$:A = \begin{bmatrix} \begin{pmatrix} 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & & 0 & 0 \\ \vdots & & \ddots & \vdots & \\ 0 & 0 & \cdots & 1 & 0 \\ 0 & 0 & & 0 & 1 \end{pmatrix} & \begin{matrix} 1 & 0 \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \\ 0 & 0 \end{matrix} \\ \begin{matrix} 0 & 1 & \ldots & 0 & 0 \\ 0 & -1 & \ldots & 0 & 0 \end{matrix} & \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix} \end{bmatrix} \quad ,B = \begin{bmatrix} \begin{pmatrix} 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & & 0 & 0 \\ \vdots & & \ddots & \vdots & \\ 0 & 0 & \cdots & 1 & 0 \\ 0 & 0 & & 0 & 1 \end{pmatrix} & \begin{matrix} 0 & 0 \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \\ 0 & 0 \end{matrix} \\ \begin{matrix} 1 & \ldots & 1\ 0 & \ldots & 0 \\ 1 & \ldots & 1\ 0 & \ldots & 0 \end{matrix} & \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix} \end{bmatrix}$$

Such that (I) is an identity matrix dimension which is equal to the dimension of $\equiv^* (C_{2q})$.We will define two matrices L_p and W_p where

$$L_5 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \quad , \quad \equiv^* (C_5) = \begin{bmatrix} 1 & 1 \\ 4 & -1 \end{bmatrix} \quad , \quad W_5 = \begin{bmatrix} 1 & 0 \\ 4 & -1 \end{bmatrix}$$

$$L_5 . \equiv^* (C_5) . W_5 = \begin{bmatrix} 5 & 0 \\ 0 & 1 \end{bmatrix}$$

By using Remark (2.3.9) and proposition (2.3.13) we get:

Such that E is a matrix of degree $\equiv^* (C_{2m})$, and if we denote c_{ij}as the elements of $\equiv^* (C_m)$ which defined by:

$(A \times L_5) . \equiv^* (Q_{4q} \times C_5) . (B \times W_5)$

the invariant factors of the matrix$(A \times L_5) . \equiv^* (Q_{4q} \times C_5) . (B \times W_5)$will be the same invariant factors of the following matrix:

$$(A\times L_5).\equiv^* (Q_{4q}\times C_5).(B\times W_5)=\begin{bmatrix}\begin{bmatrix} -5E & \begin{bmatrix}0&0\\0&2\\0&0\\\vdots&\vdots\\0&0\end{bmatrix}\\ \begin{matrix}0&\dots&0\\0&\dots&0\end{matrix} & \begin{bmatrix}-10&0\\0&-10\end{bmatrix}\end{bmatrix} & 0\\ 0 & \begin{bmatrix} E & \begin{bmatrix}0&0\\0&-1\\0&0\\\vdots&\vdots\\0&0\end{bmatrix}\\ \begin{matrix}0&\dots&0\\0&\dots&0\end{matrix} & \begin{bmatrix}2&0\\0&2\end{bmatrix}\end{bmatrix}\end{bmatrix}$$

Then $E=\begin{bmatrix}2(q-1)&2&-(q-1)&-1&(q-1)&1\\2&0&-1&0&1&0\\(q-1)&1&(q-1)&1&0&0\\1&0&1&0&0&0\\(q-1)&1&0&0&0&0\\1&0&0&0&0&0\end{bmatrix}$

$$L_q=\begin{bmatrix}-1&-2&-2&-2&-2&-2\\0&1&0&1&0&1\\0&0&0&0&1&1\\0&0&0&0&0&-1\\0&0&1&1&1&1\\0&0&0&-1&0&-1\end{bmatrix},W_q=\begin{bmatrix}2(q-1)&2&-(q-1)&-1&(q-1)&1\\2&0&-1&0&1&0\\(q-1)&1&(q-1)&1&0&0\\1&0&1&0&0&0\\(q-1)&1&0&0&0&0\\1&0&0&0&0&0\end{bmatrix}$$

Such that

$L_q\,.E.W_q=diag\{8\text{q},4,2\text{q},2\,,\text{q},1\,\},2,2$

$L_q.(-pE).W_q=diag\{-40\text{q},-20,-10\text{q},-10\,,-5\text{q},-5\,\},10,10$

$, K(Q_{4q.}\times C_p\ ) =\oplus_{i=1}^{8} C_5 \oplus_{i=1}^{6} C_q \oplus_{i=1}^{2} C_8\ \ \oplus_{i=1}^{2} C_4 \oplus_{i=1}^{8} C_2\ )$

$K\big(Q_{4q.}\times C_5\ \big) =\oplus_{i=1}^{2} K(C_{4q.}) \oplus_{i=1}^{8} C_5 \oplus_{i=1}^{6} C_q \oplus_{i=1}^{2} C_8 \oplus_{i=1}^{4} C_2\ /\ \oplus_{i=1}^{2} C_4\ )$

$K\big(Q_{4q.}\times C_5\ \big) =\oplus_{i=1}^{2} K(Q_{4q.}) \oplus_{i=1}^{8} C_5$

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