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Generalized Permuting Tri-derivations On Lattices

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ABSTRACT

In this paper , we present the notion of generalized permuting tri – derivations in lattices and looking for some related properties .

2010 mathematics subject classification : 06B35 , 06B99 , 16B70 , 16B99

1. Introduction

Information Theory , Information retrieval , information access controls and cryptanalysis are the various branches in which the lattice algebra play's a significant role [2,3,4,11], lately the peculiarities of lattices they uere researched .

L. Ferrari and X.L. Xin in [8] and [12] respectively introduced the notions of derivations on lattices and debated some affectioned peculiarities . N. alshehri in [1] introduced the concept of generalized derivations for a lattice and debated various properties , thereafter the symmetric biderivations on lattices notions are introduced in [5] by Y. Ceven and investigate some related properties .

In [6] Y. Ceven applied the notion of generalized symmetric bidervations on lattices and introduced some properties of it, while M.A. Ozturk in [10] introduce the notions of permuting tri derivations in lattices and some affectioned peculiarities are studied .

In the article, we apply the notion of generalized permuting tri derivations and looking for some related properties which is canvassed in [6] and [10] .

2. Preliminaries

Definition2.1[8]:Let Γ be anon-empty set endoued uith operations $\wedge$ and $\vee$, then $(\Gamma, \wedge, \vee)$ is said to be a lattice if it fulfilling the following requirements for everyone $\alpha, \tau, \eta \in \Gamma$

$(i) \alpha \wedge \alpha = \alpha, \alpha \vee \alpha = \alpha$

$(ii) \alpha \wedge \tau = \tau \wedge \alpha, \alpha \vee \tau = \tau \vee \alpha$

$(iii)\ (\alpha \wedge \tau) \wedge \eta = \alpha \wedge (\tau \wedge \eta), (\alpha \vee \tau) \vee \eta = \alpha \vee (\tau \vee \eta)$

$(iv)\ (\alpha \wedge \tau) \vee \alpha = \alpha, (\alpha \vee \tau) \wedge \alpha = \alpha$

Definition2.2[3]: A lattice $(\Gamma, \wedge, \vee)$ is namely distributive lattice if one of the following identities hold for all $(\Gamma, \wedge, \vee)$

$(v)\ \alpha \wedge (\tau \vee \eta) = (\alpha \wedge \tau) \vee (\alpha \wedge \eta)$

$(vi)\ \alpha \vee (\tau \wedge \eta) = (\alpha \vee \tau) \wedge (\alpha \vee \eta)$

Remark2.3[1]: In any lattice , the properties (v) and (vi) are equivalent .

Definition2.4[3]: let $(\Gamma, \wedge, \vee)$ be a lattice , a binary relation $\leq$ on Γ is defined by $\alpha \leq \tau$ if and only if $\alpha \wedge \tau = \alpha$ and $\alpha \vee \tau = \tau$

Definition2.5 [8]: A lattice $(\Gamma, \wedge, \vee)$ is namely modular if for $\alpha, \tau, \eta \in \Gamma$ fulfills the follouing requirement:

(vii) if $\alpha \leq \tau$ implies $\alpha \vee (\tau \wedge \eta) = (\alpha \vee \tau) \wedge \eta$

Lemma2.6 [7]: let $(\Gamma ,\wedge ,\vee)$ be a lattice , let the binary relation $\leq$ be as in definition 2.4 , then $(\Gamma ,\leq)$ is partially ordered set (poset) and for any $\alpha,\tau \in \Gamma$, $\alpha \wedge \tau$ is the g.l.b of $\{\alpha,\tau\}$ and $\alpha \vee \tau$ is the l.u b. of $\{\alpha,\tau\}$.

Definition2.7[10] : let $(\Gamma ,\wedge ,\vee)$ be a lattice , a mapping $T(.,.):\Gamma \times \Gamma \times \Gamma \longrightarrow \Gamma$ is namely permuting if

$$T(\alpha ,\tau,\eta) = T(\alpha,\eta,\tau) = T(\tau,\alpha,\eta) = T(\tau,\eta,\alpha) = T(\eta,\alpha,\tau) = T(\eta,\tau,\alpha)$$

for all $\alpha,\tau,\mathrm{z} \in \Gamma$.

Definition2.8[10]: let $(\Gamma ,\wedge ,\vee)$ be a lattice , a mapping $d:\Gamma \longrightarrow \Gamma$ defined by $d(\alpha) = T(\alpha,\alpha,\mathrm{x})$ is called the trace of $T(.,.,.)$ .where $T(.,.,.):\Gamma \times \Gamma \times \Gamma \longrightarrow \Gamma$ is permuting mapping .

Definition2.9[10] : let $(\Gamma ,\wedge ,\vee)$ be a lattice , a mapping $T(.,.,.):\Gamma \times \Gamma \times \Gamma \longrightarrow \Gamma$ is called permuting tri - derivation on Γ if

$$T(\alpha \wedge u ,\mathrm{y},\eta) = (T(\alpha,\mathrm{y},\eta) \wedge u) \vee (\alpha \wedge T(u,\tau,\eta))$$

for all $\alpha,\tau ,\eta,u \in \Gamma$.

obviously , a permuting tri -derivation on Γ satisfies the relation

$$T(\alpha ,\tau \wedge u,\eta) = (T(\alpha,\tau,\eta) \wedge u) \vee (\tau \wedge T(\alpha,u ,\eta))$$

$$T(\alpha ,\tau,\eta \wedge u) = (T(\alpha,\tau,\eta) \wedge u) \vee (\eta \wedge T(\alpha,\tau ,u))$$

for all $\alpha,\tau ,\eta,\mathrm{u} \in \Gamma$.

proposition2.10[10]: let $(\Gamma ,\wedge ,\vee)$ be a lattice , T be permuting tri – derivation on Γ with the trace d . then $d(\alpha) \leq \alpha$ for all $\alpha \in \Gamma$.

proposition2.11[10] : let $(\Gamma ,\wedge ,\vee)$ be a lattice , T be permuting tri – derivation on Γ. Then $T(\alpha,\tau,\eta) \leq \alpha$, $T(\alpha,\tau,\eta) \leq \tau$ and $T(\alpha,\tau,\eta) \leq \eta$ for all $\alpha,\tau,\eta \in \Gamma$

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3. Generalized Permuting Tri – derivations on lattices

Definition 3.1: let $(\Gamma,\wedge,\vee)$ be a lattice . A permuting mapping $\kappa(.,.,.):\Gamma\times\Gamma\times\Gamma\longrightarrow\Gamma$ is namely a generalized permuting tri-derivation on Γ , if there is a permuting tri-derivation $T(.,.,.):\Gamma\times\Gamma\times\Gamma\longrightarrow\Gamma$ and fulfills the follouing requirement $\kappa(\alpha\wedge \mathrm{u},\tau,\eta)=(\kappa(\alpha,\mathrm{y},\eta)\wedge u)\vee(\mathrm{x}\wedge T(u,\tau,\eta))$

for all $\alpha,\tau,\eta,\mathrm{u}\in\Gamma$.

the mapping $\vartheta:\Gamma\longrightarrow\Gamma$ defined by $\vartheta(\alpha)=\kappa(\alpha,\alpha,\alpha)$ is namely the trace of the generalized permuting tri – derivation κ.

Obviously that a generalized permuting tri derivation κ on Γ satisfies the relations

$$\kappa(\alpha,\tau\wedge \mathrm{u},\eta)=(\kappa(\alpha,\mathrm{y},\eta)\wedge u)\vee(\mathrm{y}\wedge T(\alpha,u,\eta))$$

$$\kappa(\alpha,\tau,\eta\wedge \mathrm{u})=(\kappa(\alpha,\mathrm{y},\eta)\wedge u)\vee(\mathrm{z}\wedge T(\alpha,\tau,u))$$

for all $\alpha,\tau,\eta,\mathrm{u}\in\Gamma$.

Example 3.2: let $(\mathrm{L},\wedge,\vee)$ be a lattice uith least element 0 and the mapping $\mathrm{D}(.,.,.):\mathrm{L}\times\mathrm{L}\times\mathrm{L}\longrightarrow\mathrm{L}$ defined by $\mathrm{D}(\mathrm{x},\mathrm{y},\mathrm{z})=0$ for all $\mathrm{x},\mathrm{y},\mathrm{z}\in\mathrm{L}$ is permuting tri -derivation on L .Then the mapping $\kappa(\mathrm{x},\mathrm{y},\mathrm{z}):\mathrm{L}\times\mathrm{L}\times\mathrm{L}\longrightarrow\mathrm{L}$ defined by $\kappa(\mathrm{x},\mathrm{y},\mathrm{z})=(\mathrm{x}\wedge\mathrm{y})\wedge\mathrm{z}$ for all $\mathrm{x},\mathrm{y},\mathrm{z}\in\mathrm{L}$ is generalized permuting tri - derivation on L

proposition 3.3: let κ be generalized permuting tri derivation related to a permuting tri derivation T on a lattice Γ , then the mappings $p_1:\Gamma\longrightarrow\Gamma$, $p_2:\Gamma\longrightarrow\Gamma$ and $p_3:\Gamma\longrightarrow\Gamma$ defined by $p_1(\alpha)=\kappa(\alpha,\tau,\eta)$, $p_2(\tau)=\kappa(\alpha,\tau,\eta)$ and $p_3(\eta)=\kappa(\alpha,\tau,\eta)$ respectively are generalized derivations on Γ .

proof :

$$\begin{aligned} p_1(\alpha\wedge \mathrm{u}) &= \kappa(\alpha\wedge \mathrm{u},\mathrm{y},\mathrm{z}) \\ &= (\kappa(\mathrm{x},\mathrm{y},\mathrm{z})\wedge u)\vee(\alpha\wedge T(u,\tau,\eta)) \\ &= (p_1(\alpha)\wedge u)\vee(\alpha\wedge d_1(u)) \end{aligned}$$

Where $d_1:\Gamma\longrightarrow\Gamma$ is a derivation on Γ defined by $d_1(u)=T(u,\tau,\eta)$.

Hence p_1 is a generalized derivation on Γ .

Similarly for the mappings p_2 and p_3 .

Theorem3.4 : let (Γ ,$\wedge$,$\vee$) be a lattice , κ be a generalized permuting tri- derivation related to a permuting tri derivation T on Γ, ϑ be be the trace of κ and d be the trace of T .then

i) $T(\alpha,\tau,\eta) \leq \kappa(\alpha,\tau,\eta)\ \forall\ \alpha,\tau,\eta \in \Gamma$

If Γ is distributive lattice ,then

ii) $\kappa(\alpha,\tau,\eta) \leq \alpha$, $\kappa(\alpha,\tau,\eta) \leq \beta$ and $\kappa(\alpha,\tau,\eta) \leq \eta$

iii) $\kappa(\alpha,\tau,\eta) \leq \alpha \wedge \tau$, $\kappa(\alpha,\tau,\eta) \leq \tau \wedge \eta$ and $\kappa(\alpha,\tau,\eta) \leq \alpha \wedge \eta$ and $\kappa(\alpha,\tau,\eta) \leq \alpha \wedge \tau \wedge \eta$

iv) $d(\alpha) \leq \vartheta(\alpha) \leq \alpha$

v) $d(\alpha) = \alpha$ then $\vartheta(\alpha) = \alpha$

Proof:

i) $\kappa(\alpha,\tau,\eta) = \kappa(\alpha \wedge \alpha,\tau,\eta)$

$$= (\kappa(\alpha,\tau,\eta) \wedge \alpha) \vee (\alpha \wedge T(\alpha,\tau,\eta))$$

$$= (\kappa(\alpha,\tau,\eta) \wedge \alpha) \vee T(\alpha,\tau,\eta)$$

By proposition 2.11 , then $T(\alpha,\tau,\eta) \leq \kappa(\alpha,\tau,\eta)$

ii) If Γ is distributive , then

$\kappa(\alpha,\tau,\eta) = \kappa(\alpha \wedge \alpha,\tau,\eta)$

$$= (\kappa(\alpha,\tau,\eta) \wedge \alpha) \vee (\alpha \wedge T(\alpha,\tau,\eta))$$

$= \kappa(\alpha,\tau,\eta) \wedge \alpha$ by (i) and proposition 2.11

Hence $\kappa(\alpha,\tau,\eta) \leq \alpha$

Since κ is permuting and by the same way, we get

$\kappa(\alpha,\tau,\eta) \leq \tau$ and $\kappa(\alpha,\tau,\eta) \leq \eta$

iii) Since $\kappa(\alpha,\tau,\eta) \leq \alpha$, $\kappa(\alpha,\tau,\eta) \leq \tau$ and $\kappa(\alpha,\tau,\eta) \leq \eta$

Then $\kappa(\alpha,\tau,\eta) \wedge \kappa(\alpha,\tau,\eta) \leq \alpha \wedge \tau$

So that $\kappa(\alpha,\tau,\eta) \leq \alpha \wedge \tau$

By the same way we can prove

$\kappa(\alpha,\tau,\eta) \leq \tau \wedge \eta$, $\kappa(\alpha,\tau,\eta) \leq \alpha \wedge \eta$ and $\kappa(\alpha,\tau,\eta) \leq \alpha \wedge \tau \wedge \eta$

iv) By (i) and (ii) we can conclude $d(\alpha) \leq \vartheta(\alpha) \leq \alpha$

v) It is clear by (iv)

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Corollary3.5: let $(\Gamma ,\wedge ,\vee)$ be a lattice , κ be a generalized permuting tri- derivation related to a permuting tri derivation T on Γ , if 0 is the least element and 1 is the greatest element of of Γ , then

$\kappa(\alpha,\tau,0)=0$ and $\kappa(\alpha,\tau,1)\leq\alpha$ for all $\alpha,\tau\in\Gamma$.

Proof : trivially by (ii) of theorem 3.4 .

Theorem 3.6: let $(\Gamma ,\wedge ,\vee)$ be a distributive lattice , κ be a generalized permuting tri- derivation related to a permuting tri derivation T on Γ, ϑ be be the trace of κ and d be the trace of T .then

$$\vartheta(\alpha\wedge\tau)=(\vartheta(\alpha)\wedge\tau)\vee(\alpha\wedge d(\tau))\vee T(\alpha,\alpha,\tau)\vee T(\alpha,\tau,\tau)$$

for all $\alpha,\tau\in\Gamma$.

Proof :

$\vartheta(\alpha\wedge\tau)=\kappa(\alpha\wedge\tau,\alpha\wedge\tau,\alpha\wedge\tau)$

$=(\kappa(\alpha,\alpha\wedge\tau,\alpha\wedge\tau)\wedge\tau)\vee(\alpha\wedge T(\tau,\alpha\wedge\tau,\alpha\wedge\tau))$

$=\{[\kappa(\alpha,\alpha,\alpha\wedge\tau)\wedge\tau)\vee(\alpha\wedge T(\alpha,\tau,\alpha\wedge\tau)]\wedge\tau\}$

$\vee\{\alpha\wedge[\mathrm{D}(\tau,\alpha,\alpha\wedge\tau)\wedge\tau)\vee(\alpha\wedge T(\alpha,\tau,\alpha\wedge\tau)]\}$

$=[\{[(\vartheta(\alpha)\wedge\tau)\vee(\alpha\wedge\tau\wedge T(\alpha,\alpha,\tau)]\vee[(\alpha\wedge\tau\wedge T(\alpha,\alpha,\tau)\vee(\alpha\wedge T(X,\tau,\tau)]\}\wedge\tau]$

$\vee[\alpha\wedge\{[(T(\alpha,\alpha,\tau)\wedge\tau)\vee(\alpha\wedge\tau\wedge T(\alpha,\tau,\tau)]\vee[(\alpha\wedge\tau\wedge T(\alpha,\tau,\tau))\vee(\alpha\wedge d(\tau))]\}]$

$=\{[(\vartheta(\alpha)\wedge\tau)\vee(\alpha\wedge\beta\wedge T(\alpha,\alpha,\tau)]\wedge\tau\}$

$\vee\{[(\alpha\wedge\tau\wedge T(\alpha,\alpha,\tau)\vee(\alpha\wedge T(X,\tau,\tau)]\wedge\tau\}$

$\vee\{\alpha\wedge[(T(\alpha,\alpha,\tau)\wedge\tau)\vee(\alpha\wedge\tau\wedge T(\alpha,\tau,\tau)]\}$

$\vee\{\alpha\wedge[(\alpha\wedge\tau\wedge T(\alpha,\tau,\tau))\vee(\alpha\wedge d(\tau))]\}$

$=(\vartheta(\alpha)\wedge\tau)\vee(\alpha\wedge\tau\wedge T(\alpha,\alpha,\tau)\vee(\alpha\wedge\tau\wedge T(\alpha,\alpha,\tau))$

$\vee(\alpha\wedge\tau\wedge T(\alpha,\tau,\tau))\vee(\alpha\wedge\tau\wedge T(\alpha,\alpha,\tau))\vee(\alpha\wedge\tau\wedge T(\alpha,\tau,\tau))$

$\vee(\alpha\wedge\tau\wedge T(\alpha,\tau,\tau))\vee(\alpha\wedge d(\tau))$

$=(\vartheta(\alpha)\wedge\tau)\vee T(\alpha,\alpha,\tau)\vee T(\alpha,\alpha,\tau)\vee T(\alpha,\tau,\tau)\vee T(\alpha,\alpha,\tau)$

$\vee T(\alpha,\tau,\tau)\vee T(\alpha,\tau,\tau)\vee(\alpha\wedge d(\tau))$

$= (\vartheta(\alpha) \wedge \tau) \vee (\alpha \wedge d(\tau)) \vee T(\alpha,\alpha,\tau) \vee T(\alpha,\tau,\tau)$

Since $T(\alpha,\alpha,\tau) \leq \alpha$, $T(\alpha,\alpha,\tau) \leq \tau$, $T(\alpha,\tau,\tau) \leq \alpha$ and $T(\alpha,\tau,\tau) \leq \tau$.

Corollary3.7: let $(\Gamma ,\wedge ,\vee)$ be a distributive lattice , κ be a generalized permuting tri- derivation related to a permuting tri derivation T on Γ, ϑ be be the trace of κ and d be the trace of T .then

i) $T(\alpha,\alpha,\tau) \leq \vartheta(\alpha \wedge \tau)$ and $T(\alpha,\tau,\tau) \leq \vartheta(\alpha \wedge \tau)$
ii) $\vartheta(\alpha) \wedge \tau \leq \vartheta(\alpha \wedge \tau)$
iii) $\alpha \wedge d(\tau) \leq \vartheta(\alpha \wedge \tau)$
for all $\alpha,\tau \in \Gamma$

Proof : (i) , (ii) and (iii) are directly from theorem 3.6

Theorem3.8 : let $(\Gamma ,\wedge ,\vee)$ be a distributive lattice , κ be a generalized permuting tri- derivation related to a permuting tri derivation T on Γ, ϑ be be the trace of κ and d be the trace of T .then $\vartheta(\alpha) \wedge \vartheta(\tau) \leq \vartheta(\alpha \wedge \tau)$ for all $\alpha,\tau \in \Gamma$.

Proof :
Since $\vartheta(\alpha) \wedge \tau \leq \vartheta(\alpha \wedge \tau)$ and $\vartheta(\tau) \leq \tau$ then $\vartheta(\alpha) \wedge \tau \wedge \vartheta(\tau) \leq \vartheta(\alpha \wedge \tau) \wedge \tau$

Also since $\vartheta(\alpha) \leq \alpha$, we have

$\vartheta(\alpha) \wedge \tau \wedge \vartheta(\tau) \wedge \vartheta(\alpha) \leq \vartheta(\alpha \wedge \tau) \wedge \tau \wedge \alpha$
Hence $\vartheta(\alpha) \wedge \vartheta(\tau) \wedge \vartheta(\alpha) \leq \vartheta(\alpha \wedge \tau) \wedge \alpha \wedge \tau$
So that $\vartheta(\alpha) \wedge \tau \wedge \vartheta(\tau) \leq \vartheta(\alpha \wedge \tau)$ since $(\alpha \wedge \tau) \wedge \alpha \wedge \tau = \vartheta(\alpha \wedge \tau)$.

Corollary3.9 : let $(\Gamma ,\wedge ,\vee)$ be a distributive lattice , κ be a generalized permuting tri- derivation related to a permuting tri derivation T on Γ, ϑ be be the trace of κ and d be the trace of T .then $d(\alpha) \wedge d(\tau) \leq \vartheta(\alpha \wedge \tau)$ for all $\alpha,\tau \in \Gamma$.
Proof :
Since $d(\alpha) \leq \vartheta(\alpha)$ and $d(\tau) \leq \vartheta(\tau)$
Hence $d(\alpha) \wedge d(\tau) \leq \vartheta(\alpha) \wedge \vartheta(\tau)$
By theorem 3.8 , we have $d(\alpha) \wedge d(\tau) \leq \vartheta(\alpha \wedge \tau)$ for all $\alpha,\beta \in \Gamma$.

Theorem3.10: let $(\Gamma ,\wedge ,\vee)$ be a lattice , κ be a generalized permuting tri- derivation related to a permuting tri derivation T on Γ, ϑ be be the trace of κ.then

$$\vartheta^2(\alpha) = \vartheta(\alpha) \text{ for all } \alpha \in \Gamma .$$

Proof :

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It is clear that $\vartheta^2(\alpha) = \vartheta(\vartheta(\alpha)) \leq \vartheta(\alpha) \leq \alpha$ for all $\alpha \in \Gamma$ …(1)

Nou

$$\vartheta^2(\alpha) = \vartheta(\vartheta(\alpha)) = \vartheta(\vartheta(\alpha) \wedge \alpha)$$

$$= (\vartheta(\vartheta(\alpha)) \wedge \alpha) \vee \left(\vartheta(\alpha) \wedge d(\alpha)\right) \vee \mathrm{D}(\vartheta(\alpha), \vartheta(\alpha), \alpha) \vee \mathrm{D}(\delta(\mathrm{x}), \alpha, \alpha)$$

$$= (\vartheta^2(\alpha) \wedge \alpha) \vee \left(\vartheta(\alpha) \wedge d(\alpha)\right) \vee \mathrm{D}(\vartheta(\alpha), \vartheta(\alpha), \alpha) \vee \mathrm{D}(\delta(\mathrm{x}), \alpha, \alpha)$$

$$\leq \vartheta^2(\alpha) \vee \vartheta(\alpha) \vee \delta(\mathrm{x}) \vee \delta(\mathrm{x})$$

$$= \vartheta^2(\alpha) \vee \vartheta(\alpha)$$

Hence

$\vartheta^2(\alpha) \leq \vartheta(\alpha)$ for all $\alpha \in \Gamma$ …(2)

From (1) and (2) , we have $\vartheta^2(\alpha) = \vartheta(\alpha)$.

Theorem3.10:let $(\Gamma, \wedge, \vee)$ be a distributive lattice , κ be a generalized permuting tri- derivation related to a permuting tri derivation T on Γ, ϑ be be the trace of κ and d be the trace of T .

If 0 and 1 are the least and greatest elements of Γ respectively , then

i) when $\alpha \leq \vartheta(1)$ then $\vartheta(\alpha) = \alpha$
ii) when $\vartheta(1) \leq \alpha$ then $\vartheta(\alpha) \geq \vartheta(1)$
iii) when $\alpha \leq \tau$ and $(\tau) = \tau$, ue have $\vartheta(\alpha) = \alpha$.

Proof :

i) By corollary 3.7 (2) we have
$\vartheta(1) \wedge \mathrm{x} \leq \delta(1 \wedge \mathrm{x}) = \delta(\mathrm{x})$
Now if $\alpha \leq \vartheta(1) \Rightarrow \alpha \wedge \vartheta(1) = \alpha$

Hence $\alpha \leq \vartheta(\alpha)$ but $\vartheta(\alpha) \leq \alpha$ by theorem 3.3 (iv)

So that $\vartheta(\alpha) = \alpha$

ii) when $\vartheta(1) \leq \alpha$ this implies $\vartheta(1) \wedge \alpha = \vartheta(1)$
and since $\vartheta(1) \wedge \alpha \leq \vartheta(1 \wedge \alpha) = \vartheta(\alpha)$
but $\vartheta(1) \wedge \alpha = \vartheta(1)$
hence $\vartheta(1) \leq \vartheta(\alpha)$

iii) when $\alpha \leq \tau$ then $\alpha \wedge \tau = \alpha$ and since $d(\tau) = \tau$, $\vartheta(\alpha) \leq \alpha$, $\alpha \leq \tau$ and $T(\alpha,\tau,\eta) \leq \alpha$, we have

$$\vartheta(\alpha) = \vartheta(\alpha \wedge \tau)$$
$$= (\vartheta(\alpha) \wedge \beta) \vee (\mathrm{x} \wedge d(\tau)) \vee T(\alpha,\alpha,\tau) \vee T(\alpha,\tau,\tau)$$
$$= \vartheta(\alpha) \vee \alpha \vee \alpha \vee \alpha$$
$$= \vartheta(\alpha) \vee \alpha = \alpha$$

Definition3.11 : let $(\Gamma,\wedge,\vee)$ be a lattice , the mapping $\kappa: \Gamma \times \Gamma \times \Gamma \to \Gamma$ in which satisfying $\kappa(\alpha \vee \tau, \eta, u) = \kappa(\alpha,\eta,u) \vee \kappa(\tau,\eta,u)$ for all $\alpha,\tau,\eta,u \in \Gamma$ is called joinitive mapping .

Theorem3.12: let $(\Gamma,\wedge,\vee)$ be a lattice , the mapping $\kappa: \Gamma \times \Gamma \times \Gamma \to \Gamma$ be a joinitive mapping uith the trace ϑ , then

i) $\vartheta(\alpha \vee \tau) = \vartheta(\alpha) \vee \vartheta(\tau) \vee \kappa(\alpha,\alpha,\tau) \vee \kappa(\alpha,\tau,\tau)$

ii) $\vartheta(\alpha) \vee \vartheta(\tau) \leq \vartheta(\alpha \vee \tau)$

For all $\alpha,\tau \in \Gamma$

Proof :

i) For all $\alpha,\tau \in \Gamma$ ue have

$$\vartheta(\alpha \vee \tau) = \kappa(\alpha \vee \tau, \alpha \vee \tau, \alpha \vee \tau)$$
$$= \kappa(\alpha, \alpha \vee \tau, \alpha \vee \tau) \vee \kappa(\tau, \alpha \vee \tau, \alpha \vee \tau)$$
$$= \kappa(\alpha,\alpha,\alpha \vee \tau) \vee \kappa(\alpha,\tau,\alpha \vee \tau) \vee \kappa(\tau,\alpha,\alpha \vee \tau) \vee \kappa(\tau,\tau,\alpha \vee \tau)$$
$$= \kappa(\alpha,\alpha,\alpha) \vee \kappa(\alpha,\alpha,\tau) \vee \kappa(\alpha,\tau,\alpha) \vee \kappa(\alpha,\tau,\tau)$$
$$\vee \kappa(\tau,\alpha,\alpha) \vee \kappa(\tau,\alpha,\tau) \vee \kappa(\tau,\tau,\alpha) \vee \kappa(\tau,\tau,\tau)$$
$$= \vartheta(\alpha) \vee \vartheta(\tau) \vee \kappa(\alpha,\alpha,\tau) \vee \kappa(\alpha,\tau,\tau)$$

ii) Directly from (i)

Theorem3.12: let κ_1 and κ_2 are two generalized permuting tri – derivations related to the same permuting tri – derivation T on the distributive lattice . Then the mapping $\kappa_1 \wedge \kappa_2$ defined by

$$(\kappa_1 \wedge \kappa_2)(\alpha,\tau,\eta) = \kappa_1(\alpha,\tau,\eta) \wedge \kappa_2(\alpha,\tau,\eta) \quad \text{for all } \alpha,\tau,\eta \in \Gamma$$

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Is generalized permuting tri – derivation related to apermuting tri – derivation T .

Proof : let $\alpha,\tau,\eta\ ,u \in \Gamma$

$$(\kappa_1 \wedge \kappa_2)(\alpha \wedge u,\tau,\eta) = \kappa_1(\alpha \wedge u,\tau,\eta) \wedge \kappa_2(\alpha \wedge u,\tau,\eta)$$

$$= [(\kappa_1(\alpha,\tau,\eta) \wedge u) \vee (\alpha \wedge T(u,\tau,\eta)]$$

$$\wedge [(\kappa_2(\alpha,\tau,\eta) \wedge u) \vee (\alpha \wedge T(u,\tau,\eta)]$$

$$= [(\kappa_1(\alpha,\tau,\eta) \wedge u) \wedge (\kappa_2(\alpha,\tau,\eta) \wedge u)] \vee (\alpha \wedge T(u,\tau,\eta)$$

$$= \left[\left(\kappa_1(\alpha,\tau,\eta) \wedge \kappa_2(\alpha,\tau,\eta)\right) \wedge u\right] \vee (\alpha \wedge T(u,\tau,\eta)$$

$$= [(\kappa_1 \wedge \kappa_2)(\alpha,\tau,\eta) \wedge u] \vee (\alpha \wedge T(u,\tau,\eta)$$

So that $\kappa_1 \wedge \kappa_2$ is generalized permuting tri – derivation related to a permuting tri – derivation T on a lattice .

Theorem 3.13: let κ_1 and κ_2 are two generalized permuting tri – derivations related to the same permuting tri – derivation T on the distributive lattice . Then the mapping $\kappa_1 \vee \kappa_2$ defined by

$$(\kappa_1 \vee \kappa_2)(\alpha,\tau,\eta) = \kappa_1(\alpha,\tau,\eta) \vee \kappa_2(\alpha,\tau,\eta) \quad \text{for all } \alpha,\tau,\eta \in \Gamma$$

Is generalized permuting tri – derivation related to apermuting tri – derivation T .

Proof : let $\alpha,\tau,\eta\ ,u \in \Gamma$

$$(\kappa_1 \vee \kappa_2)(\alpha \wedge u,\tau,\eta) = \kappa_1(\alpha \wedge u,\tau,\eta) \vee \kappa_2(\alpha \wedge u,\tau,\eta)$$

$$= [(\kappa_1(\alpha,\tau,\eta) \wedge u) \vee (\alpha \wedge T(u,\tau,\eta)]$$

$$\vee [(\kappa_2(\alpha,\tau,\eta) \wedge u) \vee (\alpha \wedge T(u,\tau,\eta)]$$

$$= [(\kappa_1(\alpha,\tau,\eta) \wedge u) \vee (\kappa_2(\alpha,\tau,\eta) \wedge u)] \vee (\alpha \wedge T(u,\tau,\eta)$$

$$= \left[\left(\kappa_1(\alpha,\tau,\eta) \vee \kappa_2(\alpha,\tau,\eta)\right) \wedge u\right] \vee (\alpha \wedge T(u,\tau,\eta)$$

$$= [(\kappa_1 \vee \kappa_2)(\alpha,\tau,\eta) \wedge u] \vee (\alpha \wedge T(u,\tau,\eta)$$

So that $\kappa_1 \vee \kappa_2$ is generalized permuting tri – derivation related to a permuting tri – derivation T on a lattice .

Definition3.13 : let $(\Gamma,\wedge,\vee)$ be a lattice , κ be a generalized permuting tri – derivation related to a permuting tri – derivation T , ϑ is the trace of κ , ϑ is called isotone mapping if when $\alpha \leq \tau$ implies $\vartheta(\alpha) \leq \vartheta(\tau)$.

Theorem3.14: : let κ_1 and κ_2 are tuo generalized permuting tri – derivations related to the same permuting tri – derivation T on the distributive lattice Γ , and ϑ_1 , ϑ_2 are the traces of κ_1 and κ_2 respectively .If ϑ_1 and ϑ_2 are isotone mapping , then $\vartheta_1 = \vartheta_2$ if and only if $Fi\alpha_{\vartheta_1}(\Gamma) = Fi\alpha_{\vartheta_2}(\Gamma)$.

Proof : suppose that $\vartheta_1 = \vartheta_2$ then

$$Fi\alpha_{\vartheta_1}(\Gamma) = \{\alpha \in \Gamma | \vartheta_1(\alpha) = \alpha\}$$

$$= \{\alpha \in \Gamma | \vartheta_2(\alpha) = \alpha\} \quad \text{since } \vartheta_1 = \vartheta_2$$

$$= Fi\alpha_{\vartheta_2}(\Gamma)$$

Conversely if $Fi\alpha_{\vartheta_1}(\Gamma) = Fi\alpha_{\vartheta_2}(\Gamma)$

Suppose that $\alpha \in Fi\alpha_{\vartheta_1}(\Gamma) \Rightarrow \vartheta_1(\alpha) = \alpha$

and $\vartheta_1(\vartheta_1(\alpha)) = \vartheta_1(\alpha)$

$\Rightarrow \vartheta_1(\alpha) \in Fi\alpha_{\vartheta_1}(\Gamma)$

but $Fi\alpha_{\vartheta_1}(\Gamma) = Fi\alpha_{\vartheta_2}(\Gamma)$

so that $\vartheta_1(\alpha) \in Fi\alpha_{\vartheta_2}(\Gamma)$

hence $\vartheta_2(\vartheta_1(\alpha)) = \vartheta_1(\alpha)$

by the same way we can prove that $\vartheta_1(\vartheta_2(\alpha)) = \vartheta_2(\alpha)$

since $\vartheta_1(\alpha) \leq \alpha$ and $\vartheta_2(\alpha) \leq \alpha$ and since ϑ_1 and ϑ_2 are isotone ,we can conclude

$\vartheta_2(\vartheta_1(\alpha)) \leq \vartheta_1(\alpha)$ and $\vartheta_1(\vartheta_2(\alpha)) \leq \vartheta_2(\alpha)$

but $\vartheta_2(\vartheta_1(\alpha)) = \vartheta_1(\alpha)$ and $\vartheta_1(\vartheta_2(\alpha)) = \vartheta_2(\alpha)$

hence $\vartheta_2(\vartheta_1(\alpha)) \leq \vartheta_1(\vartheta_2(\alpha))$ and $\vartheta_1(\vartheta_2(\alpha)) \leq \vartheta_2(\vartheta_1(\alpha))$

so $\vartheta_1(\alpha) = \vartheta_2(\alpha)$ for all $\alpha \in \Gamma \Rightarrow \vartheta_1 = \vartheta_2$.

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