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Some Results On Cayley Graph

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ABSTRACT

Cayley graph has been introduced by A . Cayley , which is point – symmetric . However in this paper , I have found another type of symmetric graph , which is not Cayley graph . This graph is called peterson graph with 10 vertices is proposed

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1 - Introduction:

In recent years ,graph theory has established itself as an important mathematical tool in a wide variety of subjects, ranging from operational research and chemistry to genetics and linguistics, and from electrical engineering and geography to sociology and architecture .At the same time it has also emerged as a worthwhile mathematical discipline in its own right.

A . Cayley gave a process to construct the graphes . For that we consider here the finite graph Let G be any finite group . G = { X1,X2 …Xp } G Is generated by the element S1,S2,…. Sk (i,e) each element of G can be written as a product of finite number of generators and their inverses the identity is not include in the set of generators we associate to G and S one graph , noted by Cay (G,S)

Then the set of vertices correspond to the set of G elements and defined in the following form let Vi be a vertex of cay (G,S) corresponding to Xi for i=1,2, …, p the pair (V1,V2) is an arc of cay (G,S) if

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and only if there exists si € S such that X2=X1 Si or this gives a graph of Cayley of G related to S and take different values obtain different graphes of Cayley for the same group

The sympol S is included in the notation Cay (G,S) to indicate that the ordinary graph of Cayley depends on the set S and this set S is a generator of G .

Generally let G be a finite group and A is any part of G- {1} , A is not necessary a set of generators we replace S by A when A is the set of generators the graph of Cayley is defined by A is then given by Cay (G,A) = (G,E) where E ={(X,Y) / Y € A } let Ta be a left translation defined by Ta = x → ax

Where a € G Then we have

(X,Y) € E ↔ Y € A

↔ . (ay) € A

↔ ($T_{a\,x}$, $T_{a\,y}$) € E

C.delome gave a process to constract of graphs which is point symmetric graph , let G be any finite group H subgroup of G and A is subset of G the graph [G,H,A] which G acts transively by automorphisms in the following method . A graph automorphism is a type of symmetry in which the graph is mapped onto itself

The vertices of graph are the parts of G in the form xH number is the index of H in G neighbor of xH are xaH , where a € A we say that y in neighbor of x if {x,y} is is an arc when H is reduced to a natural element of the group we find Cayley graph associate to group G and a part A,G give the automorphism of [G,H,A] if g € G , then

xH →gxH in an automorphism

2 - Definitions :

A graph G is the pair(V,E) where V(G) is the set of-vertices and E(G) is the set of arcs such that the pair (x,y) with G (x,y) = + is an arc and the G (y,x) = + , the pair {x,y} is an arc , the graph is finite if it is number of vertices is finite , the graph in which there does not have the ~~arêtes~~ is called symmetric .

The graph is point – symmetric when - for all x,y€,E (the base of G)

Their exists an automorphism of G such that f(X)=Y.

Generally , let G be a group and A is any part from G –{1}. A is not a necessary set of generators by relocated S in stade of A when A is the set of generators Cayley graph , defined by A : cay (G,A) = (G,E) where

E ={(X,Y) / Y € A

Let Ta the left translation defined by Ta = x → ax

Where a € G Then we have

(X,Y) € E ↔ Y € A

↔ . (ay) € A

↔ ($T_{a\,x}$, $T_{a\,y}$) € A

The oriented path of G T_{ax} is a sequence of a T_{ay} vertices X0,X1 , … , Xn such that G(Xi , Xi+1) = + for all I

The graph is called strong when for all x,y : there exists a path from x to y . We have the following lemma

Lemma 1

The left translations of G are the automorphisms of cay (G,S)

They cay (G,S) is point symmetric let X € G the factorization of x with respect to S is a sequence

{ Si ;1≤ I ≤ K } of elements of S such that X = $\pi_{1\leq i\leq K}$ Si

Where K is length of the factorization , we have the following lemma

Lemma 2

Let G is finite group . SC G-{1} and X € G

Then the factorization X= $\pi_{1\leq i \leq K}$

Is defined the path of length K from 1 to X

And inversely each path from 1 to X defines a factorization of X with respect to S

Proof

Put xo =1 , Xj= $\pi_{1\leq i \leq K}$ si let x= $\pi_{1\leq i \leq K}$ si . because X_j +1 =xjsj+1, (Xi,Xj +1) is an arc then (xo,x1) (X_1,X_2) … (X_K-1 , X_K) is a path reach connects 1 to x

Inversely , let (1,X_1) (X_1,X_2) … (X_K-1 , x) is a path reach connects 1 to x we have by definition an arc Xj+1=xjsj+1 when S_j+1 € S

Which given x = xk = (X_K-1)Sk and by induction we get x= $\pi_{1\leq i \leq K}$ si

Lemma 3

Let G be point symmetric graph and a € v , if for all x € G there exists a path connects a to X then the graph G is strong

In particular, Cay (G,S) is strong if and only if for all x€G , their exist a path form 1 to x

Theorem

Cay (G,A) is strong if only if A is the set of generators of G

Proof

Suppose cay (G,A) is strong , and let x € G by the above lemma , there exists the path connects 1 to x then there exist a factorization of x with respect to A (by lemma 2) x=S_1,S_2, …, S_k where Si € A for 1≤ i ≤K then A is the set of generators

.

Inversely

Suppose that A is the set of generators (we replace A by S) and let x€G.

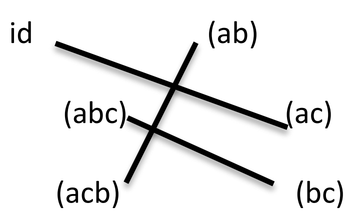
,Such that $X=\prod_{1\leq I\leq K}$ ti fnite if it put ti = There exist a sequence {ti} 1≤ i ≤K , ti

Because G is finite , there exist a natura P such that =1 we have ti= =

By changing all ti € by the expression of the form where s €S , we obtain a factorization of x with respect to S , this prove that cay (G1,S) is strong (by lemma3)

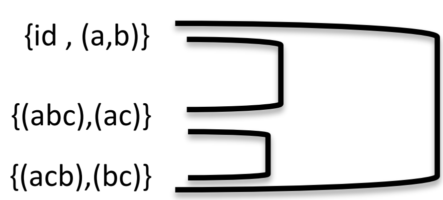
Example G is the group of permutations for the set {a,b,c} A forms the permutation (ac) : H is [id} for the first case , H is {di , (ab)}for the second

First case



[S $_{\{a,b,c\}}$, {id} , {(ac)}]

Second case

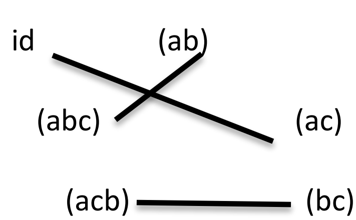


[S $_{\{a,b,c\}}$, {id , (ab)} ,{ (ac)}] .

Remark

We note that iff we connect xH to a xH we get thee? shreier graph which is the graph not in general point symmetric for example [G,H,A] are take as follow

First case



H= {id}

{id , (a,b)}

Second case {(abc),(ac)}

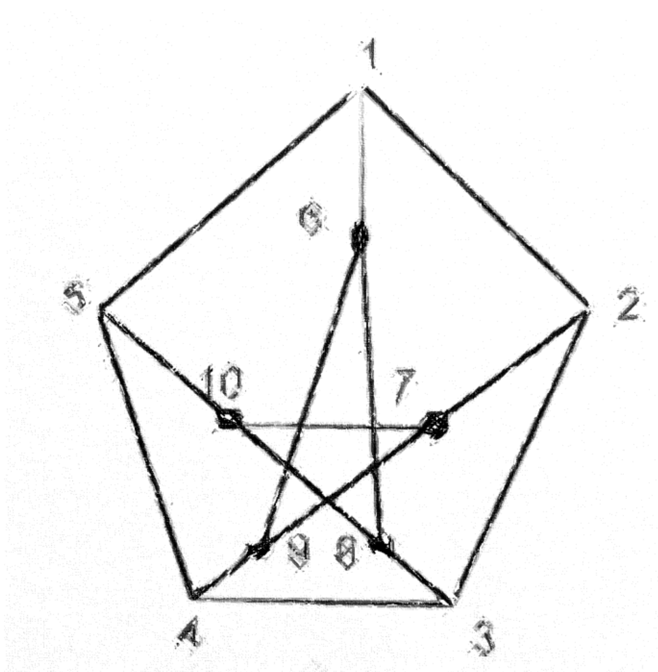
{(acb),(bc)}

[S $_{\{a,b,c\}}$, {id , (ab)} ,{ (ac)}] .

H= {id , (ab) }

Remark

There exist graphs which is point - symmetric other than cayley graph for example , Peterson graph with 10 elements .



Conclusion

We can conclude from this study that there are many graphs which are point-symmetric other than Cayley graphs.These new graph from this study are Petersen

Graph and shreier graph.

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