

8-15-2021

## $D^{**} \cap \mu$ -Closed Set in Supra topological Spaces

Bushra Jaralla Tawfeeq

Department of Mathematics, College of Education , AL-Mustansiriyah University, Baghdad, Iraq,  
bushramath1966@gmail.com

Follow this and additional works at: <https://qjps.researchcommons.org/home>

---

### Recommended Citation

Tawfeeq, Bushra Jaralla (2021) " $D^{**} \cap \mu$ -Closed Set in Supra topological Spaces," *Al-Qadisiyah Journal of Pure Science*: Vol. 26: No. 4, Article 38.

DOI: 10.29350/qjps.2021.26.4.1349

Available at: <https://qjps.researchcommons.org/home/vol26/iss4/38>

This Article is brought to you for free and open access by Al-Qadisiyah Journal of Pure Science. It has been accepted for inclusion in Al-Qadisiyah Journal of Pure Science by an authorized editor of Al-Qadisiyah Journal of Pure Science. For more information, please contact [bassam.alfarhani@qu.edu.iq](mailto:bassam.alfarhani@qu.edu.iq).



# Al-Qadisiyah Journal of Pure Science

ISSN(Printed): 1997-2490

ISSN(Online): 2411-3514

DOI: /10.29350/jops.

<http://qu.edu.iq/journalsc/index.php/IOPS>



## $D^{**\mu}$ -Closed Set in Supra topological Spaces

Authors Names	ABSTRACT
<p>Bushra Jaralla Tawfeeq</p> <p><b>Article History</b>                      Received on: 11/6/2021                      Revised on: 30/7/2021                      Accepted on: 13/8/2021</p> <p><b>Keywords:</b>                      Supra closed set                      Supra <math>D^{**}</math>-closed set                      Supra <math>D^{**}</math>-open set                      Supra topological space</p> <p><b>DOI:</b>  <a href="https://doi.org/10.29350/jops.2021.26.4.1349">https://doi.org/10.29350/jops.2021.26.4.1349</a></p>	<p>The main idea of this present work is to introduce new types of supra closed sets in a supra topological spaces called supra <math>D^{**}</math>-closed(briefly, <math>D^{**\mu}</math> –closed). Moreover, comparison this class of sets with other types of supra sets in supra topological spaces. Study and demonstrate some of their properties.</p>

## Introduction

Anuradha N, Chacko B. in [1] have provided a new type of supra sets named supra R-open sets. Arockiarani I, Pricilla M. in [2] defined supra  $\pi\Omega$  closed. Gnanambal I, Vidhya M. in [3], they have defined a suprageneralized pre regular closed sets. Later, Hussain AK, Tawfeeq BJ. in [4] they had a discussion about the definition of generalized gs-closed sets, Kamaraj M, Ramkumar G. in [5] they have showed and explored the supra sg closed sets and supra gs closed sets and clarified the ways in which they are different or similar to other kinds of closed sets in supra topological spaces.

Mashhour in 1983 [6] explained properties of supra closed set, also studied S-continuous and  $S^*$ -continuous functions. The arbitrary union is the enough condition to get a supratopological space and every topological space is a supratopological space but not conversely. A few properties of these concepts had been examined and confirmed. New results in supratopological had been reported by Meera B, Vijayalakshmi R. in [7] and Ravi O, Ramkumar G. in [8].

More recently, Saroja G, Thiripurasundari V. in [9] , Selvaraj R, Chandrasekar S. in [10] they have explained the class of  $\mu$ -closed sets and supra $\mu$ -g- closed set respectively and studied their properties and characterizations.

### **Preliminaries:**

Now , reviewing the following definitions which are important in the research, and that will be used in the next sections:

#### **Definition 1:[6].**

The spaces considered are supratopological spaces.

$(X, \mu)$  is said to be a supratopological space if it satisfies the following conditions:

(i)  $X, \varphi \in \mu$

(ii) If  $Z_i \in \mu \quad \forall i \in \mathbb{J}$  then  $\cup Z_i \in \mu$

$(X, \mu)$  is called supratopological space.

The elements of  $\mu$  are called supra open in  $(X, \mu)$  and the complement of these elements are called supra closed.

Throughout this paper  $X, \eta$  and  $\rho$  denoted the supratopological spaces

$(X, \mu), (\mathcal{Y}, \mathcal{T})$  and  $(Z, \eta)$  respectively, which no separation axioms are assumed.

#### **Definition 2:[6].**

(i) The supra closure of  $M$  is denoted by  $cl^\mu(M) = \cap \{N : N \text{ is supra closed and } M \subseteq N\}$

(ii) The supra interior of  $M$  is denoted by  $Int^\mu(M) = \cup \{N : N \text{ is supra open and } N \subseteq M\}$

**Definition 3:** A subset  $M$  of a supratopological space  $X$  is called:

(1) a supra  $\alpha$ - closed (5) if  $cl^\mu(Int^\mu(cl^\mu(M))) \subseteq M$ .

(2) a supra semi -closed (5) if  $Int^\mu(cl^\mu(M)) \subseteq M$ .

(3) a supra  $\beta$ -closed (8) (or semi-pre closed) if  $Int^\mu(cl^\mu(Int^\mu(M))) \subseteq M$ .

(4) a supra regular -closed (1) if  $M = cl^\mu(Int^\mu(M))$ , and

(5) a supra pre -closed (8) if  $cl^\mu(Int^\mu(M)) \subseteq M$ .

**Definition 4:** A subset  $M$  of a supratopological space  $X$  is called:

1) a suprageneralized  $\beta$ -closed (briefly,  $g\beta^\mu$ -closed) (8) if  $\beta cl^\mu(M) \subseteq N$  whenever  $M \subseteq N$  and  $N$  is supra open in  $X$ ,

2) a suprageneralized  $p$ -closed (briefly,  $gp^\mu$ -closed) (3) if  $pcl^\mu(M) \subseteq N$  whenever  $M \subseteq N$  and  $N$  is supra open in  $X$ ,

3) a supra  $\Omega$ - closed (briefly,  $\Omega^\mu$ - closed) (2) if  $Scl^\mu(M) \subseteq Int^\mu(N)$  (2) whenever  $M \subseteq N$  and  $N$  is supra open in  $X$ ,

4)  $sg^\mu$ -closed (5) if  $scl^\mu(M) \subseteq N$  whenever  $M \subseteq N$  and  $N$  is supra semi - open in  $X$ ,

5)  $gs^\mu$ -closed (5) if  $scl^\mu(M) \subseteq N$  whenever  $M \subseteq N$  and  $N$  is supra open in  $X$ ,

6) a supra  $\alpha g^{**}$  - closed (briefly,  $\alpha g^{**\mu}$ -closed) (7) if  $\alpha cl^\mu(M) \subseteq Int^\mu(N)$  whenever  $M \subseteq N$  and  $N$  is supra open in  $X$ ,

7) a supra  $\check{g}$  - closed (briefly,  $\check{g}^\mu$ -closed) (7) if  $cl^\mu(M) \subseteq N$  whenever  $M \subseteq N$  and  $N$  is supra  $sg$  open in  $X$ ,

8) a supra  $\omega$  - closed (briefly,  $w^\mu$ -closed) (5) if  $cl^\mu(M) \subseteq N$  whenever  $M \subseteq N$  and  $N$  is supra semi open in  $X$ ,

9)  $Pr^\mu$ -closed (9) if  $pcl^\mu(M) \subseteq N$  whenever  $M \subseteq N$  and  $N$  is supra regular semi open in  $X$ ,

10) a suprageneralized  $gs$ - closed (briefly,  $ggs^\mu$ -closed) (4) if  $cl^\mu(M) \subseteq N$  whenever  $M \subseteq N$  and  $N$  is supra  $gs$ -open in  $X$ , and

**11)** a supra  $g$ -closed (briefly,  $g^\mu$ -closed) (10) if  $cl^\mu(M) \subseteq N$  whenever  $M \subseteq N$  and  $N$  is supra  $\omega$  open in  $\mathcal{X}$ .

**12)** A subset  $M$  of a supratopological space  $(\mathcal{X}, \mu)$  is called supraregular semi open(5) if for every supraregular - open set  $N$  such that  $N \subseteq M \subseteq cl^\mu(N)$

### $D^{**\mu}$ -Closed Set:

Now, a new type of supra closed set, namely  $D^{**\mu}$ -closed set with few important properties has been tackled in the current section:

**Definition 5:** A subset  $M$  of a supratopological space  $\mathcal{X}$  is called a supraomega generalized  $\beta$ -closed (briefly,  $D^{**\mu}$ -closed) if  $\beta cl^\mu(M) \subseteq int^\mu(N)$  whenever  $M \subseteq N$  and  $N$  is  $\Omega^\mu$ -open in  $\mathcal{X}$ .

The complement of  $D^{**\mu}$ -closed is called  $D^{**\mu}$ -open.

Family of all  $D^{**\mu}$  open subsets of  $\mathcal{X}$  is denoted by  $D^{**\mu}O(\mathcal{X}, \mu)$  or  $D^{**\mu}O(\mathcal{X})$ .

### **Proposition 1:**

- I)** Every supra closed subset of  $\mathcal{X}$  is  $D^{**\mu}$ -closed,
- II)** Every  $sg^\mu$ -closed,  $gs^\mu$ -closed subset of  $\mathcal{X}$  is  $D^{**\mu}$ -closed,
- III)** Every  $ggs^\mu$ -closed subset of  $\mathcal{X}$  is  $D^{**\mu}$ -closed,
- IV)** Every  $w^\mu$ -closed subset of  $\mathcal{X}$  is  $D^{**\mu}$ -closed

So, the converse of statements are false.

**Proof. I)** Let  $S$  be any supra closed and  $\mathcal{V}$  be any  $\Omega^\mu$ -open containing  $\mathcal{M}$  in  $\mathcal{X}$ . Since  $S$  is supra closed, it follows that  $cl^\mu(S) = S$  and  $\beta cl^\mu(S) \subseteq cl^\mu(S) \subseteq S \subseteq \mathcal{V}$ . Hence  $S$  is  $D^{**\mu}$ -closed.

**Proof. II)** Let  $Z$  be any  $sg^\mu$ -closed( $gs^\mu$ -closed) subset of  $\mathcal{X}$  and  $S$  be any  $\Omega^\mu$ -open containing  $Z$  in  $\mathcal{X}$ . Since  $Z$  is  $sg^\mu$ -closed, it follows that  $scl^\mu(Z) \subseteq P$ , also  $\beta cl^\mu(Z) \subseteq scl^\mu(Z) \subseteq Z \subseteq P$ . Hence  $Z$  is  $D^{**\mu}$ -closed.

**Proof. III)** Let  $H^*$  be any  $sg^\mu$ -closed( $gs^\mu$ -closed) subset of  $\mathcal{X}$ , since every supra  $ggs^\mu$ -closed subset of  $\mathcal{X}$  is  $sg^\mu$ -closed( $gs^\mu$ -closed) (4) implies that  $H^*$  is,  $ggs^\mu$ -closed subset of  $\mathcal{X}$ , by (2), then  $H^*$  is  $D^{**\mu}$ -closed.

The proof of (IV) is similar to those of part (I), (II). Example 1 demonstrates that the converse is false.

### **Example 1:**

**1)** Let  $\mathcal{X} = \{q_1, q_2, q_3, q_4\}$  and  $\tau^\mu = \{\mathcal{X}, \emptyset, \{q_3\}, \{q_4\}, \{q_3, q_4\}\}$ , the set  $\{q_1, q_4\}$  is  $D^{**\mu}$ -closed but not supra closed

**2)** Let  $\mathcal{X} = \{g_1, g_2, g_3\}$  and  $\tau^\mu = \{\mathcal{X}, \emptyset, \{g_1, g_3\}\}$ , the subset  $\{g_1\}$  is  $D^{**\mu}$ -closed but not  $sg^\mu$ , neither  $gs^\mu$ -closed nor  $ggs^\mu$ -closed.

**3)** Let  $\mathcal{X} = \{g_1, g_2, g_3\}$  and  $\tau^\mu = \{\mathcal{X}, \emptyset, \{g_1\}, \{g_2\}, \{g_1, g_2\}, \{g_1, g_3\}\}$ , the subset  $\{g_1\}$  is  $D^{**\mu}$ -closed but not  $w^\mu$ -closed.

**Proposition 2:** If  $P$  is  $D^{**\mu}$ -closed subset of  $\mathcal{X}$ , then  $P$  is  $g\beta^\mu$ -closed.

**Proof.** Suppose that  $P$  be any  $D^{**\mu}$ -closed and  $H$  be any supra open such that  $P \subseteq H$  in  $\mathcal{X}$ . Since  $P$  is  $D^{**\mu}$ -closed. Then  $\beta cl^{\tau^\mu}(P) \subseteq H$ . Therefore  $P$  is  $g\beta^\mu$ -closed. Example 2 demonstrates that the converse is false.

**Example 2:** Let  $\mathcal{X} = \{g_1, g_2, g_3\}$  and  $\mu = \{\mathcal{X}, \varphi, \{g_2\}\}$ . Then the subset  $\{g_2, g_3\}$  is  $g\beta^\mu$ -closed but not  $D^{**\mu}$ -closed.

**Remark 1:** The concept of  $D^{**\mu}$ -closed is independent of those of supra  $\alpha g^{**\mu}$ -closed (resp.,  $gp^\mu$ -closed,  $Pr^\mu$ -closed and  $^*g^\mu$ -closed), as can be seen from Examples 3-6.

**Example 3:** Let  $\mathcal{X} = \{g_1, g_2, g_3\}$  and  $\mu = \{\mathcal{X}, \varphi, \{g_2\}, \{g_2, g_3\}\}$ . Then the subset  $\{m\}$  is  $D^{**\mu}$ -closed but it is neither  $\alpha g^{**\mu}$ -closed nor  $^*g^\mu$ -closed and the supra subset  $\{g_1, g_2\}$  is  $\alpha g^{**\mu}$ -closed and  $^*g^\mu$ -closed but is not  $D^{**\mu}$ -closed.

**Example 4:** Let  $\mathcal{X} = \{g_1, g_2, g_3, g_4\}$  and  $\mu = \{\mathcal{X}, \varphi, \{g_3\}, \{g_4\}, \{g_3, g_4\}\}$ , the supra set  $\{g_1, g_4\}$  is  $D^{**\mu}$ -closed but not  $Pr^\mu$ -closed

**Example 5:** Let  $\mathcal{X}$  and  $\mu$  be as in Example 2, the set  $\{g_1\}$  is  $D^{**\mu}$ -closed but not  $gp^\mu$ -closed.

**Example 6:** Let  $\mathcal{X} = \{g_1, g_2, g_3\}$  and  $\mu = \{\mathcal{X}, \varphi, \{g_1, g_3\}\}$ , the set  $\{g_1, g_2\}$  is  $gp^\mu$ -closed and  $Pr^\mu$ -closed but not  $D^{**\mu}$ -closed.

**Proposition 3:** If  $\mathcal{H}$  is  $g\cdot^\mu$ -closed subset of  $\mathcal{X}$  then  $\mathcal{H}$  is  $D^{**\mu}$ -closed. Example 7 demonstrates that the converse is false.

**Example 7:** Let  $\mathcal{X}$  and  $\mu$  be as in Example 3. Then the subset  $\{g_3\}$  is  $D^{**\mu}$ -closed but not  $g\cdot^\mu$ -closed

**Proposition 4:**

- I) Every supra regular closed is  $D^{**\mu}$ -closed,
  - II) Every supra regular semi-closed is  $D^{**\mu}$ -closed,
  - III) Every supra pre-closed is  $D^{**\mu}$ -closed, and
- The converse statements are false.

**Proof .I)** Let  $M^*$  be supra regular -closed in  $\mathcal{X}$ . Let  $U$  be a  $\Omega^\mu$ -open such that  $M^* \subseteq U$ . Since  $M^*$  is supra regular -closed, then  $\text{rcl}(M^*) = M^* \subseteq U$ . But,  $\beta \text{cl}(M^*) \subseteq \text{rcl}(M^*) \subseteq U$ . Therefore,  $\beta \text{cl}(M^*) \subseteq U$ . Hence  $M^*$  is  $D^{**\mu}$ -closed.

The proofs of (II) and (III) are similar. Example 8 demonstrates that the converse statements are false.

**Example 8:**

I) Let  $\mathcal{X} = \{g_1, g_2, g_3, g_4\}$  and  $\mu = \{\mathcal{X}, \varphi, \{g_1\}, \{g_2\}, \{g_1, g_2\}\}$ . Then the supra subset  $\{g_3\}$  is  $D^{**\mu}$ -closed but not supra regular closed.

II) Let  $\mathcal{X}$  and  $\mu$  be as in Example 3 Then the subset  $\{g_1\}$  is  $D^{**\mu}$ -closed but not supra regular semi-closed also the subset  $\{g_3\}$  is  $D^{**\mu}$ -closed but not supra pre-closed.

III) Let  $\mathcal{X}$  and  $\mu$  be as in Example 1 part (I). Then the subset  $\{g_1, g_4\}$  is  $D^{**\mu}$ -closed but not supra pre-closed.

**Proposition 5:** A subset  $M$  of a supratopological space  $\mathcal{X}$  is  $D^{**\mu}$ -open iff  $\mathcal{H} \subseteq \beta \text{Int}^\mu(M)$  whenever  $\mathcal{H} \subseteq M$  and  $\mathcal{H}$  is  $\Omega^\mu$ -closed.

**Proof.** Let  $H \subseteq M$  and  $H$  be a  $\Omega^\mu$ -closed of  $\mathcal{X}$ , which implies that  $H^c$  is  $\Omega^\mu$ -open and  $M^c \subseteq H^c$ . Also since  $M^c$  is  $D^{**\mu}$ -closed, then  $\beta \text{Int}^\mu(M^c) \subseteq H^c$  and  $H \subseteq \beta \text{Int}^\mu(M)$ .

Conversely: Suppose that  $A^*$  is  $\Omega^\mu$ -open of  $\mathcal{X}$  and  $M^c \subseteq A^*$ , it follows that  $A^{*c}$  is a  $\Omega^\mu$ -closed contained in  $M$  and  $A^{*c} \subseteq \beta \text{Int}^\mu(M)$ . Then  $(\beta \text{Int}^\mu(M))^c = \beta \text{cl}^\mu(M^c) \subseteq A^*$ . Therefore  $M^c$  is  $D^{**\mu}$ -closed and then  $M$  is  $D^{**\mu}$ -open.

**Proposition 6:** Let  $M$  be  $D^{**\mu}$ -closed subset of  $\mathcal{X}$ , then  $\beta\text{cl}^\mu(M)-M$  contains no non empty  $\Omega^\mu$ -closed.

**Proof.** Let  $H$  be a non empty supra  $\Omega^\mu$ -closed of  $\mathcal{X}$  such that  $H \subseteq \beta\text{cl}^\mu(M)-M$ , that mean  $H \subseteq \beta\text{cl}^\mu(M)$  and  $H \subseteq M^c$  also  $M \subseteq H^c$ . Thus  $\beta\text{cl}^\mu(M) \subseteq H^c$  and  $M$  is  $D^{**\mu}$ -closed. Then  $H \subseteq \beta\text{cl}^\mu(M) \cap (\beta\text{cl}^\mu(M))^c = \varnothing$ .

**Proposition 7:** Let  $K$  be a  $D^{**\mu}$ -closed subset of supratopological space  $\mathcal{X}$ , then  $K$  is supra  $\beta^\mu$ -closed iff  $\beta\text{cl}^\mu(K)-K$  is  $\Omega^\mu$ -closed.

**Proof.** Since  $K$  is  $\beta^\mu$ -closed, then  $\beta\text{cl}^\mu(K)=K$ , it follows  $\beta\text{cl}^\mu(K)-K=\varnothing$  which is  $\Omega^\mu$ -closed.

**Conversely:** let  $K$  be a  $D^{**\mu}$ -closed subset set of  $\mathcal{X}$  and  $\beta\text{cl}^\mu(K)-K$  is a  $\Omega^\mu$ -closed. Then by Proposition 6,  $\beta\text{cl}^\mu(K)-K=\varnothing$ . Thus  $K$  is  $\beta^\mu$ -closed.

**Proposition 8:** Let  $N \subseteq \mathcal{X}$ , if  $N$  is both  $\Omega^\mu$ -open and  $D^{**\mu}$ -closed, then  $N$  is  $\beta^\mu$ -closed in  $\mathcal{X}$ .

**Proof.** Suppose that  $N$  is a  $\Omega^\mu$ -open and  $D^{**\mu}$ -closed of  $\mathcal{X}$  then  $\beta\text{cl}^\mu(N) \subseteq N$ . Also  $N \subseteq \beta\text{cl}^\mu(N)$ . That means  $\beta\text{cl}^\mu(N) = N$ . Thus  $N$  is  $\beta$ -closed.

**Proposition 9:** If  $M$  is  $D^{**\mu}$ -closed and  $M \subseteq N \subseteq \beta\text{cl}^\mu(M)$ , then:

I)  $N$  is  $D^{**\mu}$ -closed.

II)  $\beta\text{cl}^\mu(N)-N$  contains no non empty  $\Omega^\mu$ -closed.

**Proof.**

1) Suppose that  $P$  is  $\Omega^\mu$ -open in  $\mathcal{X}$  such that  $N \subseteq P$ , it follows that  $M \subseteq P$ . Since  $M$  is  $D^{**\mu}$ -closed, which implies that  $\beta\text{cl}^\mu(M) \subseteq P$  and  $\beta\text{cl}^\mu(N) \subseteq \beta\text{cl}(\beta\text{cl}^\mu(M)) = \beta\text{cl}^\mu(M) \subseteq P$ . Hence  $\beta\text{cl}^\mu(N) \subseteq P$ .

2 Follows from Proposition 6.

**Remark 2:** The intersection of two  $D^{**\mu}$ -closed does not have to be  $D^{**\mu}$ -closed in general as explained in below example

**Example 9:** Let  $\mathcal{X} = \{g_1, g_2, g_3, g_4\}$  and  $\mu = \{\mathcal{X}, \varnothing, \{g_1\}, \{g_2\}, \{a_1, g_2\}\}$ . the subsets  $\{g_1, g_3, g_4\}$  and  $\{g_2, g_3, g_4\}$  are  $D^{**\mu}$ -closed but their intersection of  $\{g_1, g_3, g_4\}, \{g_2, g_3, g_4\} = \{g_3, g_4\}$  is not  $D^{**\mu}$ -closed.

**Remark 3:** The union of two  $D^{**\mu}$ -closed need not to be  $D^{**\mu}$ -closed in general that explained in the example below.

**Example 10:** Let  $\mathcal{X}$  and  $\mu$  be as in Example 3, the subsets  $\{g_1\}$  and  $\{g_3\}$  are  $D^{**\mu}$ -closed but their union  $\{g_1\} \cup \{g_3\} = \{g_1, g_3\}$  is not  $D^{**\mu}$ -closed.

**Proposition 10:** The intersection of  $D^{**\mu}$ -closed and  $\Omega^\mu$ -closed of  $\mathcal{X}$  is  $D^{**\mu}$ -closed.

**Proof.** Suppose that  $S$  is a  $D^{**\mu}$ -closed and  $P$  is  $\Omega^\mu$ -closed. If  $S \cap P \subseteq U^*$ ,  $U^*$  is a  $\Omega^\mu$ -open. Obtain  $S \subseteq U^* \cup P^c$  and  $U^* \cup P^c$  is  $\Omega^\mu$ -open. Since  $S$  is  $D^{**\mu}$ -closed, then  $\beta\text{cl}^\mu(S) \subseteq U^* \cup P^c$ , also  $\beta\text{cl}^\mu(S) \cap P \subseteq U^*$ . Furthermore,  $\beta\text{cl}^\mu(S \cap P) \subseteq \beta\text{cl}^\mu(S) \cap \beta\text{cl}^\mu(P) \subseteq \beta\text{cl}^\mu(S) \cap S \subseteq \beta\text{cl}^\mu(S) \cap P \subseteq U^*$ . Hence  $S \cap P$  is  $D^{**\mu}$ -closed.

**Proposition 11:** Let  $K$  be  $D^{**\mu}$ -closed and  $\Omega^\mu$ -open subset of  $\mathcal{X}$  and  $\mathcal{Y} \subseteq \mathcal{X}$  is  $\beta^\mu$ -closed then  $K \cap \mathcal{Y}$  is  $D^{**\mu}$ -closed.

**Proof.** Let  $K \subseteq \mathcal{X}$  be a  $\Omega^\mu$ -open and  $D^{**\mu}$ -closed. By Proposition 8,  $K$  is  $\beta^\mu$ -closed. This means that  $K \cap \mathcal{Y}$  is  $\beta^\mu$ -closed, it  $K \cap \mathcal{Y}$  is  $D^{**\mu}$ -closed, which completes the proof.

**proposition 12:** Let  $E^*$  be  $\alpha$ -open subspace of  $\mathcal{X}$  and  $N \subseteq E^*$ . If  $N$  is  $D^{**\mu}$ -closed in  $\mathcal{X}$ , then  $N$  is  $D^{**\mu}$ -closed in  $E^*$ .

**Proof:** Suppose that  $N \subseteq V$  where  $V$  be a  $\Omega^\mu$  –open of  $\mathcal{T}$ , which implies that  $V = E^* \cap P$  for some  $\Omega^\mu$  –open  $P$  of  $\mathcal{X}$ . Since  $N$  is  $D^{**\mu}$  -closed in  $\mathcal{X}$ , that means  $\beta cl^\mu(N) \subseteq P$  and  $\beta cl^\mu_{E^*}(N) = E^* \cap \beta cl^\mu(N) \subseteq E^* \cap P = V$ . Then  $N$  is  $D^{**\mu}$  -closed in  $E^*$ .

**Proposition 13:** If  $z^*$  is  $D^{**\mu}$  -closed, then  $\beta cl^\mu\{d\} \cap z^* \neq \varnothing$ , for every  $d \in \beta cl^\mu(z^*)$ .

**Proof.** Claiming, that  $\beta cl^\mu\{d\} \cap z^* = \varnothing$  and  $d \in \beta cl^\mu(z^*)$ , implies that  $z^* \subseteq (\beta cl^\mu\{d\})^c$  and  $(\beta cl^\mu\{d\})^c$  is  $\Omega^\mu$ -open. Since  $z^*$  is  $D^{**\mu}$  -closed, then  $\beta cl^\mu(z^*) \subseteq (\beta cl^\mu\{d\})^c$  and  $d \notin \beta cl^\mu(z^*)$  which is a contradiction. Hence  $\beta cl^\mu\{d\} \cap z^* \neq \varnothing$ .

**Remark 4:** In general if  $\mathcal{Y}$  is suprasubspace of a space  $\mathcal{X}$  and if  $S$  is  $D^{**\mu}$  - open in  $\mathcal{X}$ , then  $S \cap \mathcal{Y}$  may not be  $D^{**\mu}$  -open in  $\mathcal{Y}$ , as can be seen from Example 11.

**Example 11:** Let  $\mathcal{X} = \{u_1, u_2, u_3, u_4\}$  and  $\mu = \{\mathcal{X}, \varnothing, \{u_1\}, \{u_2\}, \{u_3\}, \{u_4\}, \{u_1, u_4\}, \{u_1, u_3\}, \{u_1, u_2, u_4\}\}$ , then  $D^{**\mu}O(\mathcal{X}) = \{\mathcal{X}, \varnothing, \{u_2\}, \{u_4\}, \{u_2, u_4\}, \{u_1, u_3, u_4\}, \{u_2, u_3, u_4\}\}$ . Let  $\mathcal{Y} = \{u_2, u_3, u_4\}$ ,  $\mathcal{T} = \{\mathcal{Y}, \varnothing, \{u_1\}\}$  and  $D^{**\mu}C(\mathcal{Y}) = \{\mathcal{Y}, \varnothing, \{u_1\}, \{u_4\}, \{u_1, u_2\}, \{u_1, u_4\}, \{u_2, u_4\}\}$ . Then  $\{u_1\} \in D^{**\mu}$  -closed in  $\mathcal{X}$  but  $\{u_2\} \cap \mathcal{Y} = \{u_2\} \notin D^{**\mu}$  -closed in  $\mathcal{Y}$ .

**Proposition 14:** For each  $g \in \mathcal{X}$ , the set  $\{g\}^c$  is  $D^{**\mu}$  -closed or  $\Omega^\mu$  –open.

**proof.** Suppose that  $\{g\}^c$  is not  $\Omega^\mu$  –open, it follows  $\mathcal{X}$  is the only  $\Omega^\mu$  –open containing  $\{g\}^c$ . Then  $\beta cl^\mu(\{g\}^c) \subseteq \mathcal{X}$ . Therefore  $\{g\}^c$  is  $D^{**\mu}$  -closed.

**Proposition 15:** Let  $S \subseteq \mathcal{X}$  be  $D^{**\mu}$  -closed, then  $\beta cl^\mu(S) - S$  is  $D^{**\mu}$  -open.

**proof.** Suppose that  $S$  is  $D^{**\mu}$  -closed,  $M$  is  $\Omega^\mu$  -closed in  $\mathcal{X}$  such that  $H \subseteq \beta cl^\mu(S) \cap M$ , by Proposition 9, then  $H = \varnothing$  and implies that  $H \subseteq \beta int^\mu(\beta cl^\mu(S) - S)$ . Then  $\beta cl^\mu(S) - S$  is  $D^{**\mu}$  -open.

**Definition 6:** The intersection of  $\Omega^\mu$ -open subsets of supra  $\mathcal{X}$  containing  $S$  is called  $\Omega^\mu$  Kernel of  $S$  and denoted by  $\ker_\Omega^\mu(S)$ .

**Proposition 16:** A subset  $S$  of  $\mathcal{X}$  is  $D^{**\mu}$  - closed iff  $\beta cl^\mu(S) \subseteq \ker_\Omega^\mu(S)$ .

**proof:** Let  $S$  be  $D^{**\mu}$  - closed in  $\mathcal{X}$  and  $n^* \in \beta cl^\mu(S)$ .

Let  $n^* \notin \ker_\Omega^\mu(S)$ , then a  $\Omega^\mu$ -open  $\mathcal{K}$  in  $\mathcal{X}$  such that  $S \subseteq \mathcal{K}$  and  $n^* \notin \mathcal{K}$ . Since  $S$  is  $D^{**\mu}$  -closed in  $\mathcal{X}$ , then  $\beta cl^\mu(S) \subseteq \mathcal{K}$ , so  $n^* \in \beta cl^\mu(S)$  which is a contradiction.

Conversely, let  $\beta cl^\mu(S) \subseteq \ker_\Omega^\mu(S)$  and  $\mathcal{K}$  is a  $\Omega^\mu$ -open containing  $S$ ,  $\ker_\Omega^\mu(S) \subseteq \mathcal{K}$  and  $\beta cl^\mu(S) \subseteq \mathcal{K}$ . Thus  $S$  is  $D^{**\mu}$  -closed.

### $D^{**}cl^\mu$ and $D^{**}int^\mu$ :

**Definition 7:** For  $S \subseteq \mathcal{X}$ ,  $D^{**}cl^\mu(S) = \bigcap \{F : S \subseteq F, F \text{ is } D^{**\mu} \text{-closed in } \mathcal{X}\}$

**Proposition 17:** Let  $S$  and  $H$  be subsets of a supratopological space  $\mathcal{X}$ , then:

- (I)  $D^{**}cl^\mu(\omega) = \omega$  and  $D^{**}cl^\mu(\varnothing) = \varnothing$
- (II) If  $S \subseteq H$ , then  $D^{**}cl^\mu(S) \subseteq D^{**}cl^\mu(H)$ .
- (III)  $D^{**}cl^\mu(S) \cup D^{**}cl^\mu(\mathcal{N}) \subseteq D^{**}cl^\mu(S \cup H)$ .
- (IV)  $D^{**}cl^\mu(S \cap H) \subseteq D^{**}cl^\mu(S) \cap D^{**}cl^\mu(H)$ .
- (V) If  $S$  is  $D^{**\mu}$  -closed, then  $D^{**}cl^\mu(S) = S$ , and

**Remark 5:** In general  $D^{**}cl^\mu(A \cup B) \neq D^{**}cl^\mu(A) \cup D^{**}cl^\mu(B)$  as demonstrated by Example 12.



**Example 12:** Let  $\mathcal{X}$  and  $\mu$  be as in Example( 1) part(II). Let  $A=\{g_1\}$ ,  $B=\{g_3\}$ , then  $D^{**}cl^\mu(A)=\{g_1\}$ ,  $D^{**}cl^\mu(B)=\{g_3\}$ , then  $D^{**}cl^\mu(A) \cup D^{**}cl^\mu(B)=\{g_1\} \cup \{g_3\}=\{g_1, g_3\}$ , but  $A \cup B=\{g_1\} \cup \{g_3\}=\{g_1, g_3\}$  and  $D^{**}cl^\mu(A \cup B)=D^{**}cl^\mu(\{g_1, g_3\})=\mathcal{X}$ . It follows that  $D^{**}cl^\mu(A \cup B) \neq D^{**}cl^\mu(A) \cup D^{**}cl^\mu(B)$

**Remark 6:** In general  $D^{**}cl^\mu(A \cap B) \neq D^{**}cl^\mu(A) \cap D^{**}cl^\mu(B)$  as demonstrated by Example 13.

**Example 13:** Let  $\mathcal{X}$  and  $\tau^\gamma$  be as in Example( 8) part(I),. If take  $A=\{g_1\}$ ,  $B=\{g_2\}$ , Then  $D^{**}cl^\mu(A)=\mathcal{X}$  and  $D^{**}cl^\mu(B)=\mathcal{X}$ , so  $D^{**}cl^\mu(A) \cap D^{**}cl^\mu(B)=\mathcal{X} \cap \mathcal{X}=\mathcal{X}$ .

but  $A \cap B=\{g_1\} \cap \{g_2\}=\varnothing$  and  $D^{**}cl^\mu(A \cap B)=D^{**}cl^\mu(\varnothing)=\varnothing$

As a consequence  $D^{**}cl^\mu(A \cap B) \neq D^{**}cl^\mu(A) \cap D^{**}cl^\mu(B)$

**Proposition 18:** Let  $\mathcal{H} \subseteq \mathcal{X}$ . Then  $m \in D^{**}cl^\mu(\mathcal{H})$ , then  $S \cap \mathcal{H} \neq \varnothing$  for every  $D^{**\mu}$ -open  $S$  containing  $m$ .  
**Proof.** Let  $m \in D^{**}cl^\mu(\mathcal{H})$  and there exists a  $D^{**\mu}$ -open  $S$  such that  $S \cap \mathcal{H}=\varnothing$ , then  $\mathcal{H} \subseteq S^c$  and  $S^c$  is  $D^{**\mu}$ -closed. It as follows  $D^{**}cl^\mu(\mathcal{H}) \subseteq S^c$ , so  $a1 \notin D^{**}cl^\mu(\mathcal{H})$ , This is a contradiction.

**Definition 8:** Let  $S \subseteq \mathcal{X}$ ,  $D^{**}int^\mu(S)=\cup\{G: G \subseteq S, G \text{ is } D^{**\mu}\text{-open in } \mathcal{X}\}$

**Proposition 19:** Let  $S$  and  $H \subseteq \mathcal{X}$ , then:

- (I)  $D^{**}int^\mu(\mathcal{X})=\mathcal{X}$  and  $D^{**}int^\mu(\varnothing)=\varnothing$ .
- (II) If  $S \subseteq H$ , then  $D^{**}int^\mu(S) \subseteq D^{**}int^\mu(H)$ .
- (III)  $D^{**}int^\mu(S) \cup D^{**}int^\mu(H) \subseteq D^{**}int^\mu(S \cup H)$ .
- (IV)  $D^{**}int^\mu(S \cap H) \subseteq D^{**}int^\mu(S) \cap D^{**}int^\mu(H)$ , and
- (V) If  $S$  is  $D^{**\mu}$ -open, then  $D^{**}int^\mu(S)=S$ .

**Remark 7:** In general  $D^{**}int^\mu(A \cup B) \neq D^{**}int^\mu(A) \cup D^{**}int^\mu(B)$  as can show in example 14

**Example 14:** Let  $\mathcal{X}$  and  $\mu$  be as in Example( 8) part I. Let  $A=\{g_1, g_2\}$ ,  $B=\{g_3\}$ , then  $D^{**}int^\mu(A)=\{g_1, g_2\}$ ,  $D^{**}int^\mu(B)=\varnothing$ , implies that  $D^{**}int^\mu(A) \cup D^{**}int^\mu(B)=\{g_1, g_2\} \cup \varnothing=\{g_1, g_2\}$ , but  $A \cup B=\{g_1, g_2\} \cup \{g_3\}=\mathcal{X}$  and  $D^{**}int^\mu(A \cup B)=D^{**}int^\mu(\mathcal{X})=\mathcal{X}$ .

It follows that  $D^{**}int^\mu(A \cup B) \neq D^{**}int^\mu(A) \cup D^{**}int^\mu(B)$

**Remark 8:** In general  $D^{**}int^\mu(A \cap B) \neq D^{**}int^\mu(A) \cap D^{**}int^\mu(B)$  as can be explained in example 15

**Example 15:** Let  $\mathcal{X}$  and  $\mu$  be as in Example( 8) part(I), if taking  $A=\{g_1, g_2\}$ ,  $B=\{g_1, g_3\}$ , Then  $D^{**}int^\mu(A)=A$  and  $D^{**}int^\mu(B)=\varnothing$ , then  $D^{**}int^\mu(A) \cap D^{**}int^\mu(B)=A \cap \varnothing=\varnothing$ .

but  $A \cap B=\{g_1, g_2\} \cap \{g_1, g_3\}=\{g_1\}$  and

$$D^{**}int^\mu(A \cap B)=D^{**}int^\mu(\{g_1\})=\{g_1\}$$

It follows that  $D^{**}int^\mu(A \cap B) \neq D^{**}int^\mu(A) \cap D^{**}int^\mu(B)$



## References

- [1] N. Amerada and B.Chock ,On Supra R-open sets and Supra R-continuity. J. of computer and Mathematical Sci., 7(8) (2016), 412-419.
- [2] I. Arockiarani and M.Pricilla ,  $\pi\Omega$  closed set and  $\pi\Omega$ s in supra topological Space. Int. J Comp Sci. Emerging Tech. ,2 ( 4) (2011),534-538.
- [3] I. Gnanambal and M. Vidhya An extensive study of supra generalized pre regular closed sets. Int. J. of Pure and Applied Mathematics, 106 (6)( 2016),57-73.
- [4] AK. Hussain and BJ. Tawfeeq, Generalized gs- closed sets. College of Education Journal. 3(2014), 213–224.
- [5] M. Kamaraj and G. Ramkumar , Ravi O. Supra sg -closed sets and gs –closed sets. Int. J. of mathematical Archive,2(11) (2011), 2413-241.
- [6] AS. Mashhour and A. Al lam , Mahmoud FS .On supra topological spaces. Indian J. Pure and Appl. Math., 4 (14) (1983), 502-510.
- [7] B. Meera and R. Vijayalakshmi , New Type of Closed Sets in Supra topological Spaces. Imperial J . of Interdisciplinary Research (IJIR), 2(9)( 2016), 503-507.
- [8] O. Ravi , G. Ramkumar and M. Kamaraj , On supra  $\beta$ -open sets and supra  $\beta$ -continuity on topological spaces. proceed, National seminar held at sivakasi, India, 4(2011), 22-31.
- [9] G. Saroja and V .Thiripurasundari,  $\text{pr}^\mu$ -Closed Sets in Supra topological Spaces. Int. J. of Sci. Engineering and Technology Research, 6 (3) (2017), 420 -424.
- [10] R. Selvaraj and S.Chandrasekar,Supra  $^*g$ -Closed Sets in Supra topological Spaces. Int. J. of Math. Trends and Technology, 56 (1) (2018),15-20.