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ABSTRACT

A new family of Salagean type harmonic univalent functions is described and investigated. For the functions in this class, we derive coefficient inequalities, extreme points, and distortion limits.

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1. INTRODUCTION

If both t and l are real harmonic in E, a continuous complex-valued function $w = t + il$ defined in a simply connected complex domain E is said to be harmonic in E. We may write $w = k + \bar{f}$ in any simply connected domain, where k and f are analytic in E. $|k'(u)| > |f'(u)|, u \in E$ is an essential and sufficient condition for w to be locally univalent and sense preserving in E.

S_T denotes the class of harmonic univalent and sense-preserving functions $w = k + \bar{f}$ in the unit disk $U = \{u : |u| < 1\}$ for which $w(0) = w_u(0)\,1 = 0$. Then, for $w = k + \bar{f} \in S_T$, the analytic functions k and f can be expressed as

$$k(u) = u + \sum_{m=2}^{\infty} d_m u^m, \quad f(u) = \sum_{m=1}^{\infty} e_m u^m, \qquad (|e_1| < 1, \ u \in U) \tag{1}$$

Clunie and Sheil Small [8] studied the class S_T and its geometric subclasses in 1984 and came up with some coefficient bounds. Since then, several papers on S_T and its subclasses have been written.

Salagean [17] introduced the differential operator E^V. More details can be seen in [2], [4], [5], [6], [7] and [20]. Jahangiri et al. [13] defined the modified Salagean operator of w as for $w = k + \bar{f}$ given by (1).

$$E^v w(u) = E^v k(u) + (-1)^v \overline{E^v f(u)} \tag{2}$$

where

$$E^v k(u) = u + \sum_{m=2}^{\infty} m^v d_m u^m, \quad E^v f(u) = \sum_{m=1}^{\infty} m^v e_m u^m. \tag{3}$$

Let $S_T(v, r, \tau, \gamma)$ denote the class of univalent harmonic functions of the form (1) that satisfy the condition for fixed positive integers $v, r, 0 \le \tau < 1$ and $\gamma \ge 0$

$$\mathcal{R}e\left\{\frac{E^v w(u)}{E^r w(u)}\right\} > \left|\gamma \frac{E^v w(u)}{E^r w(u)} - \tau\right|, \tag{4}$$

where $E^v w(u)$ is defined by (2).

The subset $\bar{S}_T(v, r, \tau, \gamma)$ is made up of harmonic functions. In $\bar{S}_T(v, r, \tau, \gamma)$, $w_v = k + \bar{f_v}$, where k and f_v are of the form

$$k(u) = u - \sum_{m=2}^{\infty} d_m u^m, \quad f_v(u) = (-1)^{v-1} \sum_{m=1}^{\infty} e_m u^m; \quad d_m, e_m \ge 0. \tag{5}$$

A number of well-known S_T subclasses are included in the class $\bar{S}_T(v, r, \tau, \gamma)$. $\bar{S}_T(1,0,\tau,0) \equiv \mathrm{F}(\tau)$ is a class of sense-preserving, harmonic univalent functions w that are starlike of order τ in U, $\bar{S}_T(2,1,\tau,0)$ is a class of sense-preserving, harmonic univalent functions w that are convex of order τ in U, and $\bar{S}_T(r+1, r, \tau, 0) \equiv \bar{T}(r, \tau)$ is a class of Salagean type harmonic univalent functions. Avcı and Zlotkiewicz [3] demonstrated that if the harmonic function w of the form (1) has $e_1 = 0$,

$$\sum_{m=2}^{\infty} m(|d_m| + |e_m|) \le 1,$$

then $w \in TS(0)$ and if

$$\sum_{m=2}^{\infty} m^2(|d_m| + |e_m|) \le 1,$$

then $w \in Tk(0)$. Silverman [18] demonstrated that if $w = k + \bar{f}$ has negative coefficients, the above coefficient condition is also needed. Later, Silverman and Silvia [19] strengthened [1], [10], [14], [15] and [16] are results for the case e_1 that isn't necessarily zero.

Jahangiri [9], [11] and [12] demonstrated that for harmonic functions w of the form (4) with $v = 1, w \in \mathrm{F}(\tau)$ if and only if

$$\sum_{m=2}^{\infty}(m-\alpha)|d_m| + \sum_{m=1}^{\infty}(m+\alpha)|e_m| \le 1-\alpha$$

and $w \in \bar{S}_T(2,1,\tau,0)$ if and only if

$$\sum_{m=2}^{\infty} m(m-\alpha)|d_m| + \sum_{m=1}^{\infty} m(m+\alpha)|e_m| \le 1-\alpha.$$

The above results are applied in this note to the families $S_T(v,r,\tau,\gamma)$ and $\bar{S}_T(v,r,\tau,\gamma)$. For $\bar{S}_T(v,r,\tau,\gamma)$, we also get extreme points, distortion bounds, convolution conditions, and convex combinations.

2. MAIN RESULTS

For harmonic functions in $S_T(v,r,\tau,\gamma)$, we introduce an adequate coefficient bound in our first theorem.

Theorem 2.1. Let $w = k + \bar{f}$ be so that k and f are given by (1). Furthermore, let

$$\sum_{m=1}^{\infty}\left(\frac{(\tau-1)m^r + (1-\gamma)m^v}{(1+\tau-\gamma)}|d_m| + \frac{(\tau-1)m^r + (-1)^{v-r}(1-\gamma)m^v}{(1+\tau-\gamma)}|e_m|\right) \le 2, \tag{6}$$

where $d_1 = 1, v \in \mathbb{N}, r \in \mathbb{N}_0, v > r$ and $0 \le \tau < 1, \beta \ge 0$; then w is sense-preserving, harmonic univalentin U and $w \in S_T(v,r,\tau,\gamma)$.

Proof. According to (2) and (4) we only need to show that

$$\left|\frac{(1+2\tau)E^r w(u) + E^v w(u) - (1+\tau)E^r w(u) - \gamma E^v w(u)}{E^r w(u)}\right| > 0 \tag{7}$$

$$= \left|\frac{(1+2\tau)\left[E^r k(u) + (-1)^r\overline{E^r f(u)}\right] + \left[E^v k(u) + (-1)^v\overline{E^v f(u)}\right] - (1+\tau)\left[E^r k(u) + (-1)^r\overline{E^r f(u)}\right] - \gamma\left[E^v k(u) + (-1)^v\overline{E^v f(u)}\right]}{E^r w(u)}\right|$$

$$= \left|\frac{(1+\tau-\gamma)u + \sum_{m=2}^{\infty}(\tau m^r + (1-\gamma)m^v)d_m u^m + (-1)^r\sum_{m=1}^{\infty}(\tau m^r + (-1)^{v-r}(1-\gamma)m^v)\overline{e_m u^m}}{u + \sum_{m=2}^{\infty} m^r d_m u^m + (-1)^r\sum_{m=1}^{\infty} m^r\overline{e_m u^m}}\right|$$

$$\ge \left(\frac{(1+\tau-\gamma)|u| + \sum_{m=2}^{\infty}(\tau m^r + (1-\gamma)m^v)|d_m||u^m| + \sum_{m=1}^{\infty}(\tau m^r + (-1)^{v-r}(1-\gamma)m^v)|e_m||u^m|}{|u| + \sum_{m=2}^{\infty} m^r|d_m||u^m| + (-1)^r\sum_{m=1}^{\infty} m^r|e_m||u^m|}\right)$$

$$\left\{\begin{array}{l} (1+\tau-\gamma)|u| + \sum_{m=2}^{\infty}(\tau m^r + (1-\gamma)m^v)|d_m||u^m| + \sum_{m=1}^{\infty}(\tau m^r - (1-\gamma)m^v)|e_m||u^m| \quad if\ v-r\ is\ odd \\ (1+\tau-\gamma)|u| + \sum_{m=2}^{\infty}(\tau m^r + (1-\gamma)m^v)|d_m||u^m| + \sum_{m=1}^{\infty}(\tau m^r + (1-\gamma)m^v)|e_m||u^m| \quad if\ v-r\ is\ even \end{array}\right\}$$

$$= (1+\tau-\gamma)|u|\left\{1+\sum_{m=2}^{\infty}\frac{\tau m^r+(1-\gamma)m^v}{(1+\tau-\gamma)}|d_m||u|^{m-1}+\sum_{m=1}^{\infty}\frac{\tau m^r+(-1)^{v-r}(1-\gamma)m^v}{(1+\tau-\gamma)}|e_m||u|^{m-1}\right\}$$

$$> (1+\tau-\gamma)\left\{1+\sum_{m=2}^{\infty}\frac{\tau m^r+(1-\gamma)m^v}{(1+\tau-\gamma)}|d_m|+\sum_{m=1}^{\infty}\frac{\tau m^r+(-1)^{v-r}(1-\gamma)m^v}{(1+\tau-\gamma)}|e_m|\right\}.$$

Since (6) makes this last expression non-negative, the proof is complete. The harmonic univalent functions

$$w\,(u) = u+\sum_{m=2}^{\infty}\frac{1+\tau-\gamma}{\tau m^r+(1-\gamma)m^v}x_m\,u^m+\sum_{m=1}^{\infty}\frac{1+\tau-\gamma}{\tau m^r+(-1)^{v-r}(1-\gamma)m^v}\overline{y_m\,u^m}, \tag{8}$$

where $v\in\mathbb{N}, r\in\mathbb{N}_0, v>r$ and $\sum_{m=2}^{\infty}|x_m|+\sum_{m=1}^{\infty}|y_m|=1$, demonstrate the sharpness of the coefficient bound given by (6). $S_T(v,r,\tau,\gamma)$ contains functions of the form (8).

$$\sum_{m=1}^{\infty}\left(\frac{\tau m^r+(1-\gamma)m^v}{(1+\tau-\gamma)}|d_m|+\frac{\tau m^r+(-1)^{v-r}(1-\gamma)m^v}{(1+\tau-\gamma)}|e_m|\right)=1+\sum_{m=2}^{\infty}|x_m|+\sum_{m=1}^{\infty}|y_m|=2.$$

The condition (6) is also required for functions $w_v = k+\bar{f_v}$ where k and f_v are of the form (5) shown in the following theorem.

Theorem 2.2. Let $w_v = k+\bar{f_v}$ be given by (5). Then $w_v\in\bar{S}_T(v,r,\tau,\gamma)$ if and only if

$$\sum_{m=1}^{\infty}[(\tau m^r+(1-\gamma)m^v)d_m+(\tau m^r+(-1)^{v-r}(1-\gamma)m^v)e_m]\le 2(1+\tau-\gamma), \tag{9}$$

where $d_1=1, 0\le\tau<1, \gamma\ge 0\ v\in\mathbb{N}, r\in\mathbb{N}_0, v>r$.

Proof. We just need to prove the only if part of the theorem since $\bar{S}_T(v,r,\tau,\gamma)\subset S_T(v,r,\tau,\gamma)$. In order to do this, for functions w_v of the form (5), we are mindful of the situation

$\mathcal{R}e\left\{\frac{E^v w(u)}{E^r w(u)}\right\} > \left|\gamma\frac{E^v w(u)}{E^r w(u)}-\tau\right|$ or equivalent to

$$\mathcal{R}e\left(\frac{(1+\tau-\gamma)u-\sum_{m=2}^{\infty}(\tau m^r+(1-\gamma)m^v)d_m\,u^m+(-1)^{v+r-1}\sum_{m=1}^{\infty}(\tau m^r+(-1)^{v-r}(1-\gamma)m^v)e_m\,\bar{u}^m}{u-\sum_{m=2}^{\infty}m^r d_m\,u^m+(-1)^{v+r-1}\sum_{m=1}^{\infty}m^r e_m\,\bar{u}^m}\right)\ge 0. \tag{10}$$

For all values of u in U, the necessary condition (10) must hold. We must have $0\le u=z<1$, when choosing the values of u on the positive real axis

$$\left(\frac{(1+\tau-\gamma)-\sum_{m=2}^{\infty}(\tau m^r+(1-\gamma)m^v)d_m\,z^{m-1}-(-1)^{v-r}\sum_{m=1}^{\infty}(\tau m^r+(-1)^{v-r}(1-\gamma)m^v)e_m\,z^{m-1}}{u-\sum_{m=2}^{\infty}m^r d_m\,z^m-(-1)^{v-r}\sum_{m=1}^{\infty}m^r e_m\,z^{m-1}}\right)\ge 0. \tag{11}$$

If condition (9) is not satisfied, the numerator in (11) is negative for z near enough to 1. As a result, for $u_0=z_0$ in $(0,1)$, the quotient in (11) is negative. The proof is complete since this contradicts the necessary condition for $w_v\in\bar{S}_T(v,r,\tau,\gamma)$.

The extreme points of closed convex hulls of $\bar{S}_T(v,r,\tau,\gamma)$denoted by clco $\bar{S}_T(v,r,\tau,\gamma)$ are then determined

Theorem 2.3. Let $w_v = k + \bar{f}_v$ be given by (5). Then $w_v \in \bar{S}_T(v,r,\tau,\gamma)$ if and only if

$$w_v(u) = \sum_{m=1}^{\infty}\left(x_m k_m(u) + y_m f_{v_m}(u)\right)$$

where

$$k_1(u) = u \ , \qquad k_m(u) = u - \frac{1+\tau-\gamma}{\tau m^r + (1-\gamma)m^v}u^m \ , \qquad (m = 2,3,\dots),$$

and

$$f_{v_m}(u) = u + (-1)^{v-1}\frac{1+\tau-\gamma}{\tau m^r + (-1)^{v-r}(1-\gamma)m^v}\bar{u}^m \ , \qquad (m = 1,2,\dots),$$

$x_m \geq 0, y_m \geq 0, x_1 = 1 - \sum_{m=2}^{\infty}(x_m + y_m) \geq 0$. *In particular, the extreme points of* $\bar{S}_T(v,r,\tau,\gamma)$ are $\{k_m\}$ and $\{f_{v_m}\}$.

Proof. Suppose

$$w_v(u) = \sum_{m=1}^{\infty}\left(x_m k_m(u) + y_m f_{v_m}(u)\right)$$

$$= \sum_{m=1}^{\infty}(x_m + y_m)u - \sum_{m=2}^{\infty}\frac{1+\tau-\gamma}{\tau m^r + (1-\gamma)m^v}x_m u^m + (-1)^{v-1}\sum_{m=1}^{\infty}\frac{1+\tau-\gamma}{\tau m^r + (-1)^{v-r}(1-\gamma)m^v}y_m \bar{u}^m.$$

Then

$$\sum_{m=2}^{\infty}\frac{\tau m^r + (1-\gamma)m^v}{1+\tau-\gamma}\left(\frac{1+\tau-\gamma}{\tau m^r + (1-\gamma)m^v}x_m\right) + \sum_{m=1}^{\infty}\frac{\tau m^r + (-1)^{v-r}(1-\gamma)m^v}{1+\tau-\gamma}\left(\frac{1+\tau-\gamma}{\tau m^r + (-1)^{v-r}(1-\gamma)m^v}y_m\right).$$

$$= \sum_{m=2}^{\infty}x_m + \sum_{m=1}^{\infty}y_m = 1 - x_1 \leq 1$$

and so $w_v \in clco\bar{S}_T(v,r,\tau,\gamma)$.

Conversely, if $w_v \in clco\bar{S}_T(v,r,\tau,\gamma)$, then $d_m \leq \frac{1+\tau-\gamma}{\tau m^r+(1-\gamma)m^v}$ and $e_m \leq \frac{1+\tau-\gamma}{\tau m^r+(-1)^{v-r}(1-\gamma)m^v}$. Set

$$x_m = \frac{\tau m^r + (1-\gamma)m^v}{1+\tau-\gamma}d_m, (m = 2,3,\dots), and\ y_m = \frac{\tau m^r + (-1)^{v-r}(1-\gamma)m^v}{1+\tau-\gamma}e_m, (m = 1,2,\dots).$$

Then note that by Theorem 2.2, $0 \leq x_m \leq 1, (m = 2,3,\dots)$ and $0 \leq y_m \leq 1, (m = 1,2,\dots)$. We define

$$x_1 = 1 - \sum_{k=2}^{\infty}x_k - \sum_{k=1}^{\infty}y_k ,$$

and note that, by Theorem 2.2, $x_1 \geq 0$. Consequently, we obtain

$$w_v(u) = \sum_{m=1}^{\infty}\left(x_m k_m(u) + y_m g f_{v_m}(u)\right).$$

The distortion limits for functions in $\bar{S}_T(v,r,\tau,\gamma)$ are given by the following theorem, which yields a covering result for this class.

Theorem 2.4. Let $w_v \in \bar{S}_T(v,r,\tau,\gamma)$. Then for $|u| = z < 1$, we have

$$|w_v(u)| \leq (1+e_1)z + \frac{1}{2^n}\left(\frac{\alpha-\beta}{(1-\beta)2^{m-n} + (\alpha-1)} - \frac{(\alpha-1) + (-1)^{m-n}(1-\beta)}{(1-\beta)2^{m-n} + (\alpha-1)}b_1\right)r^2,$$

and

$$|w_v\,(u)| \geq (1+e_1)z - \frac{1}{2^n}\left(\frac{\alpha-\beta}{(1-\beta)2^{m-n}+(\alpha-1)} - \frac{(\alpha-1)+(-1)^{m-n}(1-\beta)}{(1-\beta)2^{m-n}+(\alpha-1)}b_1\right)r^2.$$

Proof. Only the right hand inequality is proven. The left-hand inequality's proof is identical and will be omitted. Let $w_v \in \bar{S}_T(v,r,\tau,\gamma)$. be the function. Taking w_v is absolute meaning, we get

$$|w_v\,(u)| = \left|u - \sum_{m=2}^{\infty} d_m\,u^m + (-1)^{v-1}\sum_{m=1}^{\infty} e_m\,u^m\right| \leq z + \sum_{m=2}^{\infty} d_m\,z^m + (-1)^{v-1}\sum_{m=1}^{\infty} e_m\,z^m$$

$$\leq (1+e_1)z + \sum_{m=2}^{\infty}(d_m+e_m)\,z^m \leq (1+e_1)z + \sum_{m=2}^{\infty}(d_m+e_m)\,z^2$$

$$= (1+e_1)z + \frac{\tau-\gamma}{2^r[(1-\gamma)2^{v-r}+(\tau-1)]}\sum_{m=2}^{\infty}\frac{2^r[(1-\gamma)2^{v-r}+(\tau-1)]}{\tau-\gamma}(d_m+e_m)\,z^2$$

$$\leq \left((1+e_1)z + \frac{(\tau-\gamma)z^2}{2^r[(1-\gamma)2^{v-r}+(\tau-1)]}\right)\times$$
$$\sum_{m=2}^{\infty}\left(\frac{(\tau-1)m^r+(1-\gamma)m^v}{\tau-\gamma}d_m + \frac{(\tau-1)m^r+(-1)^{v-r}(1-\gamma)m^v}{\tau-\gamma}e_m\right)$$

$$\leq (1+e_1)z + \frac{1}{2^r}\left(\frac{\tau-\gamma}{(1-\gamma)2^{v-r}+(\tau-1)} - \frac{(\tau-1)m^r+(-1)^{v-r}(1-\gamma)}{(1-\gamma)2^{v-r}+(\tau-1)}e_1\right)z^2.$$

The left hand inequality in Theorem 2.4 leads to the following covering result.

Corollary 2.5. Let $w_v \in \bar{S}_T(v,r,\tau,\gamma)$, then for $|u| = z < 1$, we have

$$\left\{h: |h| < \frac{(\gamma 2^{-v}+1-\gamma)2^v+(\tau-1-\tau 2^{-r})2^r}{(1-\gamma)2^v+(\tau-1)2^r} - \frac{(\tau-1-(\tau-1)2^{-r})2^r-((-1)^{v-r}(1-\gamma)2^{-v}-1+\gamma)2^v+}{(1-\gamma)2^v+(\tau-1)2^r}\right\} \subset w_v(U).$$

Remark 2.6. If we take $v = r+1,\ \gamma = 0$, then the above covering result given in [3]. Furthermore, the results of this paper, for $\gamma = 0$ coincide with the results in [4].

REFERENCES

[1] O.P. Ahuja, A. Çetinkaya, V. Ravichandran, Harmonic univalent functions defined by post quantum calculus operators, Acta Univ. Sapientiae, Mathematica 11(1), (2019), 5–17.

[2] S. Altınkaya, S. Çakmak, S. Yalçın, On a new class of Salagean-type harmonic univalent functions associated with subordination, Honam Math J 40(3), (2018), 433–446.

[3] Y. Avcı, E. Zlotkiewicz, On harmonic univalent mappings, Ann. Univ. Mariae Curie Sklodowska Sect. A 44 (1990) 1–7.

[4] H. Bayram, S. Yalçın, A SUBCLASS OF HARMONIC UNIVALENT FUNCTIONS DEFINED BY A LINEAR OPERATOR, Palestine Journal of Mathematics, 6 (2017).

[5] H. Bayram, A Subclass of Harmonic Univalent Functions Defined by Salagean Integro Differential Operator, Al-Qadisiyah Journal Of Pure Science 25.3 (2020), 62-70.

[6] H. Bayram, S. Yalçın, A SUBCLASS OF HARMONIC UNIVALENT FUNCTIONS DEFINED BY SUBORDINATION, Acta Universitatis Apulensis, No. 60/2019, pp. 91-103.

[7] H. Bayram, S. Yalçin, On a new subclass of harmonic univalent functions, Malaysian Journal of Mathematical Sciences 14.1 (2020), 63-75.

[8] J. Clunie, T. Sheil-Small, Harmonic univalent functions, Ann. Acad. Sci. Fenn. Ser. A I Math. 9 (1984) 3–25.

[9] J. Dziok, JM. Jahangiri, H. Silverman, Harmonic functions with varying coefficients. J Inequal Appl 16,139, 2016.

[10] M.I. Hameed, C. Ozel, A. Al-Fayadh and A.R.S. Juma, Study of certain subclasses of analytic functions involving convolution operator, AIP Conference Proceedings, Vol. 2096. No. 1. AIP Publishing LLC, 2019.

[11] M. Hameed and I. Ibrahim, Some Applications on Subclasses of Analytic Functions Involving Linear Operator, 2019 International Conference on Computing and Information Science and Technology and Their Applications (ICCISTA). IEEE, 2019.

[12] J.M. Jahangiri, Harmonic functions starlike in the unit disk, J. Math. Anal. Appl. 235 (1999) 470–477.

[13] J.M. Jahangiri, G. Murugusundaramoorthy, K. Vijaya, Salagean-type harmonic univalent functions, South. J. Pure Appl.Math. 2 (2002) 77–82.

[14] S. Jameel Al-Dulaimi and M. I. Hameed, Applications Of Generalized Hypergeometric Analysis Function Of Second Order Differential Subordination, Turkish Journal of Computer and Mathematics Education Vol.12 No. 9 (2021), 3485-3490.

[15] A.R.S. Juma, R.A. Hameed and M.I. Hameed, Certain subclass of univalent functions involving fractional q-calculus operator, Journal of Advance in Mathematics 13.4, 2017.

[16] A.R.S. Juma, R.A. Hameed and M.I. Hameed, SOME RESULTS OF SECOND ORDER DIFFERENTIAL SUBORDINATION INVOLVING GENERALIZED LINEAR OPERATOR., Acta Universitatis Apulensis No. (53), pp. 19-39, 2018.

[17] G.S. Salagean, Subclass of univalent functions, Complex Analysis-Fifth Romanian Finish Seminar, Bucharest, 1, 1983,pp. 362–372.

[18] H. Silverman, Harmonic univalent function with negative coefficients, J. Math. Anal. Appl. 220 (1998) 283–289.

[19] H. Silverman, E.M. Silvia, Subclasses of harmonic univalent functions, New Zealand J. Math. 28 (1999) 275–284.

[20] S. Yalcın, A new class of Salagean-type harmonic univalent functions, Applied Mathematics Letters, Vol.18, 2(2005), 191-198.