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Rasha Ali Hussein
College of Science,University of Al-Qadisiyah Iraq, rasha.ali202085@gmail.com

Noori F. Al-Mayahi

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Authors Names
a. Rasha Ali Hussein
a. Noori F. Al-Mayahi

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ABSTRACT

In this paper, will be presented the concept of Borel sets and Baire sets on normal topological spaces and locally compact topological spaces as new types of λ – system and study the basic characteristics related to each of them and obtain results related to them..

1. Introduction

We present a study on two types of sets, the first of which is the Borel sets is any sets in a topological space that can be formed from open or closed sets through union, counted intersection and complementary sets, so it was named as Emile Borel .In the topological space Ω, the family of all sets of Borel on Ω the form σ– field is defined as Borel field or Borel σ– field , but Borel field on Ω is the smallest σ– field that contains all open sets (or equivalent all closed sets). The second type is Bayer sets were introduced by Kunihiko Kodaira (1941), Shizuo Kakutan , Kunihiko Kodaira (1944) and Halmos (1950), and named them as Baire functions, which are defined as René-Louis Baire. There are several of equivalent definitions for Baire sets, but in the most widely used, the Baire sets of a locally compact Hausdorff space form the smallest σ-field such that all compactly supported continuous functions are measurable. Borel sets are important in measure theory because any measure open or closed sets of space is such that the definition of Borel sets in that space, any measure defined on Borel sets, is called the Borel measure. Borel sets are sometimes defined as collections generated by sets compacted in topological spaces rather than open clusters. Bayer sets on a topological space are Borel sets and vice versa always incorrect, and congruence is achieved on metric spaces. This research includes three items, the first one is locally compact spaces , the second one is λ -Borel sets and the last item is λ –Baire sets on topological space.

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[a] College of Science,University of Al-Qadisiyah Iraq , E-Mail: rasha.ali202085@gmail.com

2. λ -Borel Sets

In this section, we generalized Borel sets concept to λ -Borel sets so that we got some important results for dealing with these concepts.

Definition 2.1 [2]

A nonempty family $\mathcal{F}$ of subsets of a set Ω is λ - system if it satisfies the following axioms:

1. $\Omega \in \mathcal{F}$.

2. If $A, B \in \mathcal{F}$ $A \subseteq B$ then $B/A \in \mathcal{F}$.

3. If $A_n \in \mathcal{F}$ is an increasing sequence of sets in $\mathcal{F}$, Then $\bigcup_{n=1}^{\infty} A_n \in \mathcal{F}$.

Proposition 2.2

Let $(\Omega, \mathcal{F})$ be a λ-measurable space . Then $A^c \in \mathcal{F}$, if $A \in \mathcal{F}$. Hence $\emptyset \in \mathcal{F}$.

It easy:

Let $A \in \mathcal{F}$, since $\Omega \in \mathcal{F}$ and $\Omega /A \in \mathcal{F}$, since $\Omega /A = A^c$, then $A^c \in \mathcal{F}$.

since $\Omega \in \mathcal{F}$, then $\Omega^c \in \mathcal{F}$, $\Omega^c = \emptyset$, then $\emptyset \in \mathcal{F}$.

Example 2. 3

Let $\Omega =\{ 1, 2, 3\}$ and $\mathcal{F} =\{ \Omega, \{1\},\{2\},\{3\},\{1,2\},\{1,3\},\{2,3\},\emptyset\}$ is λ-system

Proof :

$A,B \in \mathcal{F}$ and $A=\{1\}$, $B=\{1,2\}$, $A \subset B$, $B/A=\{2\} \in \mathcal{F}$

Remark 2.4

If $\mathcal{F}$ is λ – system on Ω , it is clear to show that:

1. If $A_1 , A_2 , \dots A_n \in \mathcal{F}$ then $\bigcup_{n=1}^{n} A_n \in \mathcal{F}$ *and* $\bigcap_{i=1}^{n} A_i \in \mathcal{F}$

2. If $A_n \in \mathcal{F}$, $n = 1,2, \dots$ then $\bigcap_{i=1}^{\infty} A_i \in \mathcal{F}$.

Theorem 2.5

Let Ω_1 and Ω_2 be nonempty sets, and $f: \Omega_1 \to \Omega_2$ is any function .

1.If $\mathcal{F}$ is a λ – system on Ω_1, then $\mathscr{H}=\{ A \subseteq \Omega_2 : f^{-1}(A) \in \mathcal{F}\}$ is a λ – system on Ω_2.

2.If $\mathcal{G}$ is a a λ – system on Ω_2, then $f^{-1}(\mathcal{G}) =\{ f^{-1}(A): A \in \mathcal{G}\}$ is a λ – system on Ω_2.

Proof:

1. Since $f^{-1}(\Omega_2)= \Omega_1$ and $\Omega_1 \in \mathcal{F} \Rightarrow \Omega_2 \in \mathscr{H}$

Let $A \in \mathcal{H} \Rightarrow f^{-1}(A) \in \mathcal{F} \Rightarrow (f^{-1}(A))^c \in \mathcal{F}$

But $\left(f^{-1}(A)\right)^c = f^{-1}(A^c) \Rightarrow f^{-1}(A^c) \in \mathcal{F} \Rightarrow A^c \in \mathcal{H}$

Let $A_n \in \mathcal{H}$, n = 1,2, … $\Rightarrow f^{-1}(A_n) \in \mathcal{F}$, n = 1,2, … $\Rightarrow$

$\bigcup_{n=1}^{\infty} f^{-1}(A_n) \in \mathcal{F}$, but $f^{-1}(\bigcup_{n=1}^{\infty} A_n) = \bigcup_{n=1}^{\infty} f^{-1}(A_n) \Rightarrow$

$f^{-1}(\bigcup_{n=1}^{\infty} A_n) \in \mathcal{F} \Rightarrow \bigcup_{n=1}^{\infty} A_n \in \mathcal{H}$

2. Since $\mathcal{G}$ is a λ – system on $\Omega_2 \Rightarrow \Omega_2 \in \mathcal{G} \Rightarrow f^{-1}(\Omega_2) \in f^{-1}(\mathcal{G})$

But $f^{-1}(\Omega_2) = \Omega_1 \Rightarrow \Omega_1 \in \mathcal{F}$

Let $B \in f^{-1}(\mathcal{G}) \Rightarrow B = f^{-1}$(A),where $A \in \mathcal{G} \Rightarrow A^c \in \mathcal{G}$

$\Rightarrow f^{-1}(A^c) \in f^{-1}(\mathcal{G})$, but $f^{-1}(A^c) = (f^{-1}(A))^c = B^c \Rightarrow B^c \in f^{-1}(\mathcal{G})$

Let $B_n \in f^{-1}(\mathcal{G}), n = 1,2, \ldots \Rightarrow B_n = f^{-1}(A_n)$, where $A_n \in \mathcal{G}, n = 1,2, \ldots$

$\Rightarrow \bigcup_{n=1}^{\infty} A_n \in \mathcal{G} \Rightarrow f^{-1}(\bigcup_{n=1}^{\infty} A_n) \in f^{-1}(\mathcal{G})$

But $f^{-1}(\bigcup_{n=1}^{\infty} A_n) = \bigcup_{n=1}^{\infty} f^{-1}(A_n) = \bigcup_{n=1}^{\infty} B_n \Rightarrow \bigcup_{n=1}^{\infty} B_n \in f^{-1}(\mathcal{G})$

Theorem 2.6

If $\{\mathcal{F}_\lambda\}_{\lambda\in\Lambda}$ be an arbitrary family of λ – system on a set Ω where $\Lambda \neq \emptyset$, then $\mathcal{F} = \bigcap_{\lambda\in\Lambda} \mathcal{F}_\lambda$ is a λ – system on Ω.

Proof:

Since $\Omega \in \mathcal{F}_\lambda \in \mathcal{F}_\lambda \Rightarrow \Omega \in \bigcap_{\lambda\in\Lambda} \mathcal{F}_\lambda \Rightarrow \Omega \in \mathcal{F}$.Let $A \in \mathcal{F} \Rightarrow A \in \mathcal{F}_\lambda$ for all $\lambda \in \Lambda$

$\Rightarrow A^c \in \mathcal{F}_\lambda$ for all $\lambda \in \Lambda \Rightarrow A^c \in \bigcap_{\lambda\in\Lambda} \mathcal{F}_\lambda \Rightarrow A^c \in \mathcal{F}$

Let $A_n \in \mathcal{F},, n = 1,2, \ldots \Rightarrow A_n \in \mathcal{F}_\lambda$ for all $\lambda \in \Lambda \Rightarrow \bigcup_{n=1}^{\infty} A_n \in \mathcal{F}_\lambda$ for all $\lambda \in \Lambda$

$\Rightarrow \bigcup_{n=1}^{\infty} A_n \in \bigcap_{\lambda\in\Lambda} \mathcal{F}_\lambda \Rightarrow \bigcup_{n=1}^{\infty} A_n \in \mathcal{F}$.

Definition 2.7

Let $\mathcal{G}$ be a family of sets of a set Ω . The smallest λ – system Containing $\mathcal{G}$ called the λ – system generated by $\mathcal{G}$ and it is Denoted by $\lambda(\mathcal{G})$.

Theorem 2.8

Let Ω_1 and Ω_2 be ≠ 0 sets, and $f: \Omega_1 \to \Omega_2$ is any function .If $\mathcal{G}$ is a family of set in Ω_2. Then $\lambda\left(f^{-1}(\mathcal{G})\right) = f^{-1}\left(\lambda(\mathcal{G})\right)$,Where $f^{-1}(\mathcal{G}) = \{ f^{-1}(A): A \in \mathcal{G}\}$.

Proof:

Since $\mathcal{G} \subseteq \lambda(\mathcal{G}) \Rightarrow f^{-1}(\mathcal{G}) \subseteq f^{-1}\left(\lambda(\mathcal{G})\right)$

And $f^{-1}(\lambda(\mathcal{G}))$is a $\lambda-$ system on $\Omega_1$and $\lambda(f^{-1}(\mathcal{G}))$is smallest $\lambda-$ system contain $f^{-1}(\mathcal{G})$,

then $\lambda(f^{-1}(\mathcal{G})) \subseteq f^{-1}(\lambda(\mathcal{G})) \ldots (1)$

Now put $\mathscr{H}=\{ A \subseteq \Omega_2 : f^{-1}(A) \in \lambda(f^{-1}(\mathcal{G}))\}$, by using part (1) of theorem (3.1) ,

we have $\mathscr{H}$ $\lambda-$ system on Ω_2. Let$A \in \mathcal{G} \Longrightarrow f^{-1}(A) \in f^{-1}(\mathcal{G}) \subseteq \lambda(f^{-1}(\mathcal{G})) \Longrightarrow \lambda(\mathcal{G}) \subseteq \mathcal{H}$

$f^{-1}(\lambda(\mathcal{G})) \subseteq f^{-1}(\mathscr{H}) \subseteq \lambda(f^{-1}(\mathcal{G})) \ldots (2)$

From (1) and (2), we have $\lambda(f^{-1}(\mathcal{G})) = f^{-1}(\lambda(\mathcal{G}))$.

Definition 2.9

Let $\mathcal{G}$ be a family of subsets of a set Ω, and let $A \subset \Omega$. The restriction (or trace) of $\mathcal{G}$ on A is the collection of all sets of the form $A \cap B$, were $B \in \mathcal{G}$, and it is denoted by $\mathcal{G}_A$(or $A \cap \mathcal{G}$)

$$\mathcal{G}_A = A \cap \mathcal{G} = \{ A \cap \mathrm{B} : \mathrm{B} \in \mathcal{G}\}$$

$\mathcal{G}_A$ is family of subsets of A. The fuzzy $\lambda-$ system$\sigma(\mathcal{G}_A)$ generated by $\mathcal{G}_A$. Some time denoted by $\lambda_a(A \cap \mathcal{G})$, i.e. $\lambda(\mathcal{G}_A) = \lambda_a(A \cap \mathcal{G})$.

Example 2.10

1. If $\Omega = \{a, b, c, d\}, \mathcal{G} = \{\{\mathrm{a}\}, \{\mathrm{b}, \mathrm{c}\}, \{\mathrm{c}, \mathrm{d}\}\}, \mathrm{A} = \{\mathrm{a}, \mathrm{b}\}$, then $\mathcal{G}_A = A \cap \mathcal{G} = \{\emptyset, \{\mathrm{a}\}, \{\mathrm{b}\}\}$.

2. If $\Omega = \mathbb{R}$, $\mathcal{G} = \{[\mathrm{a}, \mathrm{b}] : -\infty < a \leq b < \infty\}$, $A = [0,1]$, then

$$\mathcal{G}_A = A \cap \mathcal{G} = \{[\mathrm{a}, \mathrm{b}] : 0 \leq \mathrm{a} \leq \mathrm{b} \leq 1\}.$$

Theorem 2.11

Let $\mathcal{G}$ be a family of subsets of a set Ω, and let $A \subset \Omega$. Then $A \cap \lambda(\mathcal{G}) = \lambda(A \cap \mathcal{G})$.

Proof:

Since $\mathcal{G} \subseteq \lambda(\mathcal{G})$, then $A \cap \mathcal{G} \subseteq A \cap \lambda(\mathcal{G})$. Since $A \cap \lambda(\mathcal{G})$ is a $\lambda-$ system of subsets of A.

Therefore $\lambda(A \cap \mathcal{G}) \subseteq A \cap \lambda(\mathcal{G})$

To prove $A \cap \lambda(\mathcal{G}) \subseteq \lambda(A \cap \mathcal{G})$ we must to show that $A \cap B \in \lambda(A \cap \mathcal{G})$ for all $B \in \lambda(\mathcal{G})$.

This is not obvious, so we resort to the following basic reasoning process.

Which might be called the good sets principle.

Let $\mathcal{F}$ be the family of good sets , i.e. let $\mathcal{F}$ consist of those sets $B \in \lambda(\mathcal{G})$ such that

$A \cap B \in \lambda(A \cap \mathcal{G})$, Since $\lambda(\mathcal{G})$and $A \cap \lambda(\mathcal{G})$ are $\lambda-$ system ,

it follows quickly that $\mathcal{F}$ is a $\lambda-$ system. Since $\mathcal{G} \subseteq \mathcal{F}$, so that $\lambda(\mathcal{G}) \subseteq \mathcal{F}$, hence $\lambda(\mathcal{G}) = \mathcal{F}$

and the result follows , Briefly , every set in $\mathcal{G}$ is good set and the family of good set form a

λ – system, consequently, every set in $\lambda(\mathcal{G})$ is good.

Definition 2.12

Let (Ω,τ)be a topological space. The λ – system generated by τ is called the Borel

λ – system and it is denoted by $\beta(\Omega)$, i.e. $\beta(\Omega)=\lambda(\tau)$. The member of $\beta(\Omega)$are called Borel sets in Ω.

Remark 2.13

Let Ω be a fixed locally compact Hausdorff space. We shall denote by C the family of all compact subsets of Ω, by S the λ – system generated by C; i.e. S= λ(C) , and by G the family of all open sets belonging to S, then the sets of S are the Borel sets of Ω , so that, for instance, G may be described as the class of open Borel sets.

A real valued function on Ω is Borel measurable if it is measurable with respect to the λ – system S.

Theorem 2.14

Let Ω be a locally compact Hausdorff space

1. If $A \in$ S , then A is σ −bounded

2. If A is σ −bounded and open set in Ω , then $A \in$ S.

Proof :

1. Let $A \in$ S, then A is compact set

Since every compact set is bounded , then A is bounded set

Since every bounded set is σ −bounded, then A is σ −bounded.

2. The family of all bounded sets is λ – system

Since this λ – system includes C, it contains every set of the λ – system generated by C.

Let A be σ −bounded and open set in Ω, then there exists a sequence $\{A_n\}$ of compact sets such that $A \subseteq \bigcup_{n=1}^{\infty} A_n$, Since , for $n=1,2,..$, $A_n|A$ is compact, it follows that

$B=\bigcup_{n=1}^{\infty}(A_n|A) \in$ S, Since $B=(\bigcup_{n=1}^{\infty} A_n)|A$, it follows that

$A=(\bigcup_{n=1}^{\infty} A_n)|(\bigcup_{n=1}^{\infty}(A_n|A)) \in$ S

3. λ –Baire Sets on normal topological space

In this section, let Ω be a normal topological space. The family of all continuous real valued function on Ω will be denoted by of $C(\Omega)$.

Definition 3.1

Let Ω be a normal topological space. The family of Baire sets of Ω, denoted by $\mathcal{A}(\Omega)$ or simply by $\mathcal{A}$, is defined as the smallest λ – system of subsets of Ω making all continuous real valued functions (Borel) measurable, that is, $\mathcal{A}$ is the minimal λ – system containing all sets $f^{-1}(B)$ where B ranges over $\beta(\mathbb{R})$ and f ranges over the family $C(\Omega)$.

$$\mathcal{A}(\Omega) = \mathcal{A} = \lambda(\{f^{-1}(B): B \in \beta(\mathbb{R}), f \in C(\Omega)\})$$

Theorem 3.2

Let Ω be a normal topological space, then $\mathcal{A}$ is the smallest

λ – system making all bounded continuous functions measure, i.e. if $\mathcal{F}$ is a λ – system that makes all bounded continuous functions measurable, then $\mathcal{A} \subseteq \mathcal{F}$.

Proof:

Let $f \in C(\Omega)$

Since $f: \Omega \to \mathbb{R}$ is continuous function, then $\min\{f^+, n\}$ is bounded continuous function and $min\{f^+, n\} \uparrow f^+$ as n→ ∞.

Thus f^+ is $\mathcal{F}$- measurable and similarly f^- is $\mathcal{F}$- measurable.

Hence $f = f^+ - f^{-1}$ is $\mathcal{F}$- measurable. Thus $\mathcal{A} \subseteq \mathcal{F}$.

Remark 3.3

Let Ω be a normal topological space. The family of all bounded continuous real valued function on Ω will be denoted by $C_b(\Omega)$

Theorem 3.4

If Ω is a normal topological space. Then $\mathcal{A}(\Omega) \subseteq \beta(\Omega)$

Proof:

Let G be an open set in $\mathbb{R}$, and $f \in C(\Omega)$,Since $f: \Omega \to \mathbb{R}$ is continuous function,

then f^{-1}(G)is an open set in Ω, hence $f^{-1}(G) \in \beta(\Omega)$.

But the set f^{-1}(G) generate $\mathcal{A}(\Omega)$, from theorem (4.2)

Since any λ – system containing the sets f^{-1}(G)for all open sets G must contain the sets f^{-1}(A) for all Borel set A, It follows that $\mathcal{A}(\Omega) \subseteq \beta(\Omega)$.

Definition 3.5

Let Ω be a topological space

1. An F_δ set in Ω is a countable union of closed sets in Ω.

2.An G_δ set in Ω is a countable intersections union of closed sets in Ω .

i.e. A subset A of topological space Ω is called a G_δ if there exists a sequence $\{A_n\}$of open sets such that $A = \bigcap_{n=1}^{\infty} A_n$.

The family of all G_δ's is closed under the formation of finite union and countable intersections.

Remark 3.6

Let Ω be a fixed locally compact Hausdorff space. We shall denote by C_0 the family of all those compact subsets of Ω which are G_δ's, by S_0 the λ – system generated byC_0; i.e.

$S_0 = \lambda(C_0)$, and by C_b the family of all open sets belonging to S_0, then the sets of S_0 are the Baire sets of Ω, so that, for instance, C_b may be described as the class of open Baire sets.

A real valued function on Ω is Baire measurable if it is measurable with respect to the λ – system S_0.

Definition 3.7

Let Ω be a topological space . A set $A \subseteq \Omega$ is called a zero set if $A = f^{-1}(\{0\})$ for some continuous real valued function f on Ω ; we often write $A(f)$ for $f^{-1}(\{0\})$.

Theorem 3.8

Let Ω be a normal topological space. Then

1. Every closed G_δ set is a zero set.

2. If $A \in S_0$, then A is zero set.

3. If $A \in S_0$, then A is a Baire set.

4. Let $A(\Omega)$ is the smallest λ – system of subsets of Ω containing all zero sets in Ω.

Proof :

1. let A be closed G_δ set

Since A is G_δ set then there exists a sequence $\{A_n\}$ of open sets such that $A = \bigcap_{n=1}^{\infty} A_n$.

Since A is closed set , then $A^{c} = (\bigcap_{n=1}^{\infty} A_n)^{c} = \bigcup_{n=1}^{\infty} A_n^{c}$ is open where A_n^{c} is closed sets.

By Urysohn's lemma , there are functions $f_n \in C(\Omega)$ with $0 \leq f_n \leq 1$, $f_n(x) = 0$ for all $x \in A$ and $f(x) = 1$ for all $x \in A_n^{c} \Longrightarrow A = f_n^{-1}(\{0\})$.

2. Let $A \in S_0$, then A be compact G_δ set in Ω

Since every compact subset of Hausdorff space is closed , then A is closed, hence A be closed G_δ set, so that A is zero set.

3. Let $A \in S_0$, then A be compact G_δ set in Ω. We have $\Omega = \bigcup_{n=1}^{\infty} B_n$ where each B_n is compact . For each n, there is an we obtain compact G_δ's such that $\Omega = \bigcup_{n=1}^{\infty} B_n$.

Thus $A = \bigcup_{n=1}^{\infty}(A \cap B_n)$ is a countable union of compact G_δ 's and consider a zero set A , clearly A is a G_δ .

Theorem 3.9

Let Ω is a normal topological space. Then

1. $\mathcal{A}(\Omega)$is the is the minimal λ – system containing the openF_δ sets.
2. $\mathcal{A}(\Omega)$is the is the minimal λ – system containing the closed G_δ sets.

Proof:

1. Let $f \in C(\Omega)$; then $\{ f > a\} = \bigcup_{n=1}^{\infty}\{ f \geq a + \left(\frac{1}{n}\right)\}$ is an open F_δ sets.

As above, the sets$\{ f > a\}$, }, $a \in \mathbb{R}$, $f \in C(\Omega)$, generated $\mathcal{A}$, hence $\mathcal{A}$ is included.

2. Same proof, first point.

Corollary 3.10

If Ω is a normal topological space, the open F_δ sets are precisely the sets$\{ f > 0\}$ where

$$f \in C_b(\Omega), f \geq 0.$$

Proof :

By theorem (3.9)

Corollary 3.11

If Ω is a metric space, then $\mathcal{A}(\Omega) = \beta(\Omega)$.

Proof

$\mathcal{A}(\Omega) \subseteq \beta(\Omega)$ this direction was proven by (3.4).

Conversely , $\beta(\Omega) \subseteq \mathcal{A}(\Omega)$ $let\ F$ be a closed subset of Ω , since every closed set in a metric space is a G_δ , then F is a G_δ set. Since G_δ set in Ω is a countable intersection union of open sets in Ω, then

$$F = \bigcap_{n=1}^{\infty}\{x: d(x, F) < \frac{1}{n}\};$$

hence every open subset of Ω is an F_δ , The result now follows from (4.7).

Theorem 3.12

Let A be an open F_δ set in the normal space Ω, then

1. $A = \{ f > 0\}$ where $f \in C_b(\Omega), f \geq 0$.
2. I_A is the limit of an increasing sequence of continuous functions .

Proof

1.From corollary (3.10) ,We have $A\{ f > 0\}$ where $f \in C_b(\Omega), f \geq 0$.

2.Since $\{f > O\} = \bigcup_{n=1}^{\infty}\{f \geq l/n\}$, then by Urysohn's lemma there are functions $f\ C(\Omega)$ with $0 \leq f_n \leq 1$, $f_n(x) = 0$ for all $x \in \{ f = 0\}$ *and* $f(x) = 1$for all $\{f \geq l/n\}$.

Take g_n= max($f_1, \ldots, f_n$), then $g_n \uparrow I_{\{ f > 0\}}$.

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