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Reliable Iterative Method for solving Volterra -Fredholm Integro Differential Equations

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ABSTRACT

The aim of this paper is to propose a reliable iterative method for resolving many types of Volterra - Fredholm Integro - Differential Equations of the second kind with initial conditions. The series solutions of the problems under consideration are obtained by means of the iterative method. Four various problems are resolved with high accuracy to make evident the enforcement of the iterative method on such type of integro differential equations. Results were compared with the exact solution which exhibits that this technique was compatible with the right solutions, simple, effective and easy for solving such problems. To evaluate the results in an iterative process the MATLAB is used as a math program for the calculations.

1. Introduction

Integro and integral differential equations played a basis role in describing various fields of physical models, chemical kinetics, fluid dynamics, engineering and biological problems. At the outset the scholars utilized different convergent methods to get the approximate solution of Volterra-Fredholm Integro Differential Equations(VFIDEs) and integral equations such as the Legendre Wavelets Method [14]. The Haar Functions Method [10,11]. The Finite Difference Method [9]. Variational Iteration Method [1]. Differential Transform Method [15]. VFIDEs are difficult to solve by analytical methods. Therefore, researchers rarely dealt with analytical solution for the difficulty of solving them. In science and engineering, sometimes there are vital

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problems that can be simplified to solve these equations. The latter equation has pulled in much consideration of math scholars to work on and has been a subject of interest for them to solve VFIDEs such that Modified Adomian Decomposition Method [2,16]. The HOBW method [12]. The 2D-BPFs method [13]. Boubaker Polynomial Method [5]. Linear Programming Method [8]. In this work, we apply a new Iterative Method (IM) to solve VFIDEs. Temimi and Ansari have been suggested this method to resolving linear and non-linear functional equations [6]. This method has been extensively applied by many research works to resolve some linear and non-linear partial, ordinary differential equations, non-linear delay differential equations, higher order integro-differential equations and korteweg-de Vries equations [4,7,3,17,18]. In this research, we applied this reliable technique to solve VFIDEs. Generally, VFIDE given in the form [1]:

$$y^{(n)}(x) = f(x) + \lambda_1 \int_a^x h_1(x,t)y(t)dt + \lambda_2 \int_a^b h_2(x,t)y(t)dt, \tag{1.1}$$

$y^{(n)}(x)$ is the n[th] derivatives, $f(x)$, $h_1(x,t)$and $h_2(x,t)$ are given continuous smooth functions, $\lambda_1$and λ_2 are parameters, $y(x)$ unknown function to be determined and a, b are constants. The proposed method was applied to establish series solutions for Eq.(1.1). This method gave accurate results, overcome the difficulty of traditional methods and showing effective and perfect in handling to solve VFIDEs.

2. Fundamental Idea for the Iterative Method

The prime steps of iterative method for any differential equation can be rewritten as [6].

$$L\left(y(x)\right) + N\left(y(x)\right) + f(x) = 0, \text{ with Boundary Conditions } B(y, \frac{dy}{dx}) = 0 \tag{2.1}$$

where x is an independent variable, the known function is $f(x)$, a linear operator is L, non-linear operator is N and B is a boundary operator, the method which proposed as the following. The initial approximation is the primary step in the IM, by assuming that the initial guess y_0(x) is solution of problem $y(x)$ and solution of equation can be solving:

$$L(y_0(x)) + f(x) = 0, B(y_0, \frac{dy_0}{dx}) = 0 \tag{2.2}$$

To generate the next iteration of the solution as follows:

$$L(y_1(x)) + f(x) + N(y_0(x)) = 0, \quad B(y_1, \frac{dy_1}{dx}) = 0 \tag{2.3}$$

After several simple iterative steps of the solution, generic form of such equation is:

$$L(y_{n+1}(x)) + f(x) + N(L(y_n(x)) = 0, \quad B(y_{n+1}, \frac{dy_{n+1}}{dx}) = 0 \tag{2.4}$$

Evidently each iteration of the function y_n(x) represents effectively alone solution for Eq.(2.1).

3.Applying Iterative Method (IM) for solving Volterra - Fredholm Integro - Differential Equations of the 2nd kind.

The IM gives analytic solutions in an infinite chain of ingredients which can be repetitively specified. If such a solution exists the gained series may give the exact solution. Otherwise, the chain gives a parataxis for the solution that accord high reliability grade for resolving these problems. In addition, this method does not require to calculate restrictive assumptions or transformation like other conventional methods. VFIDE of the i^{th} order presented as the form [1]:

$$y^{(i)}(x) = f(x) + \int_0^x h_1(x,t)y(t)dt + \int_a^b h_2(x,t)y(t)dt, \tag{3.1}$$

$$y^{(i)}(x) = \frac{d^i y(x)}{dx^i} \text{ and } y(0), y'(0), \dots, y^{(i-1)}(0) \text{ are initial conditions (ICs)} \tag{3.2}$$

The right extremity includes two disjoin integrations. As for the kernel $h_2(x,t)$, we will debate $h_2(x,t)$ being severable and given via $h_2(x,t) = z(x)w(t)$ (3.3)

Or may be different kernel and given via $h_2(x,t) = h_2(x-t) = z(x) - w(t)$ (3.4)

Thus, the second integrations at the right extremity of (3.1) be

$$\int_a^b h_2(x,t)y(t)dt = \alpha z(x) \tag{3.5}$$

$$\int_a^b h_2(x,t)y(t)dt = \alpha z(x) - \beta \tag{3.6}$$

Via utilizing (3.3) and (3.4) respectively, where $\alpha = \int_a^b w(t)y(t)dt$, $\beta = \int_a^b y(t)dt$ (3.7)

We will apply the purpose method at the Eq.(3.1), so Eq.(3.1) can be express as :

$$L(y(x)) = f(\xi) + \int_0^t h_1(\xi,r)y(r)dr + \alpha z(\xi) \tag{3.8}$$

Or

$$L(y(x)) = f(\xi) + \int_0^t h_1(\xi,r)y(r)dr + \alpha z(\xi) - \beta \tag{3.9}$$

By using Eq.(3.5) and Eq.(3.6) respectively. The IM is utilized by performing two primary steps.

The differential operator $L(y\ (x))$ is the highest order derivative in the Eq.(3.8) and Eq.(3.9), by using the given ICs in Eq.(3.2) and integrating both sides of Eq.(3.8) or Eq.(3.9) from 0 to x.

We will implement the following steps at Eq.(3.8) to get :

$$y(x) = \delta_0 + \int_0^x \left(f(\xi) + \int_0^t h_1(\xi,r)y(r)dr + \alpha z(\xi) \right) d\xi \tag{3.10}$$

Where the function δ_0 is arising from integrating the source term, from applying the given ICs in Eq.(3.2) which are prescribed. To see how this method works following steps are as follows:

Step 1: to get $y_0(x)$ solving $L(y_0(x)) - f(\xi) = 0$ with ICs in Eq.(3.2) (3.11)

So integrate both extremities of Eq.(3.11) of zero to x, we obtain $y_0(x)$ as

$$y_0(x) = \delta_0 + \int_0^x (f(\xi))d\xi$$

Step 2: The next iterate is

$$L\big(y_1(x)\big) - f(\xi) - \int_0^t h_1(\xi,r)y_0(r)dr - \alpha z(\xi) = 0 \text{ with ICs in Eq.(3.2)} \tag{3.12}$$

Solving this equation and integrate both extremities of Eq.(3.12) of zero to x, leads to get $y_1(x)$as

$$y_1(x) = \delta_0 + \int_0^x \left(f(\xi) + \int_0^t h_1(\xi,r)y_0(r)dr + \alpha z(\xi)\right) d\xi$$

Step (3) : After several simple iterative steps of the solution, the generic form of such equation which is $L\big(y_{n+1}(x)\big) - f(\xi) - \int_0^t h_1(\xi,r)y_n(r)dr - \alpha z(\xi) = 0$ with ICs in Eq.(3.2) (3.13)

Solving this equation and integrate both extremities of Eq.(3.13) of zero to x, leads to obtain $y_{n+1}(x)$ as

$$y_{n+1}(x) = \delta_0 + \int_0^x \left(f(\xi) + \int_0^t h_1(\xi,r)y_n(r)dr + \alpha z(\xi)\right) d\xi$$

Evidently each iteration of the function $y_n(x)$ represents effectively solution for Eq.(3.8). Similarly, via the same steps we resolve Eq.(3.9).

4.Numerical Examples

In this section, for veracity and capability of proposed method, we solve four examples of VFIDEs[1].

Example 4.1.

Consider the VFIDE of the 2nd kind with IC as follows

$$y'(x) = 1 + \int_0^x (x-t)\, y(t)dt + \int_0^1 xty(t)dt \text{ , with IC } y(0) = 1 \tag{4.1}$$

Solution:

Via applying same steps as qualified in the prior section, we first resolve the IC to get the initial guess $y_0(x)$, IM start as follows: $L = \frac{dy}{dx}$, $N(y) = 0$,$f(x) = 1$ and

$$\alpha = \int_0^1 ty(t)dt \tag{4.2}$$

So, the primary step is

$$L(y_0) = 1 \text{ with IC } y_0(0) = 1 \tag{4.3}$$

Then, the general relation as follows

$L\big(y_{n+1}(x)\big) - 1 - \int_0^t (t-r) y_n(r) dr - \alpha t = 0$ with ICs $y_{n+1}(0) = 1$ (4.4)

By solving the problem defined in Eq.(4.3), we have $y_0(x) = 1 + x$

First iteration can be got as

$y_1'(x) = 1 + \int_0^t (t-r) y_0(r) dr + \alpha t$ with $y_1(0) = 1$ (4.5)

Thus, the solution of Eq. (4.5) as $y_1(x) = 1 + x + \frac{1}{2!}\alpha x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4$.

The second iteration is

$y_2'(x) = 1 + \int_0^t (t-r) y_1(r) dr + \alpha t$ with $y_2(0) = 1$ (4.6)

Then, the solution of Eq. (4.6) as $y_2(x) = 1 + x + \frac{1}{2!}\alpha x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \frac{1}{5!}\alpha x^5 + \frac{1}{6!}x^6 + \frac{1}{7!}x^7$.

The third iteration is

$y_3'(x) = 1 + \int_0^t (t-r) y_2(r) dr + \alpha t$ with $y_3(0) = 1$ (4.7)

Then, the solution of Eq. (4.7) as

$$y_3(x) = 1 + x + \frac{1}{2!}\alpha x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \frac{1}{5!}\alpha x^5 + \frac{1}{6!}x^6 + \frac{1}{7!}x^7 + \frac{1}{8!}\alpha x^8 + \frac{1}{9!}x^9 + \frac{1}{10!}x^{10}$$

Hence, via same steps, the another solutions can be created from calculating these problems in the generic form

$y_{n+1}'(x) = 1 + \int_0^t (t-r) y_n(r) dr + \alpha t$ with $y_{n+1}(0) = 1$ (4.8)

So, the iteration steps will be as follows

$$y_n(x) = 1 + x + \frac{1}{2!}\alpha x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \frac{1}{5!}\alpha x^5 + \frac{1}{6!}x^6 + \frac{1}{7!}x^7 + \frac{1}{8!}\alpha x^8 + \frac{1}{9!}x^9 + \frac{1}{10!}x^{10} + \cdots$$

To specified α, we replace $y_n(x)$ to Eq.(4.2)to find that $\alpha = 1$.

Substituting $\alpha = 1$ to $y_n(x)$, and utilizing $y(x) = lim_{n\to\infty} y_n(x)$.

Thus, we obtained exact solution $y(x) = e^x$ obtained upon using the Taylor series for e^x.

Example 4.2.

Consider the VFIDE of the 2nd kind with IC as follows

$y'(x) = 9 - 5x - x^2 - x^3 + \int_0^x (x-t)\, y(t) dt + \int_0^1 (x-t) y(t) dt$, with IC $y(0) = 2$ (4.9)

Solution:

To find the initial guess $y_0(x)$, $L = \frac{dy}{dx}$, $N(y) = 0$,$f(x) = 9 - 5x - x^2 - x^3$ and

$$\alpha = \int_0^1 y(t)dt\,, \beta = \int_0^1 ty(t)dt \tag{4.10}$$

So, the primary step is

$$L(y_0) = 9 - 5x - x^2 - x^3 \text{ with IC } y_0(0) = 2 \tag{4.11}$$

Then, the general relation as follows

$$L\big(y_{n+1}(x)\big) - 9 + 5x + x^2 + x^3 - \int_0^t (t-r)y_n(r)dr - \alpha t + \beta = 0\,, y_{n+1}(0) = 2 \tag{4.12}$$

Via solving the primary problem defined in Eq.(4.11),we get:

$y_0(x) = 2 + 9x - \frac{5}{2}x^2 - \frac{1}{3}x^3 - \frac{1}{4}x^4.$

The next step is to find $y_1(x)$ as follows:

$$y_1'(x) = 9 - 5x - x^2 - x^3 + \int_0^t (t-r)y_0(r)dr + \alpha x - \beta \tag{4.13}$$

And has solution: $y_1(x) = 2 + (9-\beta)x - \left(\frac{5-\alpha}{2}\right)x^2 + \frac{1}{8}x^4 - \frac{1}{24}x^5 - \frac{1}{360}x^6 - \frac{1}{840}x^7.$

Hence, via same steps, the another solutions can be created from calculating these problems in the generic form

$$y_{n+1}'(x) = 9 - 5x - x^2 - x^3 + \int_0^t (t-r)y_n(r)dr + \alpha x - \beta, \text{ with IC } y_{n+1}(0) = 2 \tag{4.14}$$

So, the iteration steps $y_2, y_3 \dots y_n$, will be as follows

$$y_2(x) = 2 + (9-\beta)x - \left(\frac{5-\alpha}{2!}\right)x^2 + \left(\frac{3-\beta}{4!}\right)x^4 - \left(\frac{5-\alpha}{5!}\right)x^5 + \frac{1}{1680}x^7 - \frac{1}{8064}x^8 - \frac{1}{181440}x^9 - \frac{1}{604800}x^{10}.$$

$$y_3(x) = 2 + (9-\beta)x - \left(\frac{5-\alpha}{2!}\right)x^2 + \left(\frac{3-\beta}{4!}\right)x^4 - \left(\frac{5-\alpha}{5!}\right)x^5 + \left(\frac{3-\beta}{7!}\right)x^7 - \left(\frac{5-\alpha}{8!}\right)x^8 + \frac{1}{1209600}x^{10} - \frac{1}{7983360}x^{11} - \frac{1}{239500800}x^{12} - \frac{1}{1037836800}x^{13}.$$

$$y_n(x) = 2 + (9-\beta)x - \left(\frac{5-\alpha}{2!}\right)x^2 + \left(\frac{3-\beta}{4!}\right)x^4 - \left(\frac{5-\alpha}{5!}\right)x^5 + \left(\frac{3-\beta}{7!}\right)x^7 - \left(\frac{5-\alpha}{8!}\right)x^8 + \cdots$$

To specified α and β,we replace $y_n(x)$ to Eq.(4.10),and utilize resulting equations to get

$\alpha = 5, \beta = 3.$

Such in turn gives exact solution $y(x) = 2 + 6x$.

Example 4.3.

Consider the VFIDE of the 2nd kind with IC as follows

$$y''(x) = -8 + 6x - 3x^2 + x^3 + \int_0^x y(t)dt + \int_{-1}^1 (1-2xt)y(t)dt, y(0) = 2, y'(0) = 6 \tag{4.15}$$

Solution:

To find the initial guess $y_0(x)$, $L = \frac{d^2y}{dx^2}$, $N(y) = 0$,$f(x) = -8 + 6x - 3x^2 + x^3$ and

$$\alpha = \int_0^1 y(t)dt\,, \beta = \int_0^1 2\,ty(t)dt \tag{4.16}$$

So, the primary step is

$$L(y_0) = -8 + 6x - 3x^2 + x^3 \text{ with IC } y_0(0) = 2\,, {y_0}'(0) = 6 \tag{4.17}$$

Then, the general relation as follows

$$L\big(y_{n+1}(x)\big) + 8 - 6x + 3x^2 - x^3 - \int_0^t y_n(r)dr - \alpha + \beta t = 0, y_{n+1}(0) = 2, y'_{n+1}(0) = 6 \tag{4.18}$$

Via solving the primary problem defined in Eq. (4.17), we get

$y_0(x) = 2 + 6x - 4x^2 + x^3 - \frac{1}{4}x^4 + \frac{1}{20}x^5.$

Hence, via same steps, the another solutions can be created from calculating these problems in the generic form

$$y''_{n+1}(x) = -8 + 6x - 3x^2 + x^3 + \int_0^t y_n(r)dr + \alpha - \beta t, y_{n+1}(0) = 2, y'_{n+1}(0) = 6 \tag{4.19}$$

So, the iteration steps $y_1, y_2 \dots\ y_n$, will be as follows

$y_1(x) = 2 + 6x + (\frac{\alpha}{2} - 4)x^2 + (\frac{4}{3} - \frac{\beta}{6})x^3 - \frac{1}{60}x^5 + \frac{1}{120}x^6 - \frac{1}{840}x^7 + \frac{1}{6720}x^8.$

$y_2(x) = 2 + 6x + (\frac{\alpha}{2} - 4)x^2 + (\frac{4}{3} - \frac{\beta}{6})x^3 + (\frac{\alpha}{120} - \frac{1}{60})x^5 + (\frac{1}{90} - \frac{\beta}{720})x^6 - \frac{1}{20160}x^8 + \frac{1}{60480}x^9 - \frac{1}{604800}x^{10} + \frac{1}{6652800}x^{11}.$

$y_3(x) = 2 + 6x + (\frac{\alpha}{2} - 4)x^2 + (\frac{4}{3} - \frac{\beta}{6})x^3 + (\frac{\alpha}{120} - \frac{1}{60})x^5 + \left(\frac{1}{90} - \frac{\beta}{720}\right)x^6 + (\frac{\alpha}{40320} - \frac{1}{20160})x^8 + (\frac{1}{45360} - \frac{\beta}{362880})x^9 - \frac{1}{19958400}x^{11} + \frac{1}{79833600}x^{12} - \frac{1}{1037836800}x^{13} + \frac{1}{14529715200}x^{14}.$

$$y_n(x) = 2 + 6x + (\frac{\alpha}{2} - 4)x^2 + \left(\frac{4}{3} - \frac{\beta}{6}\right)x^3 + (\frac{\alpha}{120} - \frac{1}{60})x^5 + \left(\frac{1}{90} - \frac{\beta}{720}\right)x^6 + \left(\frac{\alpha}{40320} - \frac{1}{20160}\right)x^8 + \left(\frac{1}{45360} - \frac{\beta}{362880}\right)x^9 + \cdots$$

To specified α and β, we replace $y_n(x)$ to Eq.(4.16),and utilize resulting equations to get

$\alpha = 2, \beta = 8$. Thus, $y(x) = 2 + 6x - 3x^2$. Then, we get exact solution for this problem readily.

Example 4.4.

Consider the VFIDE of the 2nd kind with IC as follows

$$y'''(x) = -\frac{1}{2}x^2 + \int_0^x y(t)dt + \int_{-\pi}^{\pi} xy(t)dt,\ y(0) = y'(0) = -y''(0) = 1 \tag{4.20}$$

Solution:

To find to find the initial guess $y_0(x)$, $L=\frac{d^3y}{dx^3}$, , $N(y)=0$,$f(x)=-\frac{1}{2}x^2$ and

$$\alpha=\int_{-\pi}^{\pi} y(t)dt \tag{4.21}$$

So, the primary step is

$$L(y_0)=-\tfrac{1}{2}x^2 \text{ with ICs } y_0(0)=y_0'(0)=-y_0''(0)=1 \tag{4.22}$$

Then, the general relation as follows

$$L\big(y_{n+1}(x)\big)+\tfrac{1}{2}x^2-\int_0^t y_n(r)dr-\alpha t=0\,, y_{n+1}(0)=y_{n+1}'(0)=-y_{n+1}''(0)=1 \tag{4.23}$$

Via solving the primary problem defined in Eq. (4.22), we get $y_0(x)=1+x-\frac{1}{2!}x^2-\frac{1}{5!}x^5$.

Hence, via same steps, the another solutions can be created from calculating these problems in the generic form

$$y_{n+1}'''(x)=-\tfrac{1}{2}x^2+\int_0^t y_n(r)dr+\alpha t\,, y_{n+1}(0)=y_{n+1}'(0)=-y_{n+1}''(0)=1 \tag{4.24}$$

The iterations steps y will be as follows

$$y_1(x)=1+x-\tfrac{1}{2!}x^2+\tfrac{1}{4!}(1+\alpha)x^4-\tfrac{1}{6!}x^6-\tfrac{1}{9!}x^9.$$

$$y_2(x)=1+x-\tfrac{1}{2!}x^2+\tfrac{1}{4!}(1+\alpha)x^4-\tfrac{1}{6!}x^6+\tfrac{1}{8!}(1+\alpha)x^8-\tfrac{1}{10!}x^{10}-\tfrac{1}{13!}x^{13}.$$

$$y_3(x)=1+x-\tfrac{1}{2!}x^2+\tfrac{1}{4!}(1+\alpha)x^4-\tfrac{1}{6!}x^6+\tfrac{1}{8!}(1+\alpha)x^8-\tfrac{1}{10!}x^{10}+\tfrac{1}{12!}(1+\alpha)x^{12}-\tfrac{1}{14!}x^{14}-\tfrac{1}{17!}x^{17}.$$

$$y_n(x)=1+x-\tfrac{1}{2!}x^2+\tfrac{1}{4!}(1+\alpha)x^4-\tfrac{1}{6!}x^6+\tfrac{1}{8!}(1+\alpha)x^8-\tfrac{1}{10!}x^{10}+\tfrac{1}{12!}(1+\alpha)x^{12}-\tfrac{1}{14!}x^{14}+\cdots$$

To specified α , we replace $y_n(x)$ into Eq.(4.21), and utilize resulting equations to get $\alpha=0$.

Such in turn gives chain solution

$$y(x)=x+(1-\tfrac{1}{2!}x^2+\tfrac{1}{4!}x^4-\tfrac{1}{6!}x^6+\tfrac{1}{8!}x^8-\tfrac{1}{10!}x^{10}+\tfrac{1}{12!}x^{12}-\cdots)$$

This convergent to exact solution $y(x)=x+cos\, x$.

The essence of this method, the IM in comparison with the other analytical methods does not need large computations such as Lagrange multiplier in the VIM or any complex assumptions like nonlinear Adomian's polynomials in the ADM. It also does not need a long transformation or constructive homotopy polynomials in HPM. Furthermore, this method proved that it is efficient in overcoming the difficulties in calculating and solving VFIDEs with easy steps.

5. Error Analysis

Error represent a pivotal role in approximate solutions, whenever a least error is appear, more accurate solution and closer to the exact solution is gained, that refers the accuracy and rapidity of suggested method. Approximate solution are ordinarily utilized to get solutions to several problems that cannot be resolved in analytic mathematical methods. Thus, there is an error value we have to compute. If one can locate the accurate error, hence an accurate solution will be appeared. So finding the accurate error is impossible. Then, we seek to get an approximation of the error (i.e. a worth that is not overridden via error). As for an error analysis of the outcomes, we present the successive errors as:

$$\mathbb{E}_n = \|y_{n+1} - y_n\|$$

For n= 0, 1, 2 , ... which are the variances between two successive iterate solutions. Of course we transact with an analytical continuous solution therefore to compute these differences we utilize the L_2-norm[6].

$$\|y_{n+1} - y_n\| = \sqrt{\int (y_{n+1} - y_n)^2}$$

The error idiom $\mathbb{E}_n$ between two successive for ours examples

n	$\mathbb{E}_2$ of Ex1	$\mathbb{E}_2$ of Ex2	$\mathbb{E}_2$ of Ex3	$\mathbb{E}_2$ of Ex4
0	0.2993783977579072	0.5807300991843273	0.1450473971972797	0.01601053752244971
1	$2.947354166658968E^{-003}$	$2.803297368589165E^{-002}$	$2.986049759020634E^{-003}$	$6.586732287114899E^{-006}$
2	6.706429340123424E-006	$1.219278114171868E^{-004}$	$8.564787943449617E^{-006}$	$4.462944263652475E^{-010}$

6.Conclusion

In this work, a reliable iterative method of the solution for Volterra - Fredholm Integro Differential Equations is successfully performed. Methodology and the analytical procedure are done in efficient manner and very straightforward. This method is more proficient than its traditional one as it is less complicated, needs less time to get to the solution and most importantly the exact solution is achieved in a scarce number of iterations. The precision of the obtained solution can be ameliorated via taking more terms in the solution. In many cases, series solution get by iterative method can be written in exact closed form with easy steps. Authors can expand such technique to some other problems of partial and ordinary differential equations which may be debated in further work.

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