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Estimating the Reliability of a Component between Two Stresses from Gompertz-Frechet Model

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ABSTRACT

In this paper, the reliability of the stress-strength model is derived for probability $P(Y_1<X<Y_2)$ of a component strength X between two stresses Y_1, Y_2. When X and Y_1, Y_2 flowing Gompertz Fréchet distribution with unknown shape parameters θ, λ and known parameters α, β, γ. Different methods used to estimate reliability R and Gompertz Fréchet distribution parameters which are Maximum Likelihood, Least square, Weighted Least square, Regression and Ranked set sampling methods, and compare between these estimates based on a simulation study by mean square error criteria. The comparison confirms that the performance of the maximum likelihood estimator works better than the other estimators.

1. Introduction

A significant special case where strength X should not only be greater than stress Y_1, but also lower than stress Y_2. Many electronic components, for example, are unable to operate at very high voltages or at very low voltages. Similarly, the blood pressure of a person has two systolic and diastolic thresholds, and the blood pressure of any person should be within these limits [7]. The stress-strength models of $P(Y_1<X<Y_2)$ have been studied in many fields of research, such as engineering, psychology, genetics, clinical trials and so on [10].

Some attempts have been made to define new classes of distributions to extend well-known families and at the same time provide great flexibility in modeling data in practice. Many distribution families were employing more than one parameter to generate modern distribution

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have been suggested in the statistical literature. Some renowned generators are exponentiated half-logistic generated family by Cordeiro, Alizadeh, and Ortega (2014), Lomax-G by Cordeiro, Ortega, et al. (2014), Kumaraswamy odd log-logistic-G by Alizadeh, Emadi, et al. (2015), Kumaraswamy Marshall-Olkin by Alizadeh, Tahis, et al. (2015), type 1 half- logistic family by Cordeiro et al. (2016), odd generalized exponential-G by Tahir et al. (2015), and Gompertz-Frechet by Alizadeh et al. (2016), some special models of the Go-G family Gompertz-Weibul, Gompertz-Gamma, Gompertz-beta, Gompertz-log logistic , Gompertz-Fréchet. Define the CDF of the Gompertz-G family by [4]:

$$\mathrm{F}(x) = \int_0^{-\log[1-G(x;\epsilon)]} \theta e^{\gamma t} e^{-\frac{\theta}{\gamma}(e^{\gamma t}-1)} dt = 1 - e^{\frac{\theta}{\gamma}\{1-[1-G(x;\epsilon)]^{-\gamma}\}} \quad \text{.... (1)}$$

Where $\mathrm{G}(x,\epsilon)$ is the baseline CDF depending on a parameter vector ϵ and $\gamma > 0$ and $\theta > 0$ are two shape parameter . and define the pdf of the Gompertz-G family by:

$$\mathrm{f}(x;\theta,\gamma,\epsilon) = \theta \mathrm{g}(x;\epsilon)\,[1-\mathrm{G}(x;\epsilon)]^{-\gamma-1}\, e^{\frac{\theta}{\gamma}\{1-[1-G(x;\epsilon)]^{-\gamma}\}} \quad \text{... (2)}$$

The CDF and pdf of Fréchet distribution are [12] :

$$\mathrm{G}(x,\alpha,\beta) = \exp\left[-(\tfrac{\alpha}{x})^{\beta}\right] \quad \text{... (3)}$$

and $\mathrm{g}(x,\alpha,\beta) = \beta\,\alpha^{\beta}\,x^{-\beta-1}\exp\left[-(\frac{\alpha}{x})^{\beta}\right]$ ... (4)

respectively , where $\alpha > 0$ scale parameter and $\beta > 0$ shape parameter

The CDF and pdf of the Gompertz-Frechet (GF) distribution is obtained by substituting eqution (3) in equation (1) and equations (3) and (4) in equation (2) respectively given by [11] :

$$\mathrm{F}(x,\theta,\alpha,\beta,\gamma) = 1 - \exp\left[\frac{\theta}{\gamma}\left(1 - \left\{\exp\left[-(\frac{\alpha}{x})^{\beta}\right]\right\}^{-\gamma}\right)\right] \quad \text{... (5)}$$

$$\mathrm{f}(x,\theta,\alpha,\beta,\gamma) = \theta\beta\alpha^{\beta}x^{-\beta-1}\exp\left[-(\frac{\alpha}{x})^{\beta}\right]\left\{1-\exp\left[-(\frac{\alpha}{x})^{\beta}\right]\right\}^{-\gamma-1} *$$

$$\exp\left[\frac{\theta}{\gamma}\left(1 - \left\{\exp\left[-(\frac{\alpha}{x})^{\beta}\right]\right\}^{-\gamma}\right)\right] \quad \text{... (6)}$$

where > 0 , $\beta > 0$ and $\gamma > 0$ are shape parameters and $\alpha > 0$ is the scale parameter .

The main aim of this paper, is to obtain a mathematical formula of Reliability R of probability P(Y<X), based on Gompertz Fréchet distribution in section 2. In order to find the estimators of the shape parameters (θ, λ) for the two random variables, five different estimation methods (Maximum Likelihood, Least square Method, Weighted Least square Method, Regression Method and Ranked set sampling Method) are used and then the reliability parameter is estimated in section 3. A simulation study was conducted to compare the performance of the five different estimators of the reliability in section 4, based on six experiments of shape parameter values and at different sample sizes of (15) for small, (30) for medium and (90) for large sample sizes. The comparison is made by the Mean Square Error (MSE), and the conclusions are discussed in section 6.

2. The Reliability expression:

Let $X \sim GF(\theta, \alpha, \beta, \gamma)$ be an independent random strength variable and $Y_1 \sim GF(\lambda_1, \alpha, \beta, \gamma)$, $Y_2 \sim GF(\lambda_2, \alpha, \beta, \gamma)$ are two independent random stresses variables .

Let we have $u_x = 1 - \exp\left[-(\frac{\alpha}{x})^\beta\right]$

and we can write $\exp\left[-(\frac{\alpha}{x})^\beta\right] = 1 - u_x$

Then F(x) and f(x) can be rewrite as:

$$F(x, \theta, \alpha, \beta, \gamma) = 1 - \exp\left[\frac{\theta}{\gamma}\left(1 - u_x^{-\gamma}\right)\right]$$

$$f(x, \theta, \alpha, \beta, \gamma) = \theta\beta\alpha^\beta x^{-\beta-1}\left(1 - u_x\right) u_x^{-\gamma-1} \exp\left[\frac{\theta}{\gamma}\left(1 - u_x^{-\gamma}\right)\right] \text{ and}$$

$$G(y_1, \theta, \alpha, \beta, \gamma) = 1 - \exp\left[\frac{\lambda_1}{\gamma}\left(1 - u_{y_1}^{-\gamma}\right)\right]$$

$$G(y_2, \theta, \alpha, \beta, \gamma) = 1 - \exp\left[\frac{\lambda_2}{\gamma}\left(1 - u_{y_2}^{-\gamma}\right)\right]$$

The reliability given by [1] :

$$R = p(Y_1 < X < Y_2) = \int_0^\infty p(Y_1 < X, X < Y_2)\, f(x)dx = \int_0^\infty G_{y_1}(x)\, \overline{G_{y_2}}(x) f(x)dx$$

$$= \int_0^\infty G_{y_1}(x)\left[1 - G_{y_2}(x)\right] f(x)dx$$

$$= \int_0^\infty \left(1 - \exp\left[\frac{\lambda_1}{\gamma}\left(1 - u_x^{-\gamma}\right)\right]\right) f(x)dx - \int_0^\infty \left(1 - \exp\left[\frac{\lambda_1}{\gamma}\left(1 - u_x^{-\gamma}\right)\right]\right)$$

$$* \left(1 - \exp\left[\frac{\lambda_2}{\gamma}\left(1 - u_x^{-\gamma}\right)\right]\right) f(x)dx$$

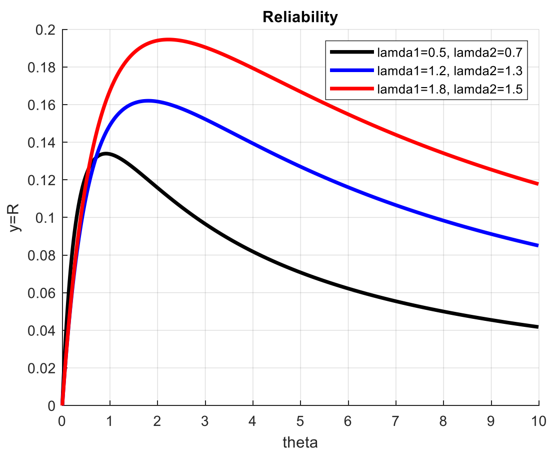
$$= 1 - \frac{\theta}{\theta+\lambda_1} - 1 + \frac{\theta}{\theta+\lambda_1} + \frac{\theta}{\theta+\lambda_2} - \frac{\theta}{\theta+\lambda_1+\lambda_2}$$

$$R = \frac{\theta}{\theta+\lambda_2} - \frac{\theta}{\theta+\lambda_1+\lambda_2} \quad \text{.... (7)}$$

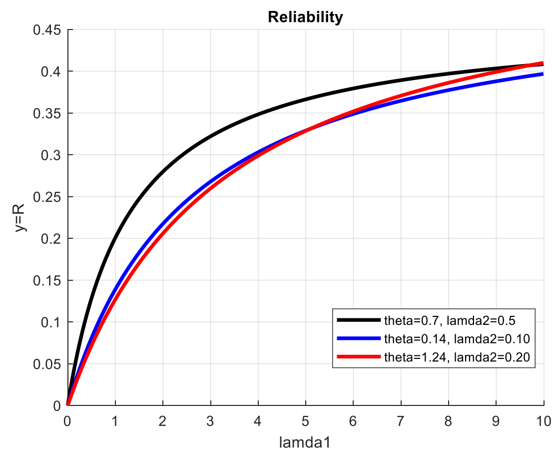
Figure (1) shows the change in the reliability curve with the effect of the strength parameter values θ in three different situations of the parameter values λ_1, λ_2 where reliability increases and begins to decrease in this case.

Figure (2) shows the change in the reliability curve with the effect of stress parameter values λ_1 in three different cases of parameter values of θ, λ_2 where reliability in this case is increasing, but it reaches a certain place and begins to decrease.

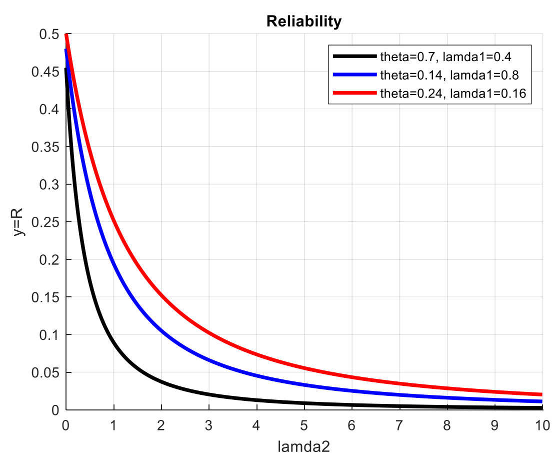
Figure (3) shows the change in the reliability curve with the effect of λ_2 strength parameter values in three different states of parameter values of $\theta, \lambda_1$where reliability in this case is decreasing.



Figure(1): the Reliability curve aginst θ



Figure(2): the Reliability curve aginst λ_1



Figure(3): the Reliability curve aginst λ_2

3. Estimation methods:

In this section, the shape parameters θ and λ of the GF and reliability are estimated using five methods of estimation: Maximum likelihood, Least square, Weighted least square, Regression and Rank set sampling methods.

3.1. Maximum likelihood (MLE):

The MLE is one method of the most important and common parameter estimation methods. In 1922 R. A. Fisher introduced the method of maximum likelihood. Let $x_1, x_2, \dots, x_n$ be strength random sample of size n from GF(θ,α,β,γ) where θ is unknown parameter and α,β,γ are known. the likelihood function given by [12] :

$$L = \prod_{i=1}^{n}[f(x_1, x_2, \dots, x_n; \theta, \alpha, \beta, \gamma)] \quad L=\prod_{i=1}^{n}\left[\theta\beta\alpha^{\beta}x_i^{-\beta-1}\exp\left[-\left(\frac{\alpha}{x_i}\right)^{\beta}\right]\left\{1-\exp\left[-\left(\frac{\alpha}{x_i}\right)^{\beta}\right]\right\}^{-\gamma-1} * \exp\left[\frac{\theta}{\gamma}(1-)\left\{1-\exp\left[-\left(\frac{\alpha}{x_i}\right)^{\beta}\right]\right\}^{-\gamma}\right]\right]$$

$$L=\theta^{n}\beta^{n}\alpha^{n\beta}\prod_{i=1}^{n}x_i^{-\beta-1}\exp\left(\sum_{i=1}^{n}\left[-\left(\frac{\alpha}{x_i}\right)^{\beta}\right]\right)\prod_{i=1}^{n}\left\{1-\exp\left[-\left(\frac{\alpha}{x_i}\right)^{\beta}\right]\right\}^{-\gamma-1} * \exp\left(\sum_{i=1}^{n}\left[\frac{\theta}{\gamma}\left(1-\left\{1-\exp\left[-\left(\frac{\alpha}{x_i}\right)^{\beta}\right]\right\}^{-\gamma}\right)\right]\right)$$

$$\ln L = n\ln\theta + n\ln\beta + n\beta\ln\alpha - (\beta+1)\sum_{i=1}^{n}\ln x_i + \sum_{i=1}^{n}\left[-\left(\frac{\alpha}{x_i}\right)^{\beta}\right] - (\gamma+1)\sum_{i=1}^{n}\ln\left\{1-\exp\left[-\left(\frac{\alpha}{x_i}\right)^{\beta}\right]\right\} + \frac{\theta}{\gamma}\sum_{i=1}^{n}\left(1-\left\{1-\exp\left[-\left(\frac{\alpha}{x_i}\right)^{\beta}\right]\right\}^{-\gamma}\right)$$

$$\frac{\partial \ln L}{\partial\theta} = \frac{n}{\hat{\theta}} + \frac{1}{\gamma}\sum_{i=1}^{n}\left(1-\left\{1-\exp\left[-\left(\frac{\alpha}{x_i}\right)^{\beta}\right]\right\}^{-\gamma}\right) = 0$$

$$\hat{\theta}_{MLE} = \frac{-n\gamma}{\sum_{i=1}^{n}\left(1-\left\{1-\exp\left[-\left(\frac{\alpha}{x_i}\right)^{\beta}\right]\right\}^{-\gamma}\right)} \qquad \dots (8)$$

In the same way, let Y_1 , Y_2 stress random variable have GF($\lambda_1, \alpha, \beta, \gamma$), GF($\lambda_2, \alpha, \beta, \gamma$), with sample size m_1 , m_2 respectively and the MLE estimator of unknown parameters λ_1, λ_2 are :

$$\hat{\lambda}_{1MLE} = \frac{-m_1\gamma}{\sum_{j_1=1}^{m_1}\left(1-\left\{1-\exp\left[-\left(\frac{\alpha}{y_{1j_1}}\right)^{\beta}\right]\right\}^{-\gamma}\right)} \qquad \dots (9)$$

$$\hat{\lambda}_{2MLE} = \frac{-m_2\gamma}{\sum_{j_2=1}^{m_2}\left(1-\left\{1-\exp\left[-\left(\frac{\alpha}{y_{2j_2}}\right)^{\beta}\right]\right\}^{-\gamma}\right)} \qquad \dots (10)$$

Then by substitute equations (8) , (9) and (10) in equation (7) we get :

$$\hat{R}_{MLE} = \frac{\hat{\theta}_{MLE}}{\hat{\theta}_{MLE}+\hat{\lambda}_{2MLE}} - \frac{\hat{\theta}_{MLE}}{\hat{\theta}_{MLE}+\hat{\lambda}_{1MLE}+\hat{\lambda}_{2MLE}} \qquad \dots (11)$$

3.2. Least Square Estimation Method (LS):-

The German mathematician Carl Friedrich Gauss studied the least square method as early as (1794), until (1809) he published the method. For model fitting, this method of estimation is very popular, especially in linear regression and non-linear regression [3]. The following equation for minimization [8] :

$$S = \sum_{i=1}^{n}(F(x_i) - E(F(x_i)))^2 \quad \text{... (12)}$$

Suppose $x_1, x_2, \dots \dots x_n$ be random sample have GF$(\theta, \alpha, \beta, \gamma)$ distribution with the sample size n.

The procedure attempts to minimize the following function with respect to θ and α, β, γ will get as:

$$S(\theta, \alpha, \beta, \gamma) = \sum_{i=1}^{n}\left(\left(1 - \exp\left[\frac{\theta}{\gamma}\left(1 - \left\{1 - \exp\left[-\left(\frac{\alpha}{x_i}\right)^{\beta}\right]\right\}^{-\gamma}\right)\right]\right) - P_i\right)^2 \quad \text{... (13)}$$

where $E(F(x_i)) = P_i$ and P_i is the plotting position, and $P_i = \frac{i}{n+1}, i = 1,2, \dots, n$

To obtain the formula of $F(x_i)$ by equation (5):

$$F(x_i) = 1 - \exp\left[\frac{\theta}{\gamma}\left(1 - \left\{1 - \exp\left[-\left(\frac{\alpha}{x_i}\right)^{\beta}\right]\right\}^{-\gamma}\right)\right]$$

$$1 - F(x_i) = \exp\left[\frac{\theta}{\gamma}\left(1 - \left\{1 - \exp\left[-\left(\frac{\alpha}{x_i}\right)^{\beta}\right]\right\}^{-\gamma}\right)\right]$$

$$\text{Ln}(1 - F(x_i)) = \left[\frac{\theta}{\gamma}\left(1 - \left\{1 - \exp\left[-\left(\frac{\alpha}{x_i}\right)^{\beta}\right]\right\}^{-\gamma}\right)\right] \quad \text{... (14)}$$

Here $x_{(i)}$ is the i:th order statistics of the random sample of the size n from GF hence for the GF. to obtain the LS estimation $\hat{\theta}_{LS}$ of the parameter θ can be define following equation (14)

$$S(\theta, \alpha, \beta, \gamma) = \sum_{i=1}^{n}\left(\left[\frac{\theta}{\gamma}\left(1 - \left\{1 - \exp\left[-\left(\frac{\alpha}{x_i}\right)^{\beta}\right]\right\}^{-\gamma}\right)\right] - q_i\right)^2 \quad \text{... (15)}$$

Where $q_i = \text{Ln}(1 - F(x_i)) = \text{Ln}(1 - P_i)$

By taking derivative to equation (15) with respect to the parameter θ and equating result to the zero :

$$\frac{dS(\theta,\alpha,\beta,\gamma)}{d\theta} = \sum_{i=1}^{n} 2\left(\left[\frac{\theta}{\gamma}\left(1 - \left\{1 - \exp\left[-\left(\frac{\alpha}{x_{(i)}}\right)^{\beta}\right]\right\}^{-\gamma}\right)\right] - q_i\right)$$

$$* \frac{1}{\gamma}\left(1 - \left\{1 - \exp\left[-\left(\frac{\alpha}{x_{(i)}}\right)^{\beta}\right]\right\}^{-\gamma}\right)$$

$$\frac{\hat{\theta}}{\gamma^2}\sum_{i=1}^{n}\left(1 - \left\{1 - \exp\left[-\left(\frac{\alpha}{x_{(i)}}\right)^{\beta}\right]\right\}^{-\gamma}\right)^2 - q_i \frac{1}{\gamma}\sum_{i=1}^{n}\left(1 - \left\{1 - \exp\left[-\left(\frac{\alpha}{x_{(i)}}\right)^{\beta}\right]\right\}^{-\gamma}\right) = 0$$

$$\hat{\theta}_{LS}=\frac{\gamma\sum_{i=1}^{n}q_i\left(1-\left\{1-\exp\left[-\left(\frac{\alpha}{x_{(i)}}\right)^{\beta}\right]\right\}^{-\gamma}\right)}{\sum_{i=1}^{n}\left(1-\left\{1-\exp\left[-\left(\frac{\alpha}{x_{(i)}}\right)^{\beta}\right]\right\}^{-\gamma}\right)^{2}} \quad \text{... (16)}$$

In the same way, let Y_1 , Y_2 stress random variable have GF($\lambda_1,\alpha,\beta,\gamma$), GF($\lambda_2,\alpha,\beta,\gamma$), with sample size m_1 , m_2 respectively and the LS estimator of unknown parameters λ_1,λ_2 are :

$$\hat{\lambda}_{1LS}=\frac{\gamma\sum_{j_1=1}^{m_1}q_{j_1}\left(1-\left\{1-\exp\left[-\left(\frac{\alpha}{y_{1(j_1)}}\right)^{\beta}\right]\right\}^{-\gamma}\right)}{\sum_{j_1=1}^{m_1}\left(1-\left\{1-\exp\left[-\left(\frac{\alpha}{y_{1(j_1)}}\right)^{\beta}\right]\right\}^{-\gamma}\right)^{2}} \quad \text{.... (17)}$$

$$\hat{\lambda}_{2LS}=\frac{\gamma\sum_{j_2=1}^{m_2}q_{j_2}\left(1-\left\{1-\exp\left[-\left(\frac{\alpha}{y_{2(j_2)}}\right)^{\beta}\right]\right\}^{-\gamma}\right)}{\sum_{j_2=1}^{m_2}\left(1-\left\{1-\exp\left[-\left(\frac{\alpha}{y_{2(j_2)}}\right)^{\beta}\right]\right\}^{-\gamma}\right)^{2}} \quad \text{.... (18)}$$

Then by substitute (16),(17) and (18) in (7), we get :

$$\hat{R}_{\mathrm{LS}}=\frac{\hat{\theta}_{LS}}{\hat{\theta}_{LS}+\hat{\lambda}_{2LS}}-\frac{\hat{\theta}_{LS}}{\hat{\theta}_{LS}+\hat{\lambda}_{1LS}+\hat{\lambda}_{2LS}} \quad \text{.... (19)}$$

3.3. Weighted Least Squares Estimation Method (WLS):-

This approach represents the action of the model's random errors, and can be used for parameter functions that are either linear or nonlinear. It works by integrating the fitting criteria with additional non-negative weights or constants associated with all data points. The size of the weight indicates the accuracy of the details found in the relevant observations [14]. The weighted least-square method can be used to minimize the following equation [8]:

$$\mathrm{Q}=\sum_{i=1}^{n}W_i\left(F\left(x_{(i)}\right)-E\left(F\left(x_{(i)}\right)\right)\right)^2 \quad \text{.... (20)}$$

Where $W_i=\frac{1}{var[F(x_{(i)})]}=\frac{(n+1)^2(n+2)}{i(n-i+1)}, i=1,2,\dots,n$

Let a random sample $(x_1,x_2,\dots\dots x_n)$ with size n take from GF$(\theta,\alpha,\beta,\gamma)$ distribution. The procedure attempts to minimize the following function with respect to θ,α,β and γ will get as :

$$\mathrm{Q}(,\alpha,\beta,\gamma)=\sum_{i=1}^{n}W_i\left(\left(1-\exp\left[\frac{\theta}{\gamma}\left(1-\left\{1-\exp\left[-\left(\frac{\alpha}{x_i}\right)^{\beta}\right]\right\}^{-\gamma}\right)\right]\right)-P_i\right)^2 \quad \text{....(21)}$$

Using the steps in equations (13) and (15) will get as:

$$Q(,\alpha,\beta,\gamma) = \sum_{i=1}^{n} W_i \left(\left[\frac{\theta}{\gamma} \left(1 - \left\{ 1 - \exp\left[-\left(\frac{\alpha}{x_{(i)}} \right)^{\beta} \right] \right\}^{-\gamma} \right) \right] - q_i \right)^2 \quad \text{...(22)}$$

By taking derivative of equation (22) with respect to the parameter θ and equating result to the zero, we get $\hat{\theta}_{WLS}$ as :

$$\hat{\theta}_{WLS} = \frac{\gamma \sum_{i=1}^{n} W_i q_i \left(1 - \left\{ 1 - \exp\left[-\left(\frac{\alpha}{x_{(i)}} \right)^{\beta} \right] \right\}^{-\gamma} \right)}{\sum_{i=1}^{n} W_i \left(1 - \left\{ 1 - \exp\left[-\left(\frac{\alpha}{x_{(i)}} \right)^{\beta} \right] \right\}^{-\gamma} \right)^2} \quad \text{...(23)}$$

In the same way, we can estimate λ_1, λ_2 as bellow:

$$\hat{\lambda}_{1WLS} = \frac{\gamma \sum_{j_1=1}^{m_1} W_{j_1} q_{j_1} \left(1 - \left\{ 1 - \exp\left[-\left(\frac{\alpha}{y_{1(j_1)}} \right)^{\beta} \right] \right\}^{-\gamma} \right)}{\sum_{j_1=1}^{m_1} W_{j_1} \left(1 - \left\{ 1 - \exp\left[-\left(\frac{\alpha}{y_{1(j_1)}} \right)^{\beta} \right] \right\}^{-\gamma} \right)^2} \quad \text{... (24)}$$

$$\hat{\lambda}_{2WLS} = \frac{\gamma \sum_{j_2=1}^{m_2} W_{j_2} q_{j_2} \left(1 - \left\{ 1 - \exp\left[-\left(\frac{\alpha}{y_{2(j_2)}} \right)^{\beta} \right] \right\}^{-\gamma} \right)}{\sum_{j_2=1}^{m_2} W_{j_2} \left(1 - \left\{ 1 - \exp\left[-\left(\frac{\alpha}{y_{2(j_2)}} \right)^{\beta} \right] \right\}^{-\gamma} \right)^2} \quad \text{... (25)}$$

Then by substitute (23),(24) and (25) in (7) we get :

$$\hat{R}_{\mathrm{WLS}} = \frac{\hat{\theta}_{WLS}}{\hat{\theta}_{WLS} + \hat{\lambda}_{2WLS}} - \frac{\hat{\theta}_{WLS}}{\hat{\theta}_{WLS} + \hat{\lambda}_{1WLS} + \hat{\lambda}_{2WLS}} \quad \text{... (26)}$$

3.4.Regression estimation method (Rg):-

Regression is the basic method of conceptually evaluating functional relations between variables. Relationships are expressed as an equation or model which relates Y response variable to X one or more expository variables. By the traditional regression equation, the simple true relations can be approximated [5]:

$$Z_i = \mathrm{a} + \mathrm{b}\, u_i + e_i \quad \text{... (27)}$$

Where Z_i is dependent variable, u_i is independent variable, e_i is the error random variable and a, b are called regression coefficients where a is the intercept and b is the slop [11].

Let $x_1, x_2, \ldots, x_n$ be random strength sample of size (n) from GF$(\theta,\alpha,\beta,\gamma)$, then the GF estimators of the unknown parameter θ ,can be obtained by taking the natural logarithm to equation (5), we get:

$$\ln(1-\mathrm{F}(x_i)) \;=\; \frac{\theta}{\gamma}\left(1-\left\{1-\exp\left[-\left(\frac{\alpha}{x_i}\right)^{\beta}\right]\right\}^{-\gamma}\right) \qquad \dots (28)$$

Substituted plotting position p_i instead of $\mathrm{F}(x_i)$ in Eq. (28), we get:

$$\ln(1-p_i) \;=\; \frac{\theta}{\gamma}\left(1-\left\{1-\exp\left[-\left(\frac{\alpha}{x_i}\right)^{\beta}\right]\right\}^{-\gamma}\right) \qquad \dots (29)$$

By comparison between Eq. (27) and Eq. (29), we can get:

$$Z_i=\ln(1-p_i)\;,\mathrm{a}=0\;,\mathrm{b}=\theta\;,u_i=\frac{1}{\gamma}\left(1-\left\{1-\exp\left[-\left(\frac{\alpha}{x_i}\right)^{\beta}\right]\right\}^{-\gamma}\right) \qquad \dots (30)$$

Where b can be estimated by minimizing summation of the squared error with respect to b then we get [9]:

$$\hat{\mathrm{b}}=\frac{n\sum_{i=1}^{n}Z_i u_i-\sum_{i=1}^{n}Z_i\sum_{i=1}^{n}u_i}{n\sum_{i=1}^{n}(u_i)^2-\left(\sum_{i=1}^{n}u_i\right)^2} \qquad \dots (31)$$

By substation Eq.(30) in Eq.(31), then GF estimator for the unknown parameter θ ,says $\hat{\theta}_{Rg}$;is

$$\hat{\theta}_{Rg}=\frac{\frac{n}{\gamma}\sum_{i=1}^{n}\mathrm{Ln}(1-p_i)\left(1-\left\{1-\exp\left[-\left(\frac{\alpha}{x_i}\right)^{\beta}\right]\right\}^{-\gamma}\right)-\frac{1}{\gamma}\sum_{i=1}^{n}\mathrm{Ln}(1-p_i)\sum_{i=1}^{n}\left(1-\left\{1-\exp\left[-\left(\frac{\alpha}{x_i}\right)^{\beta}\right]\right\}^{-\gamma}\right)}{\frac{n}{\gamma^2}\sum_{i=1}^{n}\left(\left(1-\left\{1-\exp\left[-\left(\frac{\alpha}{x_i}\right)^{\beta}\right]\right\}^{-\gamma}\right)\right)^2-\left(\frac{1}{\gamma}\sum_{i=1}^{n}\left(1-\left\{1-\exp\left[-\left(\frac{\alpha}{x_i}\right)^{\beta}\right]\right\}^{-\gamma}\right)\right)^2} \qquad \dots. (32)$$

In same way we can estimate λ_1,λ_2 as bellow:

$$\hat{\lambda}_{1Rg}=\frac{\frac{m_1}{\gamma}\sum_{j_1=1}^{m_1}\mathrm{Ln}(1-p_{j_1})\left(1-\left\{1-\exp\left[-\left(\frac{\alpha}{y_{1j_1}}\right)^{\beta}\right]\right\}^{-\gamma}\right)-\frac{1}{\gamma}\sum_{j_1=1}^{m_1}\mathrm{Ln}(1-p_{j_1})\sum_{j_1=1}^{m_1}\left(1-\left\{1-\exp\left[-\left(\frac{\alpha}{y_{1j_1}}\right)^{\beta}\right]\right\}^{-\gamma}\right)}{\frac{m_1}{\gamma^2}\sum_{j_1=1}^{m_1}\left(\left(1-\left\{1-\exp\left[-\left(\frac{\alpha}{y_{1j_1}}\right)^{\beta}\right]\right\}^{-\gamma}\right)\right)^2-\left(\frac{1}{\gamma}\sum_{j_1=1}^{m_1}\left(1-\left\{1-\exp\left[-\left(\frac{\alpha}{y_{1j_1}}\right)^{\beta}\right]\right\}^{-\gamma}\right)\right)^2}$$

....(33)

$$\hat{\lambda}_{2Rg}=\frac{\frac{m_2}{\gamma}\sum_{j_2=1}^{m_2}\mathrm{Ln}(1-p_{j_2})\left(1-\left\{1-\exp\left[-\left(\frac{\alpha}{y_{2j_2}}\right)^{\beta}\right]\right\}^{-\gamma}\right)-\frac{1}{\gamma}\sum_{j_2=1}^{m_2}\mathrm{Ln}(1-p_{j_2})\sum_{j_2=1}^{m_2}\left(1-\left\{1-\exp\left[-\left(\frac{\alpha}{y_{2j_2}}\right)^{\beta}\right]\right\}^{-\gamma}\right)}{\frac{m_2}{\gamma^2}\sum_{j_2=1}^{m_2}\left(\left(1-\left\{1-\exp\left[-\left(\frac{\alpha}{y_{2j_2}}\right)^{\beta}\right]\right\}^{-\gamma}\right)\right)^2-\left(\frac{1}{\gamma}\sum_{j_2=1}^{m_2}\left(1-\left\{1-\exp\left[-\left(\frac{\alpha}{y_{2j_2}}\right)^{\beta}\right]\right\}^{-\gamma}\right)\right)^2}$$

....(34)

Then by substitute Eq. (32), Eq. (33) and Eq. (34) in Eq. (7), we get:

$$\hat{R}_{Rg}=\frac{\hat{\theta}_{Rg}}{\hat{\theta}_{Rg}+\hat{\lambda}_{2Rg}}-\frac{\hat{\theta}_{Rg}}{\hat{\theta}_{Rg}+\hat{\lambda}_{1Rg}+\hat{\lambda}_{2Rg}} \qquad \dots (35)$$

3.5.Ranked set sample method (RSS):

In the early 1950s, in order to efficiently estimate the yield of McIntyre suggested a method of sampling pasture in Australia, which later the graded collection sampling was known as (RSS). The notion of RSS offers an efficient way of achieving an observational economy can be achieved more cheaply by rating small sets of samples with respect to the characteristic [13]. Let $(x_1, x_2, \dots, x_n)$ be random sample from GF. assumed that $(x_{(1)}, x_{(2)}, \dots, x_{(n)})$ be order statistics obtained by ordering the sample in increasing order .

The pdf of $x_{(i)}$ is:[6]

$$f(x_{(i)}) = \frac{n!}{(i-1)!(n-i)!}\left[F(x_{(i)})\right]^{i-1}\left[1 - F(x_{(i)})\right]^{n-i} f(x_{(i)}) \qquad \dots (36)$$

When substituting Eq. (5) and Eq. (6) in Eq. (36), we get :

$$f(x_{(i)}) = \frac{n!}{(i-1)!(n-i)!}\left[1 - \exp\left[\frac{\theta}{\gamma}\left(1 - u_{x_{(i)}}^{-\gamma}\right)\right]\right]^{i-1}\left[\exp\left[\frac{\theta}{\gamma}\left(1 - u_{x_{(i)}}^{-\gamma}\right)\right]\right]^{n-i}$$

$$\theta\beta\alpha^{\beta} x_{(i)}^{-B-1}\left(1 - u_{x_{(i)}}\right) u_{x_{(i)}}^{-\gamma-1} \exp\left[\frac{\theta}{\gamma}\left(1 - u_{x_{(i)}}^{-\gamma}\right)\right]$$

Where $u_x = 1 - \exp\left[-\left(\frac{\alpha}{x}\right)^{\beta}\right]$

Suppose that $Q = \frac{n!}{(i-1)!(n-i)!}$ then we get:

$$f(x_{(i)}) = Q\,\theta\beta\alpha^{\beta} x_{(i)}^{-B-1}\left[1 - \exp\left[\frac{\theta}{\gamma}\left(1 - u_{x_{(i)}}^{-\gamma}\right)\right]\right]^{i-1}\left(1 - u_{x_{(i)}}\right) u_{x_{(i)}}^{-\gamma-1}$$

$$* \left[\exp\left[\frac{\theta}{\gamma}\left(1 - u_{x_{(i)}}^{-\gamma}\right)\right]\right]^{n-i+1}$$

The likelihood function of the order sample $x_{(1)}, x_{(2)}, \dots, x_{(n)}$ is :

$$L(x_{(1)}, x_{(2)}, \dots, x_{(n)}; \theta, \alpha, \beta, \gamma) = Q^n \theta^n \beta^n \alpha^{n\beta} \prod_{i=1}^{n} x_{(i)}^{-(B+1)} \prod_{i=1}^{n}\left(1 - u_{x_{(i)}}\right)$$

$$* \prod_{i=1}^{n}\left[1 - \exp\left[\frac{\theta}{\gamma}\left(1 - u_{x_{(i)}}^{-\gamma}\right)\right]\right]^{i-1} * \prod_{i=1}^{n} u_{x_{(i)}}^{-\gamma-1}$$

$$* \prod_{i=1}^{n}\left[\exp\left[\frac{\theta}{\gamma}\left(1 - u_{x_{(i)}}^{-\gamma}\right)\right]\right]^{n-i+1} \qquad \dots (37)$$

Then the natural logarithm function for Eq. (34) can be written as :

$$\ln L = n \ln Q + n \ln \theta + n \ln \beta + n\beta \ln \alpha - (\beta + 1)\sum_{i=1}^{n} \ln x_{(i)} +$$

$$\sum_{i=1}^{n}(i-1)\ln\left[1-\exp\left[\frac{\theta}{\gamma}\left(1-u_{x_{(i)}}^{-\gamma}\right)\right]\right]+\sum_{i=1}^{n}\ln\left(1-u_{x_{(i)}}\right)$$

$$-(\gamma+1)\sum_{i=1}^{n}\ln u_{x_{(i)}}+\frac{\theta}{\gamma}\sum_{i=1}^{n}(n-i+1)\left(1-u_{x_{(i)}}^{-\gamma}\right) \quad \ldots (38)$$

To minimize Eq. (38), we must calculate the great endings by taking the partial derivative with respect to the unknown parameter θ , then we get :-

$$\frac{d\ln \mathrm{L}}{d\theta}=\frac{n}{\theta}+\sum_{i=1}^{n}(i-1)\frac{\frac{-1}{\gamma}\left(1-u_{x_{(i)}}^{-\gamma}\right)\exp\left[\frac{\theta}{\gamma}\left(1-u_{x_{(i)}}^{-\gamma}\right)\right]}{\left[1-\exp\left[\frac{\theta}{\gamma}\left(1-u_{x_{(i)}}^{-\gamma}\right)\right]\right]}+\frac{1}{\gamma}\sum_{i=1}^{n}(n-i+1)\left(1-u_{x_{(i)}}^{-\gamma}\right)$$

Equating the partial derivative to zero , thus the right –hand side will be:

$$\frac{n}{\hat{\theta}}=\frac{1}{\gamma}\sum_{i=1}^{n}\frac{(i-1)\left(1-u_{x_{(i)}}^{-\gamma}\right)\exp\left[\frac{\hat{\theta}}{\gamma}\left(1-u_{x_{(i)}}^{-\gamma}\right)\right]}{\left[1-\exp\left[\frac{\hat{\theta}}{\gamma}\left(1-u_{x_{(i)}}^{-\gamma}\right)\right]\right]}-\frac{1}{\gamma}\sum_{i=1}^{n}(n-i+1)\left(1-u_{x_{(i)}}^{-\gamma}\right)$$

$$\hat{\theta}_{RSS}=\frac{n\gamma}{\sum_{i=1}^{n}\frac{(i-1)\left(1-u_{x_{(i)}}^{-\gamma}\right)\exp\left[\frac{\theta_0}{\gamma}\left(1-u_{x_{(i)}}^{-\gamma}\right)\right]}{\left[1-\exp\left[\frac{\theta_0}{\gamma}\left(1-u_{x_{(i)}}^{-\gamma}\right)\right]\right]}-\sum_{i=1}^{n}(n-i+1)\left(1-u_{x_{(i)}}^{-\gamma}\right)} \quad \ldots (39)$$

In the same way we can estimate λ_1,λ_2 as bellow:

$$\hat{\lambda}_{1RSS}=\frac{m_1\gamma}{\sum_{j_1=1}^{m_1}\frac{(j_1-1)\left(1-u_{y_{1(j_1)}}^{-\gamma}\right)\exp\left[\frac{\lambda_0}{\gamma}\left(1-u_{y_{1(j_1)}}^{-\gamma}\right)\right]}{\left[1-\exp\left[\frac{\lambda_0}{\gamma}\left(1-u_{y_{1(j_1)}}^{-\gamma}\right)\right]\right]}-\sum_{j_1=1}^{m_1}(m_1-j_1+1)\left(1-u_{y_{1(j_1)}}^{-\gamma}\right)} \quad \ldots (40)$$

$$\hat{\lambda}_{2RSS}=\frac{m_2\gamma}{\sum_{j_2=1}^{m_2}\frac{(j_2-1)\left(1-u_{y_{2(j_2)}}^{-\gamma}\right)\exp\left[\frac{\lambda_0}{\gamma}\left(1-u_{y_{2(j_2)}}^{-\gamma}\right)\right]}{\left[1-\exp\left[\frac{\lambda_0}{\gamma}\left(1-u_{y_{2(j_2)}}^{-\gamma}\right)\right]\right]}-\sum_{j_2=1}^{m_2}(m_2-j_2+1)\left(1-u_{y_{2(j_2)}}^{-\gamma}\right)} \quad \ldots (41)$$

Then by substitute Eq. (39), Eq. (40) and Eq. (41) in Eq. (7) we get:

$$\hat{R}_{RSS}=\frac{\hat{\theta}_{RSS}}{\hat{\theta}_{RSS}+\hat{\lambda}_{2RSS}}-\frac{\hat{\theta}_{RSS}}{\hat{\theta}_{RSS}+\hat{\lambda}_{1RSS}+\hat{\lambda}_{2RSS}} \quad \ldots (42)$$

4. Simulation study:

In this section, a simulation study is used to determine the best reliability estimator with unknown Gompertz Fréchet distribution parameters, and to evaluate five different estimation methods the Maximum Likelihood, Least Square, Weighted Least Square, Regression and Ranked Set Sampling Methods, where regression estimators are used as the initial value, The mean square error criteria (MSE) for various sample sizes (15,30,90), where sample sizes (n,m_1,m_2)

are generated for each of the variables X, Y_1 and Y_2 respectively. For six different experiments in each case of the parameters value α,γ and β as shown in the table below:

Table (1): The experiments for real R value

Experiment	λ_1	λ_2	θ	α	γ	β	R
1	1.5	2.2	3	0.2	0.6	0.9	0.1292
2	1.5	2.2	1.7	0.2	0.6	0.9	0.1211
3	1.5	0.4	3	0.2	0.6	0.9	0.2701
4	1.5	2.2	3	0.3	0.3	0.6	0.1292
5	1.5	2.2	1.7	0.3	0.3	0.6	0.1211
6	1.5	0.4	3	0.3	0.3	0.6	0.2701

A simulation study is conducted using MATLAB 2020 for the six different experiments to compare the performance of reliability estimators using the following steps:

Step1: Generating the random values of the random variables by the inverse function according to the following formula:

$$x = \alpha\left[-\ln\left(1-\left\{1-\frac{\gamma}{\theta}\ln(1-F(x))\right\}^{\frac{-1}{\gamma}}\right)\right]^{\frac{-1}{\beta}}$$

Step2: Finding the MLE for reliability using Eq. (11) and LS using Eq. (19) and WLS using Eq. (26) and Rg using Eq. (35) and RSS using Eq. (42).

Step3: Finding the mean by the equation: Mean $= \frac{\sum_{i=1}^{N}\hat{R}_i}{N}$

Step4: The estimation methods are compared using the mean square error criteria: MSE $= \frac{1}{N}\sum_{i=1}^{N}\left(\hat{R}_i - R\right)^2$ where N is 500 in each experiment.

5. Discussion of the results:

The results are recorded from 1 to 6 in the following tables. The comparison of the performance of these estimators based on MSE values was noted as follows:

1-In table (2)

- For experiments (3) in the case of sample size (90,90,90) the best value of MSE is MLE, estimator followed by LS equal with RSS, Rg ,WLS.
- For experiment (1) in the case of sample size (30,30,30) and (15,30,30) the best value of MSE is MLE, estimator followed by LS, RSS, WLS equal with Rg.

2-In table (2) and (3)

- For experiment (1) in the case of sample size (15,15,15) and also in experiment (2) in sample size (30,15,15) the best value of MSE is MLE, estimator followed by LS,WLS equal with RSS, Rg .
- For experiments (1),(2),(5) in the case of sample size (30.15,15) and also in experiment (6) in sample size (15,15,15) the best value of MSE is MLE, estimator followed by LS, WLS, RSS, Rg.
- For experiments (1),(3),(4),(6) in the case of sample size (30,90,90) the best value of MSE is MLE, estimator followed by LS, Rg, WLS, RSS.
- For experiments (2),(3) in the case of sample size (15,15,15) and also in experiments (3),(6) in sample size (30,30,30) and also in experiment (4) in sample size (15,30,30) the best value of MSE is MLE, estimator followed by LS, RSS, WLS, Rg.
- For experiments (2),(5) in the case of sample size (30,30,30) the best value of MSE is MLE, followed by LS equal to RSS,WLS equal with Rg .
- For experiments (1),(2),(5) and (6) in the case of sample size (90,90,90) the best value of MSE is MLE, followed by LS, RSS, Rg, WLS.
- For experiments (2),(5) in the case of sample size (15,30,30) and also in experiments (4),(5) in sample size (15,15,15) and also in experiment (4) in sample size (30,30,30) the best value of MSE is MLE, followed by LS equal to RSS,WLS, Rg.
- For experiments (3),(6) in the case of sample size (15,30,30) and (30,15,15) the best value of MSE is MLE, followed by LS,WLS, Rg, RSS.
- For experiments (2),(5) in the case of sample size (30,90,90) and also in experiment (4) in sample size (90,90,90) there is a convergence of preference between MLE and RSS according to the MSE standard, followed by LS, Rg, WLS.

6. Conclusion

In this paper, we presented five methods for estimating reliability $P(Y_1<X<Y_2)$ when X , Y_1 and Y_2 both follow Gompertz Fréchet distribution with different parameters. Simulation results confirm that the performance of the maximum likelihood estimator is best for all experiments and for all sizes of samples, followed by the LS estimator is the second best. These studies can be used in many fields of engineering, physical and mechanical.

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Table (2): real Reliability values and it's estimators performance for Exp. 1,2,3 .

Exp1: R= 0.1292 when λ_1=1.5, λ_2=2.2, θ=3 for α=0.2, γ=0.6, β=0.9						
(n, m_1, m_2)		**MLE**	**LS**	**WLS**	**Rg**	**RSS**
(15,15,15)	Mean MSE	0.1286 0.0015	0.1292 0.0017	0.1295 0.0020	0.1300 0.0024	0.1288 0.0020
(30,30,30)	Mean MSE	0.1271 0.0007	0.1275 0.0008	0.1276 0.0012	0.1280 0.0012	0.1265 0.0009
(90,90,90)	Mean MSE	0.1290 0.0002177	0.1287 0.0002824	0.1285 0.0005989	0.1284 0.0004488	0.1294 0.0002856
(15,30,30)	Mean MSE	0.1281 0.0007	0.1284 0.0008	0.1283 0.0011	0.1281 0.0011	0.1135 0.0010
(30,15,15)	Mean MSE	0.1306 0.0013	0.1304 0.0014	0.1306 0.0017	0.1310 0.0021	0.1214 0.0020
(30,90,90)	Mean MSE	0.1291 0.0002	0.1293 0.0003	0.1296 0.0005	0.1290 0.0004	0.0965 0.0013
Exp2: R=0.1211 when λ_1=1.5, λ_2=2.2, θ=1.7 for α=0.2, γ=0.6, β=0.9						
(15,15,15)	Mean MSE	0.1201 0.0014	0.1211 0.0017	0.1222 0.0021	0.1224 0.0025	0.1199 0.0020
(30,30,30)	Mean MSE	0.1215 0.0008	0.1214 0.0010	0.1219 0.0014	0.1215 0.0014	0.1221 0.0010
(90,90,90)	Mean MSE	0.1214 0.0002236	0.1211 0.0002718	0.1207 0.0005821	0.1209 0.0004296	0.1216 0.0003135
(15,30,30)	Mean MSE	0.1225 0.0009	0.1209 0.0010	0.1207 0.0014	0.1206 0.0015	0.1280 0.0010
(30,15,15)	Mean MSE	0.1250 0.0016	0.1258 0.0018	0.1246 0.0021	0.1262 0.0025	0.1000 0.0022
(30,90,90)	Mean MSE	0.1216 0.0002677	0.1193 0.0003427	0.1195 0.0006851	0.1183 0.0005426	0.1204 0.0002835
Exp3: R=0.2701 when λ_1=1.5, λ_2=0.4, θ=3 for α=0.2, γ=0.6, β=0.9						
(15,15,15)	Mean MSE	0.2673 0.0037	0.2686 0.0044	0.2694 0.0051	0.2702 0.0062	0.2655 0.0049
(30,30,30)	Mean MSE	0.2710 0.0019	0.2711 0.0024	0.2708 0.0032	0.2718 0.0035	0.2696 0.0027
(90,90,90)	Mean MSE	0.2692 0.0006	0.2688 0.0008	0.2681 0.0015	0.2685 0.0012	0.2696 0.0008
(15,30,30)	Mean MSE	0.2669 0.0026	0.2727 0.0032	0.2711 0.0041	0.2728 0.0044	0.1826 0.0102
(30,15,15)	Mean MSE	0.2713 0.0030	0.2680 0.0035	0.2711 0.0045	0.2701 0.0053	0.3372 0.0094
(30,90,90)	Mean MSE	0.2689 0.0010	0.2746 0.0013	0.2716 0.0023	0.2752 0.0020	0.1366 0.0186

Table(3): real Reliability values and it's estimators performance for Exp. 4,5,6 .

Exp4: R=0.1292 when λ_1=1.5, λ_2=2.2, θ=3 for α=0.3, γ=0.3, β=0.6						
(n, m_1, m_2)		**MLE**	**LS**	**WLS**	**Rg**	**RSS**
(15,15,15)	Mean	0.1259	0.1264	0.1264	0.1267	0.1255
	MSE	0.0014	0.0017	0.0020	0.0024	0.0017
(30,30,30)	Mean	0.1287	0.1290	0.1295	0.1293	0.1288
	MSE	0.0006	0.0008	0.0012	0.0013	0.0008
(90,90,90)	Mean	0.1298	0.1296	0.1291	0.1294	0.1300
	MSE	0.0001966	0.0002789	0.0005817	0.0004421	0.0002398
(15,30,30)	Mean	0.1254	0.1254	0.1253	0.1250	0.1120
	MSE	0.0007	0.0008	0.0011	0.0012	0.0010
(30,15,15)	Mean	0.1296	0.1307	0.1310	0.1319	0.1177
	MSE	0.0015	0.0018	0.0021	0.0026	0.0021
(30,90,90)	Mean	0.1282	0.1286	0.1290	0.1287	0.0964
	MSE	0.0002	0.0003	0.0006	0.0004	0.0013
Exp5: R=0.1211 when λ_1=1.5, λ_2=2.2, θ=1.7 for α=0.3, γ=0.3, β=0.6						
(15,15,15)	Mean	0.1218	0.1223	0.1225	0.1234	0.1215
	MSE	0.0016	0.0019	0.0023	0.0028	0.0019
(30,30,30)	Mean	0.1222	0.1225	0.1229	0.1233	0.1218
	MSE	0.0007	0.0009	0.0013	0.0013	0.0009
(90,90,90)	Mean	0.1202	0.1200	0.1196	0.1201	0.1201
	MSE	0.0002115	0.0002777	0.0006252	0.0004630	0.0002912
(15,30,30)	Mean	0.1215	0.1215	0.1228	0.1225	0.1257
	MSE	0.0007	0.0009	0.0012	0.0013	0.0009
(30,15,15)	Mean	0.1217	0.1241	0.1247	0.1267	0.0949
	MSE	0.0015	0.0018	0.0023	0.0027	0.0024
(30,90,90)	Mean	0.1208	0.1195	0.1205	0.1192	0.1179
	MSE	0.0002662	0.0003568	0.0007064	0.0005566	0.0002755
Exp6: R=0.2701 when λ_1=1.5, λ_2=0.4, θ=3 for α=0.3, γ=0.3, β=0.6						
(15,15,15)	Mean	0.2676	0.2658	0.2646	0.2637	0.2705
	MSE	0.0039	0.0045	0.0051	0.0060	0.0054
(30,30,30)	Mean	0.2685	0.2676	0.2667	0.2669	0.2690
	MSE	0.0020	0.0023	0.0031	0.0033	0.0026
(90,90,90)	Mean	0.2701	0.2699	0.2711	0.2697	0.2710
	MSE	0.0007	0.0008	0.0016	0.0012	0.0009
(15,30,30)	Mean	0.2689	0.2741	0.2723	0.2737	0.1860
	MSE	0.0023	0.0028	0.0036	0.0039	0.0096
(30,15,15)	Mean	0.2710	0.2634	0.2648	0.2619	0.3435
	MSE	0.0031	0.0034	0.0041	0.0050	0.0109
(30,90,90)	Mean	0.2682	0.2745	0.2727	0.2758	0.1354
	MSE	0.0010	0.0012	0.0021	0.0018	0.0189

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