Al-Qadisiyah Journal of Pure Science

Volume 26 | Number 2 Article 4

4-7-2021

The New Weibull-Pareto Distribution Stress-Strength Reliability for P(T

Ali Mutair Attia
Department of Mathematics, College of Education, AL-Mustanseriyah University, Baghdad-Iraq, alimutair43@uomustansiriyah.edu.iq

Nada Sabah Karam
Department of Mathematics, College of Education, AL-Mustanseriyah University, Baghdad-Iraq, dr_nadaskaram@uomustansiriyah.edu.iq

Recommended Citation

Attia, Ali Mutair and Karam, Nada Sabah (2021) "The New Weibull-Pareto Distribution Stress-Strength Reliability for P(T," *Al-Qadisiyah Journal of Pure Science*: Vol. 26: No. 2, Article 4.
DOI: 10.29350/qjps.2021.26.2.1259
Available at: https://qjps.researchcommons.org/home/vol26/iss2/4

Al-Qadisiyah Journal of Pure Science Vol.(26) Issue (2) (2021) pp. Math. 39–51

Al-Qadisiyah Journal of Pure Science

ISSN(Printed): 1997-2490 ISSN(Online): 2411-3514

DOI: /10.29350/jops.

http://qu.edu.iq/journalsc/index.php/JOPS

The New Weibull-Pareto Distribution Stress-Strength Reliability for $P(T < X < Z)$

Authors Names

a. Ali Mutair Attia

b. Nada Sabah Karam

Article History

Received on: 24/1/2021

Revised on: 15 /2/2021

Accepted on: 17/2/2021

Keywords:

Stress-strength Reliability, Probability $P(T < X < Z)$, The New Weibull-Pareto Distribution, Estimation Method

DOI: https://doi.org/10.29350/jops.2021.26. 2.1259

ABSTRACT

In this paper, the reliability formula of the stress-strength model is derived for probability $P(T < X < Z)$ of a component having strength X between two stresses T and Z, based on the New Weibull-Pareto Distribution with unknown parameter α and known parameters β and δ. Four methods to estimate the parameters of The New Weibull-Pareto distribution are discussed by using the Maximum Likelihood, Method of Moment, Least Square Method and Weighted Least Square Method, and compare between these estimates based on a simulation study by the mean square error criteria for both small, medium and large samples. The most important results, that this comparison confirms on the performance of the maximum likelihood estimator works better for all studied experiments.

1. Introduction

In the reliability studies the stress-strength model describes the life of a component which has a random strength X and is subjected to random stress Y. This idea arises in the classical stress-strength reliability, where the person is interested in estimating the probability $P(X < Y)$ that can be interpreted as the probability of failure of the component, when the applied stress Y is greater than its strength X [5]. An important case is the estimation of $R = P(T < X < Z)$ which represents the situation where the strength X should not only be greater than stress T but also be smaller than stress Z. For example, there are many devices that do not work when the temperatures are high or when they are low. Similarly, person's blood pressure should lie within two limits, systolic and diastolic [4]. The stress-strength model of $P(T < X < Z)$ have wide applications in various subareas of engineering, psychology, genetics, clinical trials and so on [7].

[a] Department of Mathematics, College of Education, AL-Mustanseriyah University, Baghdad-Iraq, E-Mail: alimutair43@gmail.com ; alimutair43@uomustansiriyah.edu.iq

[b] Department of Mathematics, College of Education, AL-Mustanseriyah University, Baghdad-Iraq, E-Mail: karamns6760@gmail.com ; dr_nadaskaram@uomustansiriyah.edu.iq

There are many distributions for modeling lifetime data used in statistical analysis, but some of these data have not been adequately described by those distributions. This opened the way for the development of new distributions that provide greater flexibility in modeling lifetime data. As a result, in recent years many distributions have been developed by researchers. Nasiru and Luguterah in 2015 presented The New Weibull-Pareto distribution (NWPD) as a generalization of the Pareto distribution. For any random variable X that follows The New Weibull-Pareto distribution the cumulative density function (cdf) is given by: [8]

$$F(x) = 1 - e^{-\alpha\left(\frac{x}{\theta}\right)^{\beta}} \qquad ; x > 0; \alpha, \beta, \theta > 0 \qquad \text{.... (1)}$$

And the probability density function (pdf):

$$\mathrm{f}(x) = \frac{\alpha\beta}{\theta}\left(\frac{x}{\theta}\right)^{\beta-1} e^{-\alpha\left(\frac{x}{\theta}\right)^{\beta}} \qquad ; x > 0; \alpha, \beta, \theta > 0 \qquad \text{.... (2)}$$

Where α and β are the shape parameters and θ is the scale parameter.

Since $\mathrm{f}(x)$ is probability densty function, then we can rewrite equation (2) as:

$$\int_0^{\infty} \frac{\beta}{\theta}\left(\frac{x}{\theta}\right)^{\beta-1} e^{-\alpha\left(\frac{x}{\theta}\right)^{\beta}} dx = \frac{1}{\alpha} \qquad \text{.... (3)}$$

The main aim of this paper is to obtain a mathematical formula for the reliability R of the probability that a component's strength is between two stresses based on New Weibull-Pareto distribution in section 2. In order to find the estimators of the shape parameters $(\alpha, \alpha_1, \alpha_2)$ for the three random variables, four different estimation methods (Maximum Likelihood, the Moment, Least Square and Weighted Least Square Methods) are used and then the reliability parameter is estimated in section 3. A simulation study was conducted to compare the performance of the four different estimators of the reliability in section 4, based on twelve experiments of shape parameter values and at different sample sizes of (15) for small, (30) for medium and (90) for large sample sizes. The comparison is made by the mean square error criteria (MSE), and the conclusions are discussed in section 5.

2. Reliability Formulation

The reliability formula of the stress-strength models that the probability of a component strength falling in between two stresses, is given by: [1]

$$\begin{aligned} R &= P(T < X < Z) \\ &= \int_0^{\infty} P(T < x, Z > x / X = x) dF_x(x) \\ &= \int_0^{\infty} H_T(x)\left(1 - G_Z(x)\right) \mathrm{f}(\mathrm{x}) dx \end{aligned} \qquad \text{.... (4)}$$

Let T and Z be independent random stress variables with cumulative density functions $H_T(t)$, $G_Z(z)$ following $NWPD(\alpha_1, \beta, \theta)$ and $NWPD(\alpha_2, \beta, \theta)$, respectively. And let X be a random strength variable following $NWPD(\alpha, \beta, \theta)$ with (cdf) $F_X(x)$, assumed that X independent from T and Z, then:

$$H_T\,(t) = 1 - e^{-\alpha_1\left(\frac{t}{\theta}\right)^{\beta}} \quad t > 0; \alpha_1, \beta, \theta > 0 \qquad \text{.... (5)}$$

$$G_Z\,(z) = 1 - e^{-\alpha_2\left(\frac{z}{\theta}\right)^{\beta}} \quad z > 0; \alpha_2, \beta, \theta > 0 \qquad \text{.... (6)}$$

Now, from equation (3):

$$R = \int_0^{\infty} \left(1 - e^{-\alpha_1\left(\frac{x}{\theta}\right)^{\beta}}\right)\left(e^{-\alpha_2\left(\frac{x}{\theta}\right)^{\beta}}\right)\frac{\alpha\beta}{\theta}\left(\frac{x}{\theta}\right)^{\beta-1} e^{-\alpha\left(\frac{x}{\theta}\right)^{\beta}} dx$$

$$= \int_0^{\infty} \left(e^{-\alpha_2\left(\frac{x}{\theta}\right)^{\beta}}\right)\frac{\alpha\beta}{\theta}\left(\frac{x}{\theta}\right)^{\beta-1} e^{-\alpha\left(\frac{x}{\theta}\right)^{\beta}} dx$$

$$- \int_0^{\infty} \left(e^{-\alpha_1\left(\frac{x}{\theta}\right)^{\beta} - \alpha_2\left(\frac{x}{\theta}\right)^{\beta}}\right)\frac{\alpha\beta}{\theta}\left(\frac{x}{\theta}\right)^{\beta-1} e^{-\alpha\left(\frac{x}{\theta}\right)^{\beta}} dx$$

$$R = \alpha \int_0^{\infty} \frac{\beta}{\theta}\left(\frac{x}{\theta}\right)^{\beta-1} e^{-(\alpha+\alpha_2)\left(\frac{x}{\theta}\right)^{\beta}} dx - \alpha \int_0^{\infty} \frac{\beta}{\theta}\left(\frac{x}{\theta}\right)^{\beta-1} e^{-(\alpha+\alpha_1+\alpha_2)\left(\frac{x}{\theta}\right)^{\beta}} dx$$

Similarly, from equation (3), we get:

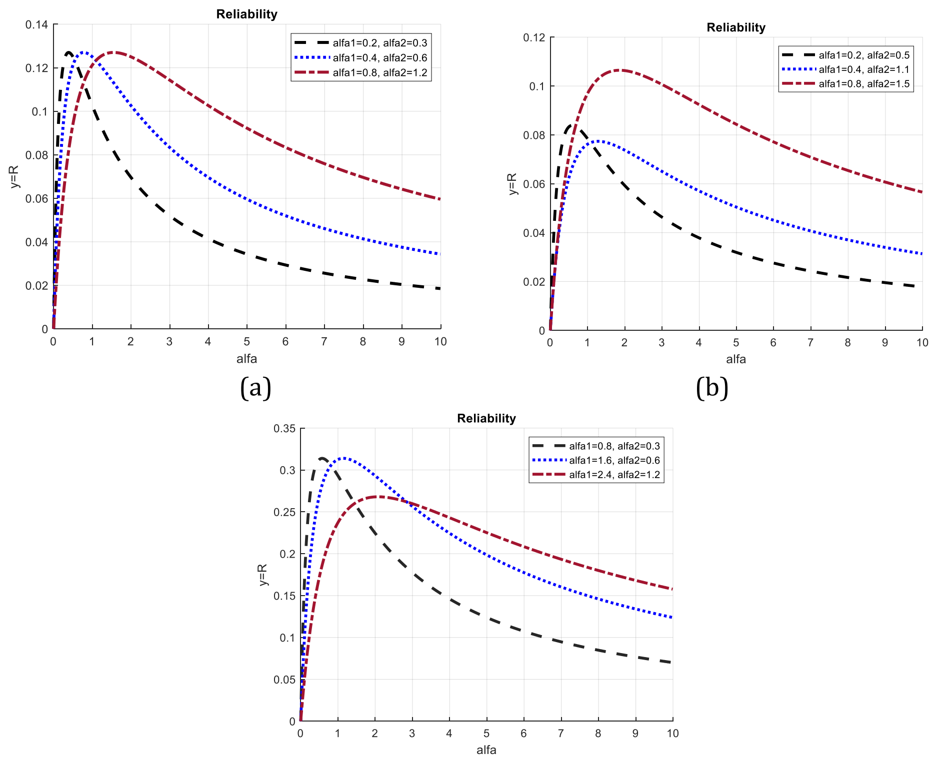
$$R = \frac{\alpha}{\alpha+\alpha_2} - \frac{\alpha}{\alpha+\alpha_1+\alpha_2} = \frac{\alpha\alpha_1}{(\alpha+\alpha_2)(\alpha+\alpha_1+\alpha_2)} \qquad \text{.... (7)}$$

The behavior of reliability is illustrated in the following figures with different values of the three distribution shape parameters. The purpose of explaining this behavior is to know how much confidence we have in the component to work under certain conditions and to continue doing this work according to the special terms of the model.

Figure (1-a,b,c) shows the change of reliability curve by the effect of different values of the strength parameter α as a function of the parameters $(\alpha, \alpha_1, \alpha_2)$, in three different cases for the values of the two parameters α_1, α_2, where reliability value increasing with the increasing in strength shape parameter α value, then it gradually decreases.

Figure (2-a,b,c) shows the effect of the stress parameter value α_1 on the change of the reliability curve as a function of the parameters $(\alpha, \alpha_1, \alpha_2)$, in three different cases for the values of the two parameters α, α_2, where reliability value increasing with increasing stress shape parameter α_1 value, but it reaches a certain point and then begins to decrease.

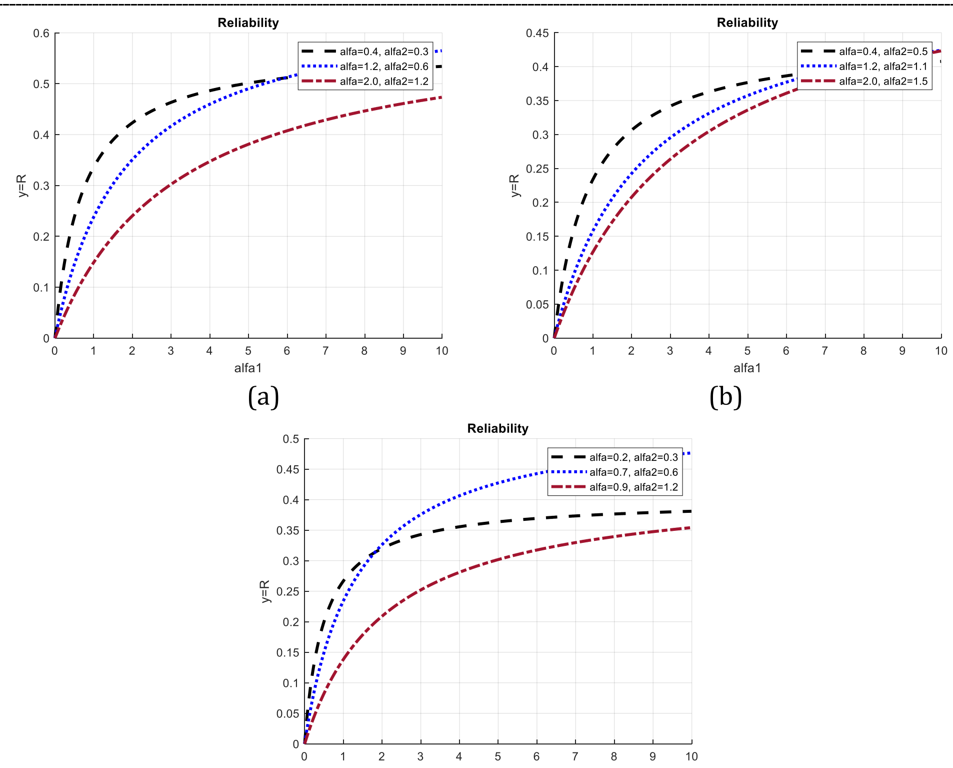
Figure (3-a,b,c) shows the effect of the stress parameter value α_2 on the change of the reliability curve as a function of the parameters $(\alpha, \alpha_1, \alpha_2)$, in three different cases for the values of the two parameters α, α_1, where reliability value decreasing and its decrease is steep with increasing stress shape parameter α_2 value.



(a) (b)

(c)

Figure 1 : Reliability curve against parameter α



(a) (b)

(c)

Figure 2 : Reliability curve against parameter α_1

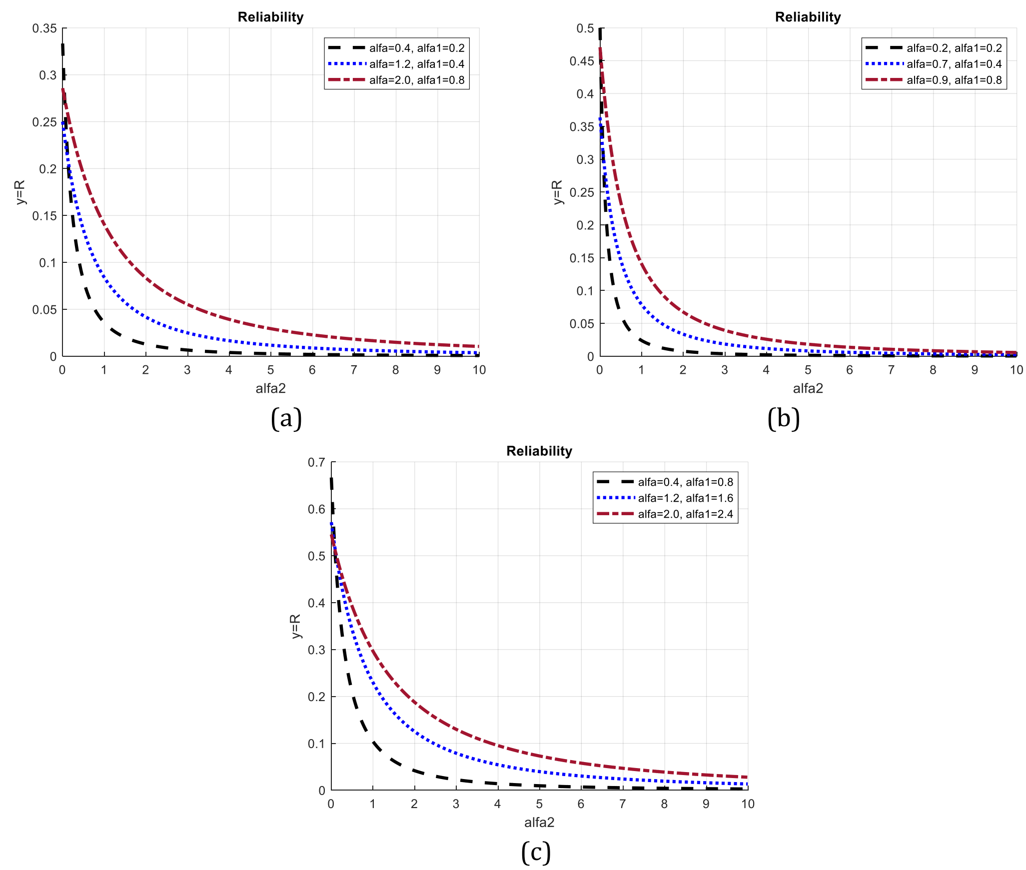


Figure 3 : Reliability curve against parameter α_2

3. Estimation Method:

In this section, four different estimation methods are used to find the estimator of The New Weibull-Pareto unknown shape parameters; $\alpha, \alpha_1, \alpha_2$ and Reliability; R of the stress-strength model. These methods are Maximum Likelihood, Method of Moment, Least Square Method and Weighted Least Square Method. These methods are used to arrive at the best reliability estimate.

3.1. Maximum Likelihood Estimator (MLE):

The method of maximum likelihood is the most widely used method for parameter estimation [3]. Let $x_1, x_2, \ldots, x_n$ be a random strength sample of size (n) from $NWPD(\alpha, \beta, \theta)$ where α is unknown parameter and β, θ are known. Then the MLE function is given by: [8], [2]

$$L(x_1, x_2, \ldots, x_n; \alpha, \beta, \theta) = \left(\frac{\alpha\beta}{\theta}\right)^n \prod_{i=1}^{n} \left(\frac{x_i}{\theta}\right)^{\beta-1} e^{-\alpha \sum_{i=1}^{n} \left(\frac{x_i}{\theta}\right)^{\beta}} \quad \ldots. (8)$$

Then the natural logarithm function for equation (8) can be written as:

$$\ln L = n \ln \alpha + n \ln \beta - n \ln \theta + (\beta - 1) \sum_{i=1}^{n} \ln\left(\frac{x_i}{\theta}\right) - \alpha \sum_{i=1}^{n} \left(\frac{x_i}{\theta}\right)^{\beta} \quad \ldots. (9)$$

By differentiate to equation (9) with respect to the unknown parameter α, and equating the result to zero, we get:

$$\frac{\partial \ln L}{\partial \alpha} = \frac{n}{\alpha} - \sum_{i=1}^{n} \left(\frac{x_i}{\theta}\right)^{\beta} \Rightarrow \frac{n}{\alpha} - \sum_{i=1}^{n} \left(\frac{x_i}{\theta}\right)^{\beta} = 0$$

$$\hat{\alpha}_{MLE} = \frac{n}{\sum_{i=1}^{n}\left(\frac{x_i}{\theta}\right)^{\beta}} \quad \text{.... (10)}$$

In the same way, let $t_1, t_2, \dots, t_m$ and $z_1, z_2, \dots, z_m$ are two random stress samples of size (m) from $NWPD(\alpha_1, \beta, \theta)$ and $NWPD(\alpha_2, \beta, \theta)$, respectively. Then the MLE estimators of the unknown parameters α_1, α_2 are:

$$\hat{\alpha}_{1MLE} = \frac{m}{\sum_{j=1}^{m}\left(\frac{t_j}{\theta}\right)^{\beta}} \quad \text{.... (11)}$$

$$\hat{\alpha}_{2MLE} = \frac{m}{\sum_{k=1}^{m}\left(\frac{z_k}{\theta}\right)^{\beta}} \quad \text{.... (12)}$$

Then the MLE estimator of Reliability R is given by substitution equations (10), (11) and (12) in equation (7), using the invariant property of this method as:

$$\hat{R}_{MLE} = \frac{\hat{\alpha}_{MLE}\hat{\alpha}_{1MLE}}{(\hat{\alpha}_{ML}+\hat{\alpha}_{2MLE})(\hat{\alpha}_{MLE}+\hat{\alpha}_{1MLE}+\hat{\alpha}_{2MLE})}$$

3.2. Method of Moment (MOM):

The Method of Moment was introduced by Pearson in 1894. It was one of the first methods used to estimate a population parameter [6]. To derived the method of moment estimators of parameters of The New Weibull-Pareto, let $x_1, x_2, \dots, x_n$ be a strength random sample of size (n) from $NWPD(\alpha, \beta, \theta)$ and let $t_1, t_2, \dots, t_m$ and $z_1, z_2, \dots, z_m$ be a stress random samples of size (m) from $NWPD(\alpha_1, \beta, \theta)$ and $NWPD(\alpha_2, \beta, \theta)$, respectively. Then their population means are given by: [8]

$$E(x) = \theta\, \alpha^{\frac{-1}{\beta}}\, \Gamma\left(\frac{\beta+1}{\beta}\right), E(t) = \theta\, \alpha_1^{\frac{-1}{\beta}}\, \Gamma\left(\frac{\beta+1}{\beta}\right), E(z) = \theta\, \alpha_2^{\frac{-1}{\beta}}\, \Gamma\left(\frac{\beta+1}{\beta}\right)$$

According to the method of moment, equating the samples mean with the corresponding populations mean, then the moment estimators of $\alpha, \alpha_1, \alpha_2$ are:

$$\hat{\alpha}_{MOM} = \left(\frac{\bar{x}}{\theta\, \Gamma\left(\frac{\beta+1}{\beta}\right)}\right)^{-\beta} \quad \text{.... (13)}$$

$$\hat{\alpha}_{1MOM} = \left(\frac{\bar{t}}{\theta\, \Gamma\left(\frac{\beta+1}{\beta}\right)}\right)^{-\beta} \quad \text{.... (14)}$$

$$\hat{\alpha}_{2MOM} = \left(\frac{\bar{z}}{\theta\, \Gamma\left(\frac{\beta+1}{\beta}\right)}\right)^{-\beta} \quad \text{.... (15)}$$

By substitution equation (13), (14) and (15) in equation (7), we can obtain the approximate estimator of R as bellow:

$$\hat{R}_{MOM} = \frac{\hat{\alpha}_{MOM}\hat{\alpha}_{1MOM}}{(\hat{\alpha}_{MOM}+\hat{\alpha}_{2MOM})(\hat{\alpha}_{MOM}+\hat{\alpha}_{1MOM}+\hat{\alpha}_{2MOM})}$$

3.3. Least Square Method (LS):

Least square method was originally suggested by Swain, Venkatraman and Wilson in 1988 to estimate the parameters of Beta distribution. The least square estimators are obtained by minimizing the sum between the value and it's expected value [9]. Suppose $x_{(1)}, x_{(2)}, \dots, x_{(n)}$ be the order statistics strength random sample of size n from $NWPD(\alpha, \beta, \theta)$. The least square estimator can be obtained by minimizing the following equation: [2]

$$S = \sum_{i=1}^{n}\left[F(x_i) - E\big(F(x_i)\big)\right]^2 \quad \text{.... (16)}$$

Where $E\big(F(x_i)\big) = Pi$ the plotting position and $P_i = \frac{i}{n+1}$, $i = 1,2,\dots,n$

Putting the cdf of NWPD in equation (16), we get:

$$S = \sum_{i=1}^{n}\left[1 - e^{-\alpha\left(\frac{x_i}{\theta}\right)^{\beta}} - Pi\right]^2 \quad \text{.... (17)}$$

So then,

$$S = \sum_{i=1}^{n}\left[\alpha\left(\frac{x_i}{\theta}\right)^{\beta} - q_i\right]^2 \quad \text{.... (18)}$$

Where $q_i = -\ln\big(1 - F(x_i)\big) = -\ln(1 - P_i)$

By differentiate to equation (19) with respect to the unknown shape parameter α and equating the result to zero, we will get:

$$\frac{\partial S}{\partial \alpha} = 2\sum_{i=1}^{n}\left[\alpha\left(\frac{x_i}{\theta}\right)^{\beta} - qi\right] * \left(\frac{x_i}{\theta}\right)^{\beta} \Rightarrow \hat{\alpha}\sum_{i=1}^{n}\left(\frac{x_i}{\theta}\right)^{2\beta} - \sum_{i=1}^{n} q_i\left(\frac{x_i}{\theta}\right)^{\beta} = 0$$

$$\hat{\alpha}_{LS} = \sum_{i=1}^{n} q_i\left(\frac{x_i}{\theta}\right)^{\beta} \Big/ \sum_{i=1}^{n}\left(\frac{x_i}{\theta}\right)^{2\beta} \quad \text{.... (19)}$$

In the same way, the lest square estimators for α_1 and α_2, are given by:

$$\hat{\alpha}_{1LS} = \sum_{j=1}^{m} q_j\left(\frac{t_j}{\theta}\right)^{\beta} \Big/ \sum_{j=1}^{m}\left(\frac{t_j}{\theta}\right)^{2\beta} \quad \text{.... (20)}$$

$$\hat{\alpha}_{2LS} = \sum_{k=1}^{m} q_k\left(\frac{z_k}{\theta}\right)^{\beta} \Big/ \sum_{k=1}^{m}\left(\frac{z_k}{\theta}\right)^{\beta} \quad \text{.... (21)}$$

Where $P_j = \frac{j}{m+1}$, $j = 1,2,\dots,m$ and $P_k = \frac{k}{m+1}$, $k = 1,2,\dots,m$

By substitution equations (19), (20) and (21) in (7), we obtain the LS estimator for the reliability R, will be approximately as:

$$\hat{R}_{LS} = \frac{\hat{\alpha}_{LS}\hat{\alpha}_{1LS}}{(\hat{\alpha}_{LS}+\hat{\alpha}_{2LS})(\hat{\alpha}_{LS}+\hat{\alpha}_{1LS}+\hat{\alpha}_{2LS})}$$

3.4. Weighted Least Square Method (WLS):

The weighted least square estimator can be obtained by minimizing the following equation: [8]

$$WS = \sum_{i=1}^{n} w_i\left[F(x_i) - E\big(F(x_i)\big)\right]^2 \quad .... (22)$$

Where $w_i = \frac{1}{var[F(x_i)]} = \frac{(n+1)^2+(n+2)}{i(n-i+1)}$

The weighted least square estimator of the unknown shape parameter α can be obtained by minimizing the following equation:

$$WS = \sum_{i=1}^{n} w_i\left[1 - e^{-\alpha\left(\frac{x_i}{\theta}\right)^{\beta}} - Pi\right]^2 \quad .... (23)$$

So then,

$$WS = \sum_{i=1}^{n} w_i\left[\alpha\left(\frac{x_i}{\theta}\right)^{\beta} - q_i\right]^2 \quad .... (24)$$

By differentiate to equation (24) with respect to the unknown shape parameter α and equating the result to zero, we will get:

$$\frac{\partial WS}{\partial \alpha} = 2\sum_{i=1}^{n} w_i\left[\alpha\left(\frac{x_i}{\theta}\right)^{\beta} - q_i\right] * \left(\frac{x_i}{\theta}\right)^{\beta} \Longrightarrow \hat{\alpha}\sum_{i=1}^{n} w_i\left(\frac{x_i}{\theta}\right)^{2\beta} - \sum_{i=1}^{n} w_i q_i\left(\frac{x_i}{\theta}\right)^{\beta} = 0$$

$$\hat{\alpha}_{WLS} = \sum_{i=1}^{n} w_i q_i\left(\frac{x_i}{\theta}\right)^{\beta} \Big/ \sum_{i=1}^{n} w_i\left(\frac{x_i}{\theta}\right)^{2\beta} \quad .... (25)$$

In the same way, the weighted lest square estimators for α_1 and α_2, are given by:

$$\hat{\alpha}_{1WLS} = \sum_{j=1}^{m} w_j q_j\left(\frac{t_j}{\theta}\right)^{\beta} \Big/ \sum_{j=1}^{m} w_j\left(\frac{t_j}{\theta}\right)^{2\beta} \quad .... (26)$$

$$\hat{\alpha}_{2WLS} = \sum_{k=1}^{m} w_k q_k\left(\frac{z_k}{\theta}\right)^{\beta} \Big/ \sum_{k=1}^{m} w_k\left(\frac{z_k}{\theta}\right)^{2\beta} \quad .... (27)$$

Where $w_j = \frac{1}{var[F(t_j)]} = \frac{(m+1)^2+(m+2)}{j(m-j+1)}$ and $w_k = \frac{1}{var[F(z_k)]} = \frac{(m+1)^2+(m+2)}{k(m-k+1)}$

By substitution equations (25), (26) and (27) in (7), we obtain the WLS estimator for the reliability R, will be approximately as:

$$\hat{R}_{WLS} = \frac{\hat{\alpha}_{WLS}\hat{\alpha}_{1WLS}}{(\hat{\alpha}_{WLS}+\hat{\alpha}_{2WLS})(\hat{\alpha}_{WLS}+\hat{\alpha}_{1WLS}+\hat{\alpha}_{2WLS})}$$

4. Simulation study:

In this section, a simulation study is used to determine the best estimate of the reliability with unknown parameters of The New Weibull-Pareto distribution, and to performance the four different estimates from the maximum likelihood, method of moment, least square method and weighted least square method, are evaluated by using the mean square error criteria (MSE), with different sample sizes (15,30,90) and ($\beta = 2, 0.4, 2; \theta = 2, 2, 0.6$), for three different experiments in each case of the parameters value β and θ.

For the nine different experiments, a simulation study is conducted by using MATLAB 2020 to compare the performance of the reliability estimators by the following steps:

Step1: Generating the random values of the random variables by the inverse function according to the following formula: $x = \theta\left[-\ln\left(1 - F(x)\right)/\alpha\right]^{1/\beta}$

Step2: Calculate the mean by the equation: Mean $= \frac{\sum_{i=1}^{N} \hat{R}_i}{N}$

Step3: The comparison of estimation methods is done by using the mean square error criteria: MSE$= \frac{1}{N}\sum_{i=1}^{N}\left(\hat{R}_i - R\right)^2$, where N the number of replication in each experiment is 1000.

The results are recorded in the tables from 1 to 3. The comparison of these estimator's performance based on the MSE values, where the value of MSE decreases with increasing sample sizes for MLE, MOM, LS and WLS for each of the experiments in the three tables, the best value of MSE is MLE estimator, followed by MOM, LS,WLS. Therefore, the estimators of the maximum likelihood method give better performance than those of the method of moment, least square and weighted least square methods, through the small values of the MSE for all experiments and for all sample sizes.

5. Conclusion

In this paper, we presented four methods for estimating reliability P (T< X< Z) as each of T, Z and X follow the New Weibull-Pareto distribution with different parameters. Simulation results that appeared confirm that the performance of the maximum likelihood estimator is much better than the estimators of the moment, least square and weighted least square for all experiments and for all sample sizes.

6. Acknowledgments

The authors would like to thank Mustansiriyah University (www.uo Mustansiriyah.edu.iq) Baghdad- Iraq for its support in the present work.

Table 1: Estimate for Reliability when $\beta = 3,\ \theta = 3$

n,m		**MLE**	**MOM**	**LS**	**WLS**
Exp. 1: $\alpha = 1.3, \alpha_1 = 0.7, \alpha_2 = 0.7, R = 0.1685$					
15,15	Mean	0.1673	0.1674	0.1673	0.1671
	MSE	0.0018	0.0021	0.0021	0.0026
30,30	Mean	0.1669	0.1666	0.1671	0.1669
	MSE	0.0009	0.0011	0.0011	0.0015
90,90	Mean	0.1676	0.1674	0.1676	0.1671
	MSE	2.8660e-04	3.4110e-04	3.5510e-04	7.3270e-04
30,15	Mean	0.1696	0.1693	0.1689	0.1691
	MSE	0.0018	0.0022	0.0019	0.0023
15,90	Mean	0.1668	0.1666	0.1686	0.1685
	MSE	3.4100e-04	3.9690e-04	4.0310e-04	7.6320e-04
30,90	Mean	0.1673	0.1671	0.1685	0.1684
	MSE	3.1830e-04	3.8450e-04	3.8990e-04	7.9990e-04
Exp. 2: $\alpha = 1.2, \alpha_1 = 1.2, \alpha_2 = 0.8, R = 0.2250$					
15,15	Mean	0.2252	0.2258	0.2246	0.2241
	MSE	0.0028	0.0032	0.0033	0.0039
30,30	Mean	0.2235	0.2238	0.2232	0.2223
	MSE	0.0014	0.0016	0.0017	0.0025
90,90	Mean	0.2255	0.2252	0.2261	0.2263
	MSE	0.0005	0.0005	0.0006	0.0012
30,15	Mean	0.2279	0.2285	0.2277	0.2274
	MSE	0.0027	0.0033	0.0031	0.0037
15,90	Mean	0.2220	0.2212	0.2207	0.2202
	MSE	0.0005	0.0006	0.0007	0.0013
30,90	Mean	0.2225	0.2224	0.2215	0.2210
	MSE	0.0005	0.0006	0.0006	0.0014
Exp. 3: $\alpha = 1.5, \alpha_1 = 0.9, \alpha_2 = 1.5, R = 0.1154$					
15,15	Mean	0.1171	0.1170	0.1170	0.1170
	MSE	0.0013	0.0015	0.0015	0.0018
30,30	Mean	0.1162	0.1162	0.1165	0.1166
	MSE	0.0007	0.0008	0.0008	0.0012
90,90	Mean	0.1160	0.1160	0.1162	0.1166
	MSE	2.0660e-04	2.4880e-04	2.5860e-04	5.7430e-04
30,15	Mean	0.1147	0.1158	0.1146	0.1141
	MSE	0.0012	0.0014	0.0013	0.0016
15,90	Mean	0.1136	0.1134	0.1117	0.1121
	MSE	2.2150e-04	2.5610e-04	2.9810e-04	5.5400e-04
30,90	Mean	0.1155	0.1154	0.1147	0.1154
	MSE	1.9470e-04	2.3620e-04	2.5600e-04	5.4590e-04

Table 2: Estimate for Reliability when $\beta = 0.6,\ \theta = 3$

n,m		MLE	MOM	LS	WLS
Exp. 4: $\alpha = 1.3,\ \alpha_1 = 0.7, \alpha_2 = 0.7,\ R = 0.1686$					
15,15	Mean	0.1645	0.1652	0.1654	0.1659
	MSE	0.0018	0.0019	0.0020	0.0024
30,30	Mean	0.1670	0.1672	0.1673	0.1673
	MSE	0.0009	0.0010	0.0011	0.0016
90,90	Mean	0.1689	0.1689	0.1689	0.1689
	MSE	2.9520e-04	3.3080e-04	3.7930e-04	8.0130e-04
30,15	Mean	0.1689	0.1697	0.1685	0.1685
	MSE	0.0018	0.0020	0.0021	0.0025
15,90	Mean	0.1659	0.1654	0.1677	0.1677
	MSE	3.0500e-04	3.4800e-04	3.8170e-04	8.0220e-04
30,90	Mean	0.1672	0.1666	0.1676	0.1663
	MSE	2.9780e-04	3.3520e-04	3.8350e-04	7.7090e-04
Exp. 5: $\alpha = 1.2,\ \alpha_1 = 1.2, \alpha_2 = 0.8,\ R = 0.2250$					
15,15	Mean	0.2211	0.2214	0.2215	0.2218
	MSE	0.0028	0.0030	0.0033	0.0040
30,30	Mean	0.2218	0.2220	0.2220	0.2219
	MSE	0.0014	0.0015	0.0018	0.0025
90,90	Mean	0.2253	0.2251	0.2249	0.2241
	MSE	0.0005	0.0006	0.0006	0.0013
30,15	Mean	0.2223	0.2220	0.2222	0.2212
	MSE	0.0027	0.0029	0.0031	0.0037
15,90	Mean	0.2219	0.2220	0.2211	0.2213
	MSE	0.0005	0.0005	0.0006	0.0013
30,90	Mean	0.2237	0.2240	0.2237	0.2246
	MSE	0.0005	0.0005	0.0006	0.0012
Exp. 6: $\alpha = 1.5,\ \alpha_1 = 0.9, \alpha_2 = 1.5,\ R = 0.1154$					
15,15	Mean	0.1158	0.1158	0.1158	0.1158
	MSE	0.0014	0.0014	0.0016	0.0019
30,30	Mean	0.1170	0.1170	0.1171	0.1172
	MSE	0.0006	0.0007	0.0008	0.0011
90,90	Mean	0.1159	0.1159	0.1160	0.1164
	MSE	2.0940e-04	2.3070e-04	2.6330e-04	5.5110e-04
30,15	Mean	0.1157	0.1160	0.1171	0.1172
	MSE	0.0012	0.0012	0.0014	0.0017
15,90	Mean	0.1137	0.1137	0.1121	0.1129
	MSE	2.2060e-04	2.3180e-04	2.9120e-04	5.6030e-04
30,90	Mean	0.1159	0.1162	0.1155	0.1165
	MSE	2.1370e-04	2.2740e-04	2.6080e-04	5.6650e-04

Table 3: Estimate for Reliability when $\beta = 3,\ \theta = 0.8$

n,m		MLE	MOM	LS	WLS
Exp. 7: $\alpha = 1.3, \alpha_1 = 0.7, \alpha_2 = 0.7, R = 0.1686$					
15,15	Mean	0.1675	0.1666	0.1678	0.1681
	MSE	0.0019	0.0022	0.0021	0.0025
30,30	Mean	0.1673	0.1673	0.1672	0.1673
	MSE	0.0009	0.0011	0.0012	0.0016
90,90	Mean	0.1684	0.1688	0.1682	0.1680
	MSE	3.1620e-04	3.7000e-04	3.9160e-04	7.9440e-04
30,15	Mean	0.1699	0.1702	0.1682	0.1682
	MSE	0.0018	0.0020	0.0021	0.0025
15,90	Mean	0.1657	0.1658	0.1668	0.1654
	MSE	3.2510e-04	3.7550e-04	4.0770e-04	8.3510e-04
30,90	Mean	0.1682	0.1681	0.1691	0.1689
	MSE	3.1840e-04	3.7740e-04	4.0760e-04	8.5170e-04
Exp. 8: $\alpha = 1.2, \alpha_1 = 1.2, \alpha_2 = 0.8, R = 0.2250$					
15,15	Mean	0.2237	0.2236	0.2228	0.2222
	MSE	0.0027	0.0032	0.0031	0.0036
30,30	Mean	0.2232	0.2228	0.2234	0.2231
	MSE	0.0014	0.0017	0.0017	0.0024
90,90	Mean	0.2241	0.2242	0.2238	0.2231
	MSE	0.0005	0.0006	0.0006	0.0013
30,15	Mean	0.2266	0.2271	0.2260	0.2253
	MSE	0.0025	0.0032	0.0028	0.0033
15,90	Mean	0.2217	0.2211	0.2204	0.2203
	MSE	0.0005	0.0006	0.0006	0.0013
30,90	Mean	0.2239	0.2236	0.2235	0.2234
	MSE	0.0004	0.0005	0.0006	0.0012
Exp. 9: $\alpha = 1.5, \alpha_1 = 0.9, \alpha_2 = 1.5, R = 0.1154$					
15,15	Mean	0.1173	0.1179	0.1171	0.1171
	MSE	0.0013	0.0015	0.0015	0.0018
30,30	Mean	0.1160	0.1157	0.1163	0.1165
	MSE	0.0006	0.0007	0.0008	0.0012
90,90	Mean	0.1157	0.1158	0.1153	0.1146
	MSE	1.9910e-04	2.3530e-04	2.4710e-04	4.9920e-04
30,15	Mean	0.1164	0.1164	0.1173	0.1173
	MSE	0.0012	0.0014	0.0014	0.0016
15,90	Mean	0.1147	0.1143	0.1132	0.1141
	MSE	2.1240e-04	2.4840e-04	2.9380e-04	6.0180e-04
30,90	Mean	0.1163	0.1159	0.1158	0.1166
	MSE	2.1870e-04	2.6570e-04	2.7080e-04	5.8860e-04

References

[1] Abd AL-Fattah, A. M., EL-Helbawy, A. A. and AL-Dayian, G.R., Inverted Kumaraswamy Distribution: Properties and Estimation, Pakistan Journal of Statistics, Vol. 33(1) (2017), 37-61.

[2] Almetwally, E. M., and Almongy, H. M, Estimation Methods for the New Weibull-Pareto Distribution: Simulation and Application, Journal of Data Science, 17(3) (2019), 610-630.

[3] Casella, G., and Berger, R. L, Statistical Inference. Brooks/Cole Publishing Company, (1990).

[4] Hameed, B. A., Salman, A. N., and Kalaf, B. A, On Estimation of $P\ (Y_1 < X < Y_2)$ in Cased Inverse Kumaraswamy Distribution, Iraqi Journal of Science, (2020), 845-853.

[5] Hassan, A. S., Elsayed, A. E., and Rania, M. S, On the Estimation of $P\ (Y < X < Z)$ for Weibull Distribution in the Presence of k Outliers, International Journal of Engineering Research and Applications, 3(6) (2013), 1728-1734.

[6] Karam, N. S., and Jani, H. H, Estimation of Reliability in Multi-Component Stress-Strength Model Following Burr-III Distribution, Journal of College of Education, (1) (2016), 329-342.

[7] Kotz, S., and Pensky, M, The stress-strength model and its generalizations: theory and applications, World Scientific, (2003).

[8] Nasiru, S., and Luguterah, A, The new weibull-pareto distribution, Pakistan Journal of Statistics and Operation Research, (2015), 103-114.

[9] Swain J., Venkatraman S., and Wilson J., Least Squares Estimation of Distribution Function in Johnson's Translation System, Journal of Statistical Computation and Simulation, Vol. 29(1988), 271-297.