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ABSTRACT

In this paper , the symmetric left(right) reverse Γ^*-4-centralizer of a Γ-ring M with involution is presented and studied . Then we proved that the 4-additive mapping $T : M \times M \times M \times M \rightarrow M$ is a reverse Γ^*-4-centralizer of M if it satisfies one of these conditions :

(i) $T((r \circ y)_\gamma, r_2, r_3, r_4) = (T(r, r_2, r_3, r_4) \circ y^*)_\gamma = (r^* \circ T(y, r_2, r_3, r_4))_\gamma$

(ii) $T(r^3, r_2, r_3, r_4) = r^* \gamma T(r, r_2, r_3, r_4) \beta r^*$

(iii) $T(r, r_2, r_3, r_4) \gamma y^* = r^* \gamma T(y, r_2, r_3, r_4)$

for all $y, r, r_2, r_3, r_4 \in M$ and $\gamma, \beta \in \Gamma$.

1. Introduction

Majeed and Al-Taay in 2010 [1] proved many results of symmetric reverse $*$-centralizer of $*$-ring . Saed in 2016 [2] introduced the notion of double reverse θ^*- centralizer of rings with involution and proved basic properties of this type of mapping . Again Saed in 2016 [3] introduced the notion of Jordan $(\theta,\theta)^*$ - derivation pairs of rings with involution and proved basic properties of this type of mapping . Faraj and Super in 2020 [4] these results are studied by using the concept of symmetric reverse $*$-n-centralizer .

Let M be a Γ-ring with involution . This paper is organized as follows . In section two , we recall some well-known definitions , examples and results that will be used in this paper .In section three , we present the notion of symmetric left(right) reverse Γ^*-4-centralizer of M , and we worked on some results , which give the two cases that (left and right) reverse Γ^*-4-centralizer have come up with a

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concept symmetric reverse Γ^*-4-centralizer and showed some related results of the present concept under certain conditions .

2. Basic Concept

Definition 2.1:[5]

Assume M and Γ be additive abelian groups. If there exists a mapping M x Γ x M → M : (a , α , b) → a α b which satisfies the conditions :

for every a ,b ,c ∈ M , α , β ∈ Γ :

(i) (a +b) α c=a αc + b α c
a(α+ β)b = a αb + aβb
a α (b+c) = a αb + a αc

(ii) (a αb) β c=a α (b βc)

Where M is refer to as a Γ-ring .

Example 2.2:

Let M = $\left\{\begin{pmatrix} a & b & 0 \\ 0 & c & 0 \\ 0 & 0 & 0 \end{pmatrix} : a, b, c \in Z\right\}$, and Γ = $\left\{\begin{pmatrix} n & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} : n \in Z\right\}$

We use the usual addition and multiplication on matrices of

M x Γ x M → M , then M is a Γ-ring .

Definition 2.3: [6]

Let M , be a Γ-ring . The set Z (M) = { x∈ M : xγy = yγx , for all y∈ M and γ ∈ Γ} is called the center of the Γ- ring M .

Definition 2.4: [6]
A Γ-ring M is called semiprime if a Γ M Γ a = {0} implies that a=0 , for a ∈M .

Definition2.5: [6]

A Γ-ring M is called 2-torsion-free if 2a=0 implies a=0 , for a ∈M .

Definition 2.6: [6]

A Γ-ring M is called a commutative if aγb = bγa , for all a, b ∈ M and γ ∈ Γ .

Definition 2.7: [7]

Let M be a Γ-ring , for any x , y ∈ M and α ∈ Γ , the symbol
$[x,y]_\alpha$= x αy – y αx , will denote the commutator .
$(x \circ y)_\alpha$ = x αy + y αx , will denote the additive group commutator .

Lemma 2.8: [7]

If M is a Γ-ring , for all a ,b, c ∈ M and $\alpha, \beta \in \Gamma$ then :

(i) $[a,b]_\alpha + [b,a]_\alpha = 0$

(ii) $[a+b,c]_\alpha = [a\,,c]_\alpha + [b,c]_\alpha$

(iii) $[a\,,b+c]_\alpha = [a\,,b]_\alpha + [a,c]_\alpha$

(iv) $[a\,,b]_{\alpha+\beta} = [a,b]_\alpha + [a,b]_\beta$

(v) $[a\,\beta b, c]_\alpha = a\,\beta[b,c]_\alpha + [a,c]_\alpha \beta b + a\,\beta c\,\alpha b - a\,\alpha c\,\beta b$.

Definition 2.9: [7]

An additive mapping $(x\alpha x) \to (x\alpha x)^*$ on a Γ-ring M is called an involution if $(x\alpha y)^* = y^*\alpha x^*$ and $(x\alpha x)^{**} = x\alpha x$ for all x , y∈ M and $\alpha \in \Gamma$. A Γ-ring M equipped with an involution is called a Γ-ring M with involution (also known as Γ^*-ring) .

Definition 2.10: [7]

An additive mapping T: M → M is left (right) reverse Γ^*-centralizer of a Γ-ring M with involution if $T(y\alpha x) = T(x)\,\alpha\, y^*$ $(T(y\alpha x) = x^*\alpha T(y))$ for all x , y∈ M and $\alpha \in \Gamma$.

Definition 2.11: [7]

A reverse Γ^*-centralizer of a Γ-ring M with involution is an additive mapping which is both a left and right reverse Γ^*-centralizer .

Example 2.12:

Let F be a field and $D_3(F)$ be a set of all diagonal matrices of order 3 with respect to the usual operation of addition and multiplication , then $D_3(F)$ is a commutative ring .

$T : D_3(F) \to D_3(F)$ be an additive mapping defined as

$$T(x) = T\left(\begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & 0 \end{bmatrix}\right) = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad , x\,,y \in F$$

Define $y = \begin{bmatrix} x_1 & 0 & 0 \\ 0 & y_1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $y^* = \begin{bmatrix} x_1 & 0 & 0 \\ 0 & y_1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $\alpha = \left\{ \begin{bmatrix} \alpha_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} : \alpha_1 \in \Gamma \right\}$, then

$T(y\alpha x) = T(x)\,\alpha\, y^*$. Hence T is a reverse Γ^*-centralizer .

3. Symmetric Reverse Γ^*- 4- centralizers

First , we introduce the basic definitions in this paper

Definition 3.1:

Let M , be the Γ-ring with involution . An 4-additive mapping T : M x M x M x M → M is said to be left reverse Γ^*-4-centralizer if the following equations hold for all $y, r_1, r_2, r_3, r_4 \in M$ and $\gamma \in \Gamma$.

$T_1(r_1\,\gamma\, y\,, r_2\,, r_3\,, r_4) = T_1(y\,, r_2\,, r_3\,, r_4)\,\gamma\, r_1^*$

$T_2(r_1, r_2 \gamma y, r_3, r_4) = T_2(r_1, y, r_3, r_4)\gamma r_2^*$

$T_3(r_1, r_2, r_3 \gamma y, r_4) = T_3(r_1, r_2, y, r_4)\gamma r_3^*$

$T_4(r_1, r_2, r_3, r_4 \gamma y) = T_4(r_1, r_2, r_3, y)\gamma r_4^*$

T is said to be a symmetric left reverse Γ^*-4-centralizer if all the above equations are equivalent to each other . That is ,

$T(r_1 \gamma y, r_2, r_3, r_4) = T(y, r_2, r_3, r_4)\gamma r_1^*$

for all $y, r_1, r_2, r_3, r_4 \in M$ and $\gamma \in \Gamma$.

Definition 3.2:

Let M , be the Γ-ring with involution . An 4-additive mapping T : M x M x M x M → M is said to be right reverse Γ^*-4-centralizer if the following equations hold for all $y, r_1, r_2, r_3, r_4 \in M$ and $\gamma \in \Gamma$.

$T_1(r_1 \gamma y, r_2, r_3, r_4) = y^* \gamma T_1(r_1, r_2, r_3, r_4)$
$T_2(r_1, r_2 \gamma y, r_3, r_4) = y^* \gamma T_2(r_1, r_2, r_3, r_4)$

$T_3(r_1, r_2, r_3 \gamma y, r_4) = y^* \gamma T_3(r_1, r_2, r_3, r_4)$

$T_4(r_1, r_2, r_3, r_4 \gamma y) = y^* \gamma T_4(r_1, r_2, r_3, r_4)$

T is said to be a symmetric right reverse Γ^*-4-centralizer if all the above equations are equivalent to each other . That is ,

$T(r_1 \gamma y, r_2, r_3, r_4) = y^* \gamma T(r_1, r_2, r_3, r_4)$

for all $y, r_1, r_2, r_3, r_4 \in M$ and $\gamma \in \Gamma$.

So , M is called symmetric reverse Γ^*-4-centralizer if M is symmetric left reverse Γ^*-4-centralizer and right reverse Γ^*-4-centralizer together .

Example 3.3:

Consider $M = \left\{ \begin{pmatrix} 0 & a & b \\ 0 & 0 & c \\ 0 & 0 & 0 \end{pmatrix} : a, b, c \in \mathbb{C} \right\}$, where $\mathbb{C}$ is a ring of complex numbers . Clearly , M is a non-commutative ring under the usual addition and multiplication of matrices .

A map T : M x M x M x M → M is defined by :

$$T\left\{ \begin{pmatrix} 0 & a_1 & b_1 \\ 0 & 0 & c_1 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & a_2 & b_2 \\ 0 & 0 & c_2 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & a_3 & b_3 \\ 0 & 0 & c_3 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & a_4 & b_4 \\ 0 & 0 & c_4 \\ 0 & 0 & 0 \end{pmatrix} \right\} =$$

$$\begin{pmatrix} 0 & 0 & c_1c_2c_3c_4 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \text{ for all } \begin{pmatrix} 0 & a_1 & b_1 \\ 0 & 0 & c_1 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & a_2 & b_2 \\ 0 & 0 & c_2 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & a_3 & b_3 \\ 0 & 0 & c_3 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & a_4 & b_4 \\ 0 & 0 & c_4 \\ 0 & 0 & 0 \end{pmatrix} \in M$$

Such that $\begin{pmatrix} 0 & a & b \\ 0 & 0 & c \\ 0 & 0 & 0 \end{pmatrix}^* = \begin{pmatrix} 0 & c & b \\ 0 & 0 & a \\ 0 & 0 & 0 \end{pmatrix}$

And $\Gamma = \left\{ \begin{pmatrix} 0 & n & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} : n \in Z \right\}$

Then , T is a symmetric left reverse Γ^*-4-centralizer and also it is a right reverse Γ^*-4-centralizer

Lemma 3.4:

Let M be a semiprime Γ-ring with involution , let a∈ M be a fixed element , and let $T(r, r_2, r_3, r_4) = a\gamma r^* + r^*\gamma a$ satisfy

$T((r \circ y)_\beta, r_2, r_3, r_4) = (T(r, r_2, r_3, r_4) \circ y^*)_\beta = (r^* \circ T(y, r_2, r_3, r_4))_\beta$ for all $y, r, r_2, r_3, r_4 \in$ M and $\gamma, \beta \in \Gamma$ then a∈Z(M) .

Proof :

$T((r \circ y)_\beta, r_2, r_3, r_4) = (T(r, r_2, r_3, r_4) \circ y^*)_\beta = (r^* \circ T(y, r_2, r_3, r_4))_\beta$ (3.1)

By hypothesis $T(r, r_2, r_3, r_4) = a\gamma r^* + r^*\gamma a$

$T((r \circ y)_\beta, r_2, r_3, r_4) = (a\gamma r^* + r^*\gamma a)\beta y^* + y^*\beta(a\gamma r^* + r^*\gamma a)$ (3.2)

Let $r = r\beta y + y\beta r$ in hypothesis , we get

$T(r, r_2, r_3, r_4) = a\gamma (r\beta y + y\beta r)^* + (r\beta y + y\beta r)^*\gamma a$ (3.3)

Then , by (3.1) and (3.2) , we have

$a\gamma y^*\beta r^* + r^*\beta y^*\gamma a - r^*\gamma a\beta y^* - y^*\beta a\gamma r^* = 0$

$(a\gamma y^* - y^*\gamma a)\beta r^* + r^*\beta(y^*\gamma a - a\gamma y^*) = 0$

This implies that $[[a, y^*]_\gamma, r^*]_\beta = 0$

By [Lemma 2.3 , 4] , we get a∈Z(M) .

Lemma 3.5:

Let M be a semiprime Γ-ring with involution **.** If every mappings T of M satisfy
$T((r \circ y)_\gamma, r_2, r_3, r_4) = (T(r, r_2, r_3, r_4) \circ y^*)_\gamma = (r^* \circ T(y, r_2, r_3, r_4))_\gamma$
for all $y, r, r_2, r_3, r_4 \in$ M and $\gamma \in \Gamma$. Then T maps Z(M) into Z(M) .

Proof :

Let $a = T(c, r_2, r_3, r_4)$ for $c \in Z(M)$ and $r, r_2, r_3, r_4 \in M$.

$2T(c\beta r, r_2, r_3, r_4) = T(c\beta r + r\beta c, r_2, r_3, r_4) = T(c, r_2, r_3, r_4)\beta r^* +$

$r^* \beta T(c, r_2, r_3, r_4) = a \beta r^* + r^* \beta a$

$T(r\gamma y + y\gamma r, r_2, r_3, r_4) = 2T(c\beta(r\gamma y + y\gamma r), r_2, r_3, r_4)$

$= 2[T(c\beta r, r_2, r_3, r_4)\gamma y^* + y^*\gamma T(c\beta r, r_2, r_3, r_4)]$

$= 2[T(c\gamma y, r_2, r_3, r_4)\beta r^* + r^* \beta T(c\gamma y, r_2, r_3, r_4)]$

$= T(c\beta r, r_2, r_3, r_4)\gamma y^* + y^*\gamma T(c\beta r, r_2, r_3, r_4)$

$= T(c\gamma y, r_2, r_3, r_4)\beta r^* + r^* \beta T(c\gamma y, r_2, r_3, r_4)$

$= T(r, r_2, r_3, r_4)\gamma y^* + y^*\gamma T(r, r_2, r_3, r_4)$

$= T(y, r_2, r_3, r_4)\gamma r^* + r^* \gamma T(y, r_2, r_3, r_4)$

For all $y, r, r_2, r_3, r_4 \in M$ and $\gamma, \beta \in \Gamma$.

By Lemma 3.4 , we get : $a \in Z(M)$.

Theorem 3.6:

Let M be a 2-torsion free semiprime Γ-ring with involution , and $T : M \times M \times M \times M \to M$ be an 4-additive mapping which satisfies :

$T((r \circ y)_\gamma, r_2, r_3, r_4) = (T(r, r_2, r_3, r_4) \circ y^*)_\gamma = (r^* \circ T(y, r_2, r_3, r_4))_\gamma$ for all $y, r, r_2, r_3, r_4 \in M$ and $\gamma \in \Gamma$. Then , T is a reverse Γ^*-4-centralizer of M .

Proof :

$T((r \circ y)_\gamma, r_2, r_3, r_4) = (T(r, r_2, r_3, r_4) \circ y^*)_\gamma = (r^* \circ T(y, r_2, r_3, r_4))_\gamma$
$= T(r\gamma y + y\gamma r, r_2, r_3, r_4) =$

$T(r, r_2, r_3, r_4)\gamma y^* + y^*\gamma T(r, r_2, r_3, r_4) = T(y, r_2, r_3, r_4)\gamma r^* + r^* \gamma T(y, r_2, r_3, r_4)$

Replacing $y = (r \circ y)_\beta$ in the last relation where $\beta \in \Gamma$, we have :

$T(r, r_2, r_3, r_4)\gamma (r\beta y + y\beta r)^* + (r\beta y + y\beta r)^* \gamma T(r, r_2, r_3, r_4) =$

$T(r, r_2, r_3, r_4)\gamma y^*\beta r^* + y^*\gamma T(r, r_2, r_3, r_4)\beta r^* + r^* \beta T(r, r_2, r_3, r_4)\gamma y^* + r^* \beta y^* \gamma T(r, r_2, r_3, r_4)$

This implies that :

$T(r, r_2, r_3, r_4)\gamma r^* \beta y^* + y^*\beta r^* \gamma T(r, r_2, r_3, r_4) =$

$y^*\gamma T(r, r_2, r_3, r_4)\beta r^* + r^* \beta T(r, r_2, r_3, r_4)\gamma y^*$. Then :

$(T(r, r_2, r_3, r_4) \circ ((r \circ y)_\beta{}^*)_\gamma = (T(r, r_2, r_3, r_4) \circ y^*)_\gamma \beta r^*$

Also , we have :

$[T(r, r_2, r_3, r_4), r^*]_\beta \gamma y^* = y^*\gamma [T(r, r_2, r_3, r_4), r^*]_\beta$, we get :

$[T(r, r_2, r_3, r_4), r^*]_\beta \in Z(M)$.

Now , one will show that $[T(r, r_2, r_3, r_4), r^*]_\beta = 0$, and let c ∈ Z(M)

$2\,T(c\delta\,r, r_2, r_3, r_4) = T(c\delta\,r + r\delta\,c, r_2, r_3, r_4)$

$= T(c, r_2, r_3, r_4)\delta\,r^* + r^*\,\delta\,T(c, r_2, r_3, r_4) = 2\,T(r, r_2, r_3, r_4)\delta\,c^*$

By Lemma 3.5 , we have

$T(c\delta\,r, r_2, r_3, r_4) = T(r, r_2, r_3, r_4)\delta\,c^* = T(c, r_2, r_3, r_4)\delta\,r^*$

Also , for all c ∈ Z(M) , one takes that

$[T(r, r_2, r_3, r_4), r^*]_\beta\,\delta\,c^* = T(r, r_2, r_3, r_4)\,\beta\,r^*\,\delta\,c^* - r^*\,\beta\,T(r, r_2, r_3, r_4)\,\delta\,c^* = T(r, r_2, r_3, r_4)\,\delta\,c^*\,\beta\,r^* - r^*\,\beta\,T(r, r_2, r_3, r_4)\,\delta\,c^* = T(c, r_2, r_3, r_4)\,\beta\,r^{*2} - r^*\,\beta\,T(c, r_2, r_3, r_4)\,\delta\,r^*$

$= T(c, r_2, r_3, r_4)\,\beta\,r^*\,\delta\,r^* - r^*\,\beta\,T(c, r_2, r_3, r_4)\,\delta\,r^*$

$= [T(c, r_2, r_3, r_4), r^*]_\beta\,\delta\,r^*$

For all c ∈ Z(M) , also one gets $T(c, r_2, r_3, r_4) \in Z(M)$, then

$= T(c, r_2, r_3, r_4)\,\beta\,r^*\,\delta\,r^* - T(c, r_2, r_3, r_4)\,\beta\,r^*\,\delta\,r^*$

$= T(c, r_2, r_3, r_4)\,\beta\,r^{*2} - T(c, r_2, r_3, r_4)\,\beta\,r^{*2}$

One other hand , one will show that ,

$2\,T(r^2, r_2, r_3, r_4) = T(r\gamma r + r\gamma r, r_2, r_3, r_4)$

$= T(r, r_2, r_3, r_4)\gamma\,r^* + r^*\,\gamma\,T(r, r_2, r_3, r_4)$

$= 2\,r^*\,\gamma\,T(r, r_2, r_3, r_4) = 2\,T(r, r_2, r_3, r_4)\gamma\,r^*$

Theorem 3.7:

Assume that M be a 2-torsion free semiprime Γ-ring with involution with an identity element , and T : M x M x M x M → M be an 4- additive mapping such that : $T(r^3, r_2, r_3, r_4) = r^*\,\gamma\,T(r, r_2, r_3, r_4)\beta\,r^*$. Then T is a reverse Γ^*-4-centralizer of M . for all r , r_2 , r_3 , r_4 ∈ M and $\gamma, \beta \in \Gamma$

Proof :

Since $T(r^3, r_2, r_3, r_4) = r^*\,\gamma\,T(r, r_2, r_3, r_4)\beta\,r^*$ (3.4)

Multiply involution both sides to (3.4) to get the following

$(T(r^3, r_2, r_3, r_4))^* = r\,\gamma\,(T(r, r_2, r_3, r_4))^*\beta\,r$

for all r , r_2 , r_3 , r_4 ∈ M and $\gamma, \beta \in \Gamma$.

Suppose that $F : M \times M \times M \times M \rightarrow M$, then

$F(r, r_2, r_3, r_4) = (T(r, r_2, r_3, r_4))^*$, and also we get

$F(r^3, r_2, r_3, r_4) = (T(r^3, r_2, r_3, r_4))^* = (r^* \gamma T(r, r_2, r_3, r_4) \beta r^*)^*$

$= r \gamma (T(r, r_2, r_3, r_4))^* \beta r = r \gamma F(r, r_2, r_3, r_4) \beta r$

We have F is Γ-4-centralizer

$F(r\gamma y, r_2, r_3, r_4) = r \gamma F(y, r_2, r_3, r_4) = F(r, r_2, r_3, r_4) \gamma y$. Then ,

$(T(r\gamma y, r_2, r_3, r_4))^* = F(r\gamma y, r_2, r_3, r_4) = r \gamma F(y, r_2, r_3, r_4)$

$= r \gamma (T(y, r_2, r_3, r_4))^*$, for all $y, r, r_2, r_3, r_4 \in M$ and $\gamma \in \Gamma$. (3.5)

Also ,

$(T(r\gamma y, r_2, r_3, r_4))^* = F(r\gamma y, r_2, r_3, r_4) = F(r, r_2, r_3, r_4) \gamma y$

$= (T(r\gamma y, r_2, r_3, r_4))^* \gamma y$, for all $y, r, r_2, r_3, r_4 \in M$ and $\gamma \in \Gamma$. (3.6)

Multiply involution both sides to (3.5) and (3.6) to get

$T(r\gamma y, r_2, r_3, r_4) = T(y, r_2, r_3, r_4) \gamma r^* = y^* \gamma T(r, r_2, r_3, r_4)$

Theorem 3.8:

Suppose that M is a semiprime Γ-ring with involution , and $T : M \times M \times M \times M \rightarrow M$ is an 4-additive mapping . If $T(r, r_2, r_3, r_4)\gamma y^* = r^* \gamma T(y, r_2, r_3, r_4)$ for all $y, r, r_2, r_3, r_4 \in M$ and $\gamma \in \Gamma$, then M is a left reverse Γ^*-4-centralizer of M .

Proof :

$T(r, r_2, r_3, r_4)\gamma y^* = r^* \gamma T(y, r_2, r_3, r_4)$ (3.7)

Calculating the following equation and by (3.7) , we have

$T(r + y, r_2, r_3, r_4)\gamma z^* - T(r, r_2, r_3, r_4)\gamma z^* - T(y, r_2, r_3, r_4)\gamma z^*$

$= (r + y)^* \gamma T(z, r_2, r_3, r_4) - r^* \gamma T(z, r_2, r_3, r_4) - y^* \gamma T(z, r_2, r_3, r_4)$

$= ((r + y)^* - r^* - y^*) \gamma T(z, r_2, r_3, r_4)$

$= (r^* + y^* - r^* - y^*) \gamma T(z, r_2, r_3, r_4)$

This implies that $[T(r + y, r_2, r_3, r_4) - T(r, r_2, r_3, r_4) - T(y, r_2, r_3, r_4)] \gamma z^* = 0$ (3.8)

Now , let $z^* = z$ in (3.8) to get

$[T(r + y, r_2, r_3, r_4) - T(r, r_2, r_3, r_4) - (y, r_2, r_3, r_4)] \gamma z = 0$

for all $y, r, r_2, r_3, r_4 \in M$ and $\gamma \in \Gamma$ (3.9)

Since M is semiprime ring , one obtains that

$T(r + y , r_2 , r_3, r_4) = T(r , r_2 , r_3, r_4) + T(y , r_2 , r_3, r_4)$

Similarly , one calculates the relation

$(T(y\gamma r , r_2 , r_3, r_4) - T(r , r_2 , r_3, r_4)\, \gamma\, y^*)\, \beta z^*$, then T is a left reverse Γ^*-4-centralizer of M .

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