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On n-Convergence of Filters

Raad Aziz Hussain Al-Abdulla
College of Science, Department of Mathematics, University of Al-Qadisiyah, Iraq, raad.hussain@qu.edu.iq

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On n-Convergence of Filters

Authors Names

a Raad Aziz Hussain Al-Abdulla

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ABSTRACT

In this paper, we introduced a new form of sets named n- open sets by studing and describing some of the properties and relationships between the features of these sets. We obtained some results and relationships between by the theories inferring through the style set (n-open). Furthermore, we presented an analysis of the classes of convergence in topological spaces. We called it a name n—convergence of filters beneficiaries of the idea of n-open sets and its possibilities of n- cluster point of filters has been studied.

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1. Introduction

The main idea of this analysis is to define the generalization of some forms of topological concepts. (Leven, 1963 [7]) define semi open(s-open) set, semi closed (s-closed) set and study their characterization. He has defined a set A called (s-open)set in topological space Ҳ if find an open set O : $O \subseteq A \subseteq \overline{O}$ where $\overline{O}$ denotes by the closure of O in Ҳ, the complement semi-open (s-open) set named semi-closed (s-closed) set. (Maheswari and Tapi ,1978 [8]) defined the feebly closed (f-closed), feebly open (f-open) set. A set A$\subseteq$ Ҳ where(Ҳ, ţ) topological space called feebly open (f-open) set in Ҳ if there is a V$\subseteq$ţ such as $V \subseteq A \subseteq \overline{V}^{s}$. As known, convergence is a basic concept in analysis. General topology contains important type of convergence, that reverting to(Cartan , 1937 [3]; Willard ,1970 [15] ; and Bourbaki , 1989 [1]) depends on the filter's concept. Also, through [11] and [14], we can get some of the concept that were dealt with in this research. Through the previous concepts we were able to get some theories, properties and important relations among them. We also in this work found some examples of some reverse cases that are generally incorrect.

a.College of Science, Department of Mathematics, University of Al-Qadisiyah, Iraq
e-mail: raad.hussain@qu.edu.iq

Notation: Whenever in the research the symbol(tp-s) is mentioned it, will be used to denote the topological space, (s-open) to semi open set,(s-closed) to semi closed set,(f-open) to feebly open set and (f-closed) to feebly closed set.

2. Preliminaries

Definition(2.1): **[7]**. Assume that $(\mathfrak{X}, \mathfrak{t})$ is a tp-s & $A \subseteq \mathfrak{X}$. Then A is named s-open in $\mathfrak{X}$ if there exists $w \in \mathfrak{t} : w \subseteq A \subseteq \overline{w}$. Or equivalent **[7]**, A called s-open in $\mathfrak{X}$ if $A \subseteq \overline{A^{\circ}}$, equivalent $\overline{A} = \overline{A^{\circ}}$, the complement of s-open is named s-closed.Then A is called s-closed in $\mathfrak{X}$ if there is a closed set B so that $B^{\circ} \subseteq A \subseteq B$, or equivalent **[4]**, A is s-closed in $\mathfrak{X}$ if $\overline{A}^{\circ} \subseteq A$, equivalent $A^{\circ} = \overline{A}^{\circ}$. It is clear that when A open set then it is s-open and when A closed set then it is s-closed.

Proposition (2.2): **[7].** Let $\{A_{\lambda}\}_{\lambda \in \Lambda}$ be a collection of s-open in a tp-s $\mathfrak{X}$ then $\bigcup_{\lambda \in \Lambda} A_{\lambda}$ is s-open .

Remark (2.3): [7] . Let A, B s-$open$ in tp-s $\mathfrak{X}$, then $A \cap B$ does not need to be s-open in $\mathfrak{X}$.

Example(2.4): Assume that $\mathfrak{X} = \{a, b, c\}$, $\mathfrak{t} = \{\{a\}, \{b\}, \{a, b\}, \mathfrak{X}, \emptyset\}$ then each of $\{a, c\}, \{b, c\}$ are s-open, however$\{a, c\} \cap \{b, c\} = \{c\}$ not s-open.

Lemma(2.5): **[5].** For every singleton $\{x\}$ in a tp-s $\mathfrak{X}$ is either $\overline{\{x\}}^{\circ} = \emptyset$ or $\{x\} \subseteq \overline{\{x\}}^{\circ}$.

Proposition(2.6): Let $\overline{A}^{\circ} = \emptyset$ in a tp-s $\mathfrak{X}$. Then A is s-closed.

Definition(2.7): **[6]**. Assume that $\mathfrak{X}$ is a tp-s & $B \subseteq X$. An s −neighborhood of B is any subset of $\mathfrak{X}$ that includes an s-open set that contains B the s-neighborhood of a subset $\{x\}$ is named s-neighborhood of point x.

We always denoted by $N_s(x)$to the family of all s −neighborhoods of x.

Definition(2.8): **[8].** Let $(\mathfrak{X}, \mathfrak{t})$be tp-s& $A \subseteq \mathfrak{X}$, then $\cap \{B; B$ s-$closed$ $subset$ of $\mathfrak{X}$ and $A \subseteq B\}$ named (s-closure) of A .

And $\cup \{U; U$ s-$open$ $subset$ of $\mathfrak{X}$ and $A \subseteq U\}$ named (s-interior) of A and are shortened by $\overline{A}^{s}$, $A^{\circ s}$ respectively.

Proposition(2.9): Let $\mathfrak{X}$ be a (tp-s) & A, B are subset of $\mathfrak{X}$ with B s-open set . If $x \in B$ and $B \cap A = \emptyset$,thus $x \notin \overline{A}^{s}$.

Proposition(2.10): Assume that $(\mathfrak{X}, \mathfrak{t})$is a tp-s & A, B subsets of $\mathfrak{X}$ then :

(i)A is s-closed if and $only$ if $A = \overline{A}^{s}$ **[15]** (ii)if $A \subseteq B$, then $\overline{A}^{s} \subseteq \overline{B}^{s}$ **[15]** (iii) $A^{\circ s} = A^{c^{-s^{c}}}$ **[8].**

Definition(2.11): **[5].** A set A in a tp-s $\mathfrak{X}$ called f-open in $\mathfrak{X}$ where is an open set V so that $V \subseteq A \subseteq \overline{V}^{s}$, or equivalent, A set A named f-open in $\mathfrak{X}$ if and only if $A \subseteq \overline{A}^{\circ}$, however A^{c} called f-closed that $\overline{A}^{\circ} \subseteq A$. It is clear that each open is f-open set ,each closed is f-closed set .

Example(2.12)**:**Assume that $\mathfrak{X} = \{s, z, q, r, w\}$ and $\mathfrak{t} = \{\{z\}, \{q, r\}, \emptyset, \{z, q, s\}, \mathfrak{X}\}$ then, f-open sets are $\{\mathfrak{X}, \{q, s\}, \{z, q, s\}, \emptyset, , \{z\}, \{z, q, r, s\}\}$, f-closed sets are $\{\mathfrak{X}, \{q, r\}, \{z, r, w\}, \emptyset, \{r, w\}, \{w\}\}$.

Take $A = \{q, r, s, z, \}$ is f-open , but $A \not\ni \mathfrak{t}$ it set. Also $A^c = \{w\}$ is f-closed , but it is not closed.

Proposition(2.13): **[12].** Let $\mathfrak{X}$ be $tp\text{-}s$ then the union of all f-open sets in $\mathfrak{X}$ is also f- open . And the finite intersection of f-open sets in $\mathfrak{X}$ is f-open set in $\mathfrak{X}$.

Definition(2.14): **[13].** Assume that $\mathfrak{X}$ is a $tp\text{-}s$& $B \subseteq \mathfrak{X}$ .Any subset of $\mathfrak{X}$ that includes an f-open set containing B is named f-neighborhood of B, the f-neighborhood of a subset $\{x\}$ is often named f-neighborhood of point x.

The family of all f-neighborhoods of x always denoted by $\mathrm{N}_f(x)$.

Definition(2.15) **: [2].** Assume that $\mathfrak{X}$ is a $tp\text{-}s$ & $A \subseteq \mathfrak{X}$ .Therefore the intersection of all f-closed of $\mathfrak{X}$ which includes A is called f-closure of A and shortened by $\overline{A}^f$, that means $\overline{A}^f = \cap \{F: F\ is\ f\text{-}closed\ in\ X\ such\ that\ A \subseteq F\}$. However f-interior of A shortened by $\mathrm{A}^{\circ f}$, that means $\mathrm{A}^{\circ f} = \cup \{\mathrm{G} : \mathrm{G}\ is\ f$-open $in\ X$ such that $\mathrm{G} \subseteq \mathrm{A}\}$.

Example(2.16): Let $(\mathfrak{X}, \mathfrak{t})$ be a $tp\text{-}s$ and a $\epsilon\ \mathfrak{X}$, $\mathfrak{t} = \{ \mathrm{A} \subseteq \mathfrak{X} : \mathrm{a} \epsilon \mathrm{A}\} \cup \{\emptyset\}$. Then the f-open and open sets in $\mathfrak{X}$ are equivalent. Let $\mathrm{D} \subseteq \mathfrak{X}$. Then $\overline{\mathrm{D}}^f = \begin{cases} \mathfrak{X} & a \epsilon \mathrm{D} \\ \mathrm{D} & a \notin \mathrm{D} \end{cases}$ and $\mathrm{D}^{\circ f} = \begin{cases} \mathrm{D} & a \epsilon \mathrm{D} \\ \emptyset & a \notin \mathrm{D} \end{cases}$

Proposition(2.17): **[9].** Let $\mathfrak{X}$ be a $tp\text{-}s$ & A, B subset of $\mathfrak{X}$ where B is f-open, If $x \in B$ and $A \cap B = \emptyset$ then $x \notin \overline{A}^f$.

Proposition(2.18): **[9].** Assume that $(\mathfrak{X}, \mathfrak{t})$is a $tp\text{-}s$ & A, B subsets of $\mathfrak{X}$ then : A is f-closed$if\ and\ only\ if\ A = \overline{A}^f$.

Definition(2.19)**:** The intersection of every feebly open subset of $tp\text{-}s$ $\mathfrak{X}$ contained a set A is named feebly kernel of A and shortened by ($f$ker(A)). Means that : $f$ker (A) = $\cap$ {U : U f-open and A $\subseteq$ U}.

Lemma(2.20)**:** A subset A of a $tp\text{-}s$ $\mathfrak{X}$ is n-open if and only if $\overline{A}^s \subseteq fker\ (A)$.

Proof: Let A be n-open in $\mathfrak{X}$, then $\overline{A}^s \subseteq U$ when $A \subseteq U$ and U is f-open in $\mathfrak{X}$, this emplace $\overline{A}^s \subseteq \cap$ {U : A $\subseteq$ U and U$\in$ f-open(X)} = $f\ ker\ (A)$. Conversely, assume that $\overline{A}^s \subseteq f\ ker\ (A)$ $.Then\overline{A}^s \subseteq \cap$ {U : A $\subseteq$ U and U$\in$ f-open in (X)}.Therefore $\overline{A}^s \subseteq U$ for all f-open set U in $\mathfrak{X}$.

Definition(2.21)**:** Assume that $\mathfrak{X}$ is a $tp\text{-}s$ then a subset A in a space $\mathfrak{X}$ is named narrow open (n-open) set in a space $\mathfrak{X}$ if $A \subseteq U$ where U feebly open set in $\mathfrak{X}$ then $\overline{A}^s \subseteq U$. The complement of narrow open is named narrow closed (n-closed) it is as follows $U \subseteq \mathrm{A}^{\circ s}$ where U f-closed set in $\mathfrak{X}$.

Remark(2.22)**:** Each s-closed is n-open. However the converse is generally not true.

Proof : Suppose A is s-closed set in a $tp\text{-}s$ $\mathfrak{X}$ and $A \subseteq U$ such that U be f-open set . We have A is s-closed set, then $A = \overline{A}^s$ and $A = \overline{A}^s \subseteq U$. Therefore A is n-open set.

Example(2.23): Assume that(Ӽ, ț) is a tp-s when Ӽ $= \{z, q, r, s, w\}$, ț $= \{$Ӽ$, \emptyset, \{z\}, \{q, s\}, \{z, q, s\}\}$ and $A = \{q, r, w\}$, Then A is n-open but not s-closd.

Proposition(2.24): Let Ӽ be tp-s, then the arbitrary intersection of n-open sets in Ӽ is also n-open set .

Proof: Assume that $\{A_\lambda : \lambda \epsilon \Lambda\}$ is any collection of n-open sets in a space Ӽ and let $A = \cap A_{\lambda_{\lambda \in \Lambda}}$, suppose x$\in \overline{A}^s$ thus by Lemma 2.5, there are two cases.

Case 1: $\overline{\{x\}}^\circ = \emptyset$ .If $x \notin A$ then $x \notin A_\alpha$ for some Λ . By Proposition(2.6), {x} is s-closed. Then $\{x\}^c$ *is* $s-$ *open set such that* $A_\alpha \subseteq \{x\}^c$. *Then* $x \notin fker\left(A_\alpha\right)$.On the other hand by Lemma 2.20, A_α is n-open. Then $x \in \overline{A}^s \subseteq \overline{A_\alpha}^s \subseteq f\ ker\left(A_\alpha\right)$ paradoxically $x \in A$ and hence $x \in f\ ker\ (A)$. *Then* $\overline{A}^s \subseteq f\ ker\ (A)$. *Hence* A *is* n-open set.

Case 2: $\{x\} \subseteq \overline{\{x\}}^\circ$. Let $F = \overline{\{x\}}^\circ$ and $x \notin f\ ker\ (A)$, ∃ feebly closed set C does contain x, so that $C \cap A = \emptyset$. Then $x \in F = \overline{\{x\}}^\circ \subseteq \overline{C}^\circ \subseteq C$. As F is an open set containing x, then F is s-open set and $x \in \overline{A}^s$ therefore, $F \cap A \neq \emptyset$. Since F $\subseteq C$, then $C \cap A \neq \emptyset$ paradoxically. Then $x \epsilon f\ ker\ (A)$ and hence *isA* n-open set.

In Example(2.25) we can show that the union of n-open sets is not necessarily n-open:

Example(2.25): Let Ӽ $= \{z, q, r\}$ with ț $= \{$Ӽ$, \emptyset, \{z\}, \{q\}, \{z, q\}\}$ be a topology on Ӽ. Then $\{z\}, \{q\}$ are n-open sets but$\{z\} \cup \{q\} = \{z, q\}$ is not n-open set.

Definition(2.26): Assume that Ӽ is a tp-s& $B \subseteq$ Ӽ . Any subset of Ӽ that contains an n-open set containing B is named n-neighborhood of , the n-neighborhood of a subset $\{x\}$ is named n-neighborhood of point x.

The family of all n −neighborhoods of x always denoted by $\mathrm{N_n}(x)$.

Definition(2.27): Let Ӽ be a tp-s , $p \in$ Ӽ & $A \in$ Ӽ .The point p is named n-limit point of A if each n-open set containing U, includes a point of A distinct from p .We are going to name the family of all n-limit point of A the n- derivative set of A and shortened by A'^n. Then $p \in A'^n$ if for each n-open set U in Ӽ such $p \in V$ this indicates that $(U \cap A) - \{p\} \neq \emptyset$.

Definition(2.28): Assume that Ӽ is a tp-s & $A \subseteq$ Ӽ ,then $\cap\{F\text{: } F \text{ is } n\text{-closed in } X \text{ such that } A \subseteq F\}$ is named n-closure of A and shortened by $\overline{A}^n$.

Proposition(2.29): Let Ӽ be a tp-s & A, B are subset of Ӽ with B n-open set . If $x \in B$ and $B \cap A = \emptyset$,thus $x \notin \overline{A}^n$.

Proof: Assume that G is n-open set , x $\in G$ and $G \cap A = \emptyset$. Then $A \subseteq G^c$, G^c is n-closed and x $\notin G^c$. Therefore $x \notin \overline{A}^n$.

Remark (2.30): Assume that (Ӽ, ț)is a tp-s & A, B subsets of Ӽ . If A is n-closed , *then* $A = \overline{A}^n$.

Example(2.31): In Example (2.25), $\overline{\{z,q\}}^{n}$ = { z, q }, but { z , q } is not n-closed set.

Definition (2.32): Let Ҳ and Ꝁ be two *tp-s*. Then a function f: Ҳ → Ꝁ is called n-irresolute function if for all n-open set A in Ꝁ. Therefore $f^{-1}(A)$ is an n-open set in Ҳ.

Theorem(2.33): Let Ҳ and Ꝁ be two *tp-s*. If a function f: Ҳ → Ꝁ is n-irresolute function, then for each $x \in$ Ҳ and each n-neighborhood W of $f(x)$ in Ꝁ, there is a n-neighborhood G of x in Ҳ such that $f(G) \subseteq W$.

Proof: Let f: Ҳ → Ꝁ be a n-irresolute function and W be a n-neighborhood of $f(x)$ in Ꝁ. To prove that there is a n-neighborhood G of x in Ҳ such that $f(G) \subseteq W$. Since f is a n-irresolute then $f^{-1}(W)$ is a n-neighborhood of x in Ҳ. Let $G = f^{-1}(W)$, then $f(G) = f(f^{-1}(W)) \subseteq W$ and hence $f(G) \subseteq W$.

3.On n- Convergence of Filters

This section introduces an explanation and some theorems about a modern class of convergence, which is (n-convergence) of filters.

Definition (3.1): [15]. Let Ҳ ≠ ∅ , ∑ ≠ ∅ ⊆ Ҳ, that ∑ is a filter on Ҳ if :

1. $\mathcal{F}_1 \cap \mathcal{F}_2 \in \sum$ for all $\mathcal{F}_1$, $\mathcal{F}_2 \in \sum$.
2. When $\mathcal{F}_1 \in \sum$, $\mathcal{F}_1 \subseteq \mathcal{F}_2$ then $\mathcal{F}_2 \in \sum$.

Definition (3.2): [10]. Let Ҳ ≠ ∅ , ∑ be a filter on Ҳ and ∑∘ be sub family of a filter ∑ , then ∑∘ is named a filter base shortened by (F_b) if and only if every element of ∑ includes some element of ∑∘ . This means that for each $\mathcal{F} \in \sum$ there is $\mathcal{F}_\circ \in \sum_\circ$ such that $\mathcal{F}_\circ \subseteq \mathcal{F}$.

Proposition(3.3): [10]. Let Ҳ ≠ ∅ and ∑ be a filter on Ҳ. If ∑∘ is F_b for a filter ∑, then∑ = { $\mathcal{F} \subseteq$ Ҳ : $\mathcal{F}_\circ \subseteq \mathcal{F}$, *for some* $\mathcal{F}_\circ \in \sum_\circ$} is named filter generated (F_g) by ∑∘.

Proposition(3.4): [10]. Assume that Ҳ ≠ ∅ , let ∑∘ ≠ ∅ be a family of Ҳ and ∑ = { $A \subseteq$ Ҳ , $B \subseteq A$ *for some* $B \in \sum_\circ$} , then ∑ is a filter on Ҳ if and only if for all V, W ∈ ∑∘, ∃ $\mathcal{F} \in \sum_\circ$ such that $\mathcal{F} \subseteq$ V ∩ W.

Theorem (3.5):[15]. Assume that Ҳ ≠ ∅ and ∅ ≠ Ҳ* ⊆ Ҳ, if ∑∘ is F_b on Ҳ*, then ∑∘ is F_b of a filter on Ҳ.

Definition (3.6): [10]. A filter ∑ on a *tp-s* (Ҳ, ƫ) is named convergent to a point $y \in$ Ҳ a abbreviated by ($\sum \to y$) if and only if $N(y) \subseteq \sum$. The point $y \in$ Ҳ is named a limit point of ∑.

It is said that $y \in$ Ҳ a (cluster point) of ∑ and this one is shortened by ($\sum \propto y$) if and only if $\mathcal{F} \cap V \neq \emptyset$, for all $\mathcal{F} \in \sum$ and $V \in N(y)$.

Proposition (3.7): [10]. Let Ҳ$_1$, Ҳ$_2$ be two *tp-s* and f: Ҳ$_1$ ⟶ Ҳ$_2$ be a function from Ҳ$_1$ in to Ҳ$_2$, then:

1. If ∑ is a filter on Ҳ$_1$.Then $f(\sum)$ is a filter on Ҳ$_2$ which has for a base the sets $f(\mathcal{F})$, $\mathcal{F} \in \sum$ for a base .
2. If ∑∘ is a F_b on Ҳ$_1$. Then $f(\sum_\circ)$ is a F_b on Ҳ$_2$.

Proposition 3.8: [15] **.** Let Ҳ$_1$, Ҳ$_2$ be two $tp - s$ and f: Ҳ$_1$ ⟶ Ҳ$_2$ be a function from Ҳ$_1$ in to Ҳ$_2$ **,** $y \in$ Ҳ$_1$. Then f is a continuous function if and only if for $\Sigma \to y$ in Ҳ$_1$, then $f(\Sigma) \to f(y)$ in Ҳ$_2$

Definition (3.9): A filter ∑ on a *tp-s* (Ҳ, ƫ)is named n-convergent to a point $y \in$ Ҳ $\left(\text{written } \sum \overset{n}{\to} y\right)$ if $N_n(y) \subseteq \sum$, a filter ∑ on a *tp-s* (Ҳ, ƫ) has $y \in$ Ҳ as n-cluster point abbreviated by $\left(\sum \overset{n}{\propto} y\right)$ if $\mathcal{F} \in \sum$ meets each $G \in N_n(y)$ **.**

Theorem(3.10): A filter ∑ located on a *tp-s* (Ҳ, ƫ) has $y \in$ Ҳ as an n−cluster point if $y \in \cap \overline{\mathcal{F}}^{n}$, for all $\mathcal{F} \in \sum$.

Proof : Let $\mathcal{F} \in \sum$ and $y \in \cap \overline{\mathcal{F}}^{n}$. Then for all $G \in N_n(y)$, for all $\mathcal{F} \in \sum$, F ∩ G ≠ ∅ . Therefore $\sum \overset{n}{\propto} y$.

Example (3.11): (i)Let Ӿ $= \{a, b, c, d, e\}$ with $T = \{\emptyset$, Ӿ, $\{a\}$,{b,d},{a,b,d}} .Then fo(Ӿ) $= \{\emptyset$, Ӿ, $\{a\}, \{b, d\}, \{a, b, d\}, \{a, b, d, e\}\{a, b, c, d\}\}$, and so(Ӿ)={ ∅, Ӿ,{a},{a,c},{a,e},{b,d},{a,b,d},{a,c,e},{b,c,d}{b,d,e},{b,c,d,e},{a,b,c,d},{a,b,d,e}}and no(Ӿ)=

{∅, Ӿ, $\{a\}, \{b\}, \{c\}, \{d\}, \{e\}, \{b, c\}, \{b, d\}, \{a, c\}, \{c, d\}, \{a, e\}, \{b, e\}, \{d, e\}, \{c, e\}, \{b, c, d\}, \{b, d, e\}, \{a, c, e\}, \{b, c, e\}$, $\{a, c, d, e\}, \{b, c, d, e\}\}$. Let $\sum = \{$Ӿ$\}$ be a filter on X. Since $N(c) = \{$Ӿ$\}$, then $N(c) \subseteq \sum$, thus $\sum$ converges to c. Since

$N_n(c) =$
$\{X\}, \{c\}, \{a, c\}, \{b, c\}, \{c, d\}, \{c, e\}, \{b, c, d\}, \{a, c, e\}, \{b, c, e\}, \{c, d, e\}\{a, c, d, e\}, \{a, b, c, e\}, \{b, c, d, e\}\}$, then $N_n(c) \not\subset \sum$, therefore $\sum$is not n −converges to c.

(ii) Let Ӿ $= \{a, b\}$ with $T = \{\emptyset$, Ӿ, $\{a\}\}$ .Then fo(Ӿ) $= \{\emptyset$, Ӿ, $\{a\}\}$, and so(Ӿ)={ ∅, Ӿ,{a}}and $no(X) = \{\emptyset, X, \{b\}\}$. Let $\sum = \{$Ӿ$\}$ be a filter on Ӿ. Since $N_n(a) = \{$Ӿ$\}$, then $N_n(a) \subseteq \sum$, thus $\sum$ is n-converges to c. Since $N(a) = \{$Ӿ$, \{a\}\}$, then $N(a) \not\subset \sum$, therefore $\sum$ is not converges to a.

(iii)Let Ӿ $= \{a, b, c\}$ with T={∅, X, $\{a\}$}. Then so(Ӿ)={ ∅, Ӿ,{a},{a,b},{a,c}}, fo(Ӿ)={∅, Ӿ,{a},{a,b},{a,c}} and no(Ӿ)={ ∅, Ӿ,{b},{c},{b,c}} .Let $\sum = \{$Ӿ$, \{b\}, \{a, b\}, \{a, c\}\}$ be a filter on Ӿ. Then $\sum \overset{n}{\propto} a$ but a is not cluster point to $\sum$.

(iv)Let Ӿ $= \{a, b, c, d\}$ *with T={∅, Ӿ, {a}}, then so(Ӿ)={∅, Ӿ,{a},{a,b},{a,c},{a,d},{a,b,c},{a,b,d},{a,c,d}},* fo(X)={∅,X,{a},{a,b},{a,c},{a,d},{a,b,c},{a,b,d},{a,c,d}} and no(X)={∅, Ӿ,{b},{c},{d},{b,c},{b,d},{c,d},{b,c,d}} .Let $\sum = \{$Ӿ$, \{a, c, d\}\}$ be a filter on Ӿ. Then $\sum \propto b$, but b is not *n-cluster* point of $\sum$.

Theorem (3.12): Assume that $\sum$ be a filter on a *tp-s* (Ӿ, ʈ) and $y \in$ Ӿ. If $\sum \overset{n}{\to} y$,then $\sum \overset{n}{\propto} y$.

Proof: Let $\sum \overset{n}{\to} y$, then $N_n(y) \subseteq \Sigma$, consequently $F \cap G \in \sum$, for all $G \in N_n(y)$ and for all F belonging to$\sum$. Since $\sum$ is a filter on Ӿ, therefore $F \cap G \neq \emptyset$ for all $G \in N_n(y)$ and for all $F \in \sum$, Therefore $\sum \overset{n}{\propto} y$.

However, in general the opposite of Theorem (3.12) is not valid as the next example:

Example(3.13): In Example **(**3.11(i)), where $\sum = \{$Ӿ$, \{b, c, d, e\}\}$. *Then* $\sum \overset{n}{\propto} e$ but $\sum$ is not n −converges to e.

Theorem(3.14): Assume that $\sum$ be a filter on a *tp-s* (Ӿ, ʈ) and $y \in$ Ӿ. If $\sum \overset{n}{\to} y$, then every filter smaller than $\sum$ is also n −converges to y.

Proof: ((Clear)).

However, in general the opposite of Theorem (3.14) is not valid as the next example:

Example (3.15): In Example [**3.11(i)**] , let $\sum' = \{$Ӿ,{b,c,d,e}}and $\sum = \{$Ӿ$\}$. Thus $\sum'$ is n−converges to b. But $\sum \subseteq \sum'$ and $\sum$ is not n−converges to b.

Definition(3.16): Let(Ӿ, ʈ)$tp - s$ and $\sum_\circ$ be F_b on a (Ӿ, ʈ), then $\sum_\circ$ is said to be n-convergence to $y \in$ Ӿ (written $\sum_\circ \overset{n}{\to} y$) if and only if the filter generated by $\sum_\circ$ it is n−convergent to y.

Furthermore, suppose the F_b $\sum_\circ$ has $y \in$ Ӿ as an n−cluster point (written $\sum_\circ \overset{n}{\propto} y$) if and only if each $\mathcal{F}_\circ \in \sum_\circ$ matches $G \in N_n(y)$.

Definition(3.17): Let $\sum_\circ$ be a F_b on a *tp-s* $(X, \mathfrak{t})$ and $y \in X$. Then:

1. If a point $k \in \overline{\mathcal{F}_\circ}^{n}$, for every $F_\circ \in \sum_\circ$, then k is defined as an n-adherent point of $\sum_\circ$.
2. If a point $k \in \cap\overline{\mathcal{F}_\circ}^{n}$, for every $\mathcal{F}_\circ \in \sum_\circ$, then k is said to be n-accumulation point of $\sum_\circ$.

Theorem(3.18): A F_b $\sum_\circ$ on a *tp-s* $(X, \mathfrak{t})$ is $n-$convergence to a point y belongs to X if and only if for each G belongs to $N_n(y)$, there exists $\mathcal{F}_\circ \in \sum_\circ$ such as $\mathcal{F}_\circ \subseteq G$.

Proof: Let $\sum_\circ \overset{n}{\rightarrow} y$, then a filter $\sum$ generated by $\sum_\circ$ and $\sum \overset{n}{\rightarrow} y$. Then $N_n(y) \subseteq \sum$, therefore for every $G \in N_n(y)$, $V \in \sum$. Therefore there is $\mathcal{F}_\circ \in \sum_\circ$ such that $\mathcal{F}_\circ \subseteq G$.

On the other hand,, to prove that $\sum_\circ \overset{n}{\rightarrow} y$ means , $\sum$ to be a filter on X generated by $\sum_\circ$ with $\Sigma \overset{n}{\rightarrow} y$. Let $G \in N_n(y)$ then, by hypothesis, there is $\mathcal{F}_\circ \in \sum_\circ$ such that $\mathcal{F}_\circ \subseteq G$. Because $\sum$ is a filter on X, then $G \in \sum$. Then $G \in \sum$ and $N_n(y) \subseteq \sum$. Then $\sum_\circ \overset{n}{\rightarrow} y$.

Proposition(3.19): A filter $\sum$ on a *tp-s* $(X, \mathfrak{t})$ has $y \in X$ as an n-cluster point if and only if there exists a filter $\sum^*$ smaller than $\sum$ which n-convergence to y.

Proof: Assume the $\sum \overset{n}{\propto} y$. So by Definition **(3.16)** every $\mathcal{F} \in \sum$ meets every $G \in N_n(y)$. So $\sum^*{}_\circ = \{\mathcal{F} \cap G: \mathcal{F} \in \Sigma, G \in N_n(y)\}$ is a filter base for some filter $\sum^*$ which is finer , means $\sum$ and $n-$convergence to y.

On the other hand, give $\sum \subseteq \sum^*$and $\sum^* \overset{n}{\rightarrow} y$, then $\sum^* \overset{n}{\rightarrow} y$ and $N_n(y) \subseteq \sum^*$. Hence each $\mathcal{F} \in \sum$and each $G \in N_n(y)$ belongs to $\sum^*$. Because $\sum^*$is a filter, then $\mathcal{F} \cap G \neq \emptyset$.

Theorem (3.20): Assume that $(X, \mathfrak{t})$ be a *tp-s* and $A \subseteq X$, $y \in X$. If $y \in \overline{A}^{n}$, then we have a filter $\sum$ on X so that $A \in \sum$ and $\sum \overset{n}{\rightarrow} y$.

Proof: If $y \in \overline{A}^{n}$, then $A \cap G \neq \emptyset$ is necessary for all $G \in N_n(y)$. Therefore $\sum_\circ = \{A \cap G: G \in N_n(y)\}$ is a F_b for some filter $\sum$. The result filter includes A and $\Sigma \overset{n}{\rightarrow} y$.

Corollary (3.21): Let $(X, \mathfrak{t})$ be a *tp-s* and $A \subseteq X$, $y \in X$. If $\in \overline{A}^{n}$, then there is F_b $\sum_\circ$ on X such that $A \in \sum_\circ$ and $\sum \overset{n}{\rightarrow} y$.

Theorem (3.22): Let $f: X_1 \rightarrow X_2$ be a function and $\sum$ is a filter $\sum$ on X_1, $y \in X_1$. If f is $n-$irresolute, then whenever $\sum \overset{n}{\rightarrow} y$ in X_1, then $f(\sum) \overset{n}{\rightarrow} f(y)$ in X_2.

Proof: Assume that f is $n-$irresolute function and $\sum \overset{n}{\rightarrow} y$. To prove $f(\Sigma) \overset{n}{\rightarrow} f(y)$ in X_2. Let $W \in N_n(f(y))$, as f is $n-$irresolute , then there is $G \in N_n(y)$ such that $f(G) \subseteq W$. Since $\sum \overset{n}{\rightarrow} y$, then $G \in \Sigma$. But $W \in f(\sum)$, hence $f(\sum) \overset{n}{\rightarrow} f(y)$.

Theorem (3.23): Suppose $(X, \mathfrak{t})$ is a *tp-s* and $A \subseteq X$. A point $y \in X$ is $n-$limit point of A if and only if $A - \{y\}$ is a subset of some filter $\sum$ which is $n-$convergence to y.

Proof: Assume y is $n-$limit point of A, then $A - \{y\} \cap G \neq \emptyset$ for all $G \in N_n(y)$. $\sum_\circ = \{A - \{y\} \cap G: G \in N_n(y)\}$is F_b for some filter $\sum$. The outcoming filter includes $A - \{y\}$ and $\sum \overset{n}{\rightarrow} y$. In the other hand,, If $A - \{y\} \in \sum$ with $\sum \overset{n}{\rightarrow} y$, then $A - \{y\} \in \sum$ and $N_n(y) \subseteq \sum$. Since $\sum$ is a filter, then $A - \{y\} \cap G \neq \emptyset$ for all $G \in N_n(y)$. Consequently y is the $n-$limit point of a set A.

Definition (3.24): [15]. Suppose $\{h_a: a \in D\}$ is a net in a *tp-s* $(X, \mathfrak{t})$, $\sum$ is a filter generated by a F_b $\sum_\circ$ made up of the sets $A_{a_\circ} = \{h_a: a \geq a_\circ, a_\circ \in D\}$ is named a filter generated by $\{h_a: a \in D\}$ meaning , $\sum_\circ = \{A_{a_\circ} \subseteq X: h_a \textit{ is eventually in } A_{a_\circ}\}$ is a F_b, $\sum$ is a filter on X and its named a filter associated with the net $\{h_a: a \in D\}$.

Propostion (3.25):A net $\{h_a : a \in D\}$ in a *tp-s* (Ӿ, ţ) is $n-$convergence to y ∈ Ӿ if and only if a filter ∑ generated by $\{h_a : a \in D\}$ is n−convergent to y.

Proof: A net $\{h_a : a \in D\}$ is an n−convergent to y ∈ Ӿ if and only if each V ∈ N_n(y) will has a tail of $\{h_a : a \in D\}$, we have the tails of $\{h_a : a \in D\}$ are ba se for a filter generated by $\{h_a : a \in D\}$.

Definition (3.26): [15]. Let ∑∘ be a F_b on a *tp*-s (Ӿ, ţ), for all $B_1, B_2 \in$ ∑∘ ,we put $B_1 \geq B_2$ if and only if $B_1 \subseteq B_2$, then (∑∘, ≥) is a directed set. For all $\mathcal{F} \in$ ∑∘, defines h: ∑∘ →∪ $\mathcal{F}$, $\mathcal{F} \in$ ∑∘ such that for any $\mathcal{F} \in$ ∑∘ takes (fixed) $h_F \in \mathcal{F}$ such that $h(\mathcal{F}) = h_{\mathcal{F}}$. Therefore $\{h_B : B \in$ ∑∘$\}$ is a net in Ӿ and is named a net associated with F_b ∑∘.

Theorem (3.27): Assume that $\{h_B : B \in$ ∑∘$\}$ is a net associated with a F_b∑∘ on a *tp-s* (Ӿ, ţ) and y ∈ Ӿ. If ∑∘ $\xrightarrow{n} y$, then $h_B \xrightarrow{n} y$.

Proof: Assume that ∑∘ $\xrightarrow{n} y$ and G ∈ N_n(y). Thus there is $B_\circ \in$ ∑∘ such that $B_\circ \subseteq$ G, then $h_{B_\circ} \in$ G, so $h_B \in$ G for all $B \geq B_\circ$.this emplace $h_B \xrightarrow{n} y$.

Definition(3.28): [15]. Let ∑∘ be F_b on a *tp-s* (Ӿ, ţ). Take $D = \{(k, B): k \in B, B \in \Sigma_\circ\}$, $(D, \geq)$ is directed set by the relation, $(k_1, B_1) \geq (k_2, B_2)$ if and only if $B_1 \subseteq B_2$, then set the function $h: D \to$ Ӿ, by $h(a) = h_a \in$ Ӿ, where $a = (k, B)$. Then $\{h_a : a \in D\}$ is named the canonical net (net based) of ∑∘.

Proposition (3.29): Assume that(Ӿ, ţ) *tp-s* ,∑∘ be F_b on a (Ӿ, ţ) then∑∘ is n−convergence to a point k ∈ Ӿ if and only if the canonical net of ∑∘ is $n-$convergence to k.

Proof : Assume that ∑∘ $\xrightarrow{n} k$ with G ∈ N_n(k), then there is $\mathcal{F}_\circ \in$ ∑∘ so that $\mathcal{F}_\circ \subseteq$ G. Since $\mathcal{F}_\circ \neq \emptyset$, there is $k_\circ \in \mathcal{F}_\circ$. Use $a_\circ = (k_\circ, \mathcal{F}_\circ)$, then $h_a \in$ G for all $a \geq a_\circ$. Hence $h_a \xrightarrow{n} k$. On the other hand,, let $h_a \xrightarrow{n} k$ and G ∈ N_n(k), there is $a_\circ \in$ D such that $h_a \in$ G for all $a \geq a_\circ$. Therefore there are $\mathcal{F}_\circ \in$ ∑∘ and $k_\circ \in \mathcal{F}_\circ$ so that $a_\circ = (k_\circ, \mathcal{F}_\circ)$. To prove $\mathcal{F}_\circ \subseteq$ G. Assume that this is $y_\circ \in \mathcal{F}_\circ$. So $a = (k, \mathcal{F}) \geq (k_\circ, \mathcal{F}_\circ) = a_\circ$, thus $h_a \in$ G. Hence $\mathcal{F}_\circ \subseteq$ G,this emplace $\Sigma_\circ \xrightarrow{n} k$.

Corollary (3.30): A F_b∑∘ on a tp-s (Ӿ, ţ) has k∈ Ӿ as an n −cluster point if and only if the canonical net on ∑∘ has k as an n −cluster point.

Definition(3.31):A *tp-s* (Ӿ, ţ) is n-T_2- space if for each a , b ∈ Ӿ with a ≠ b, there are V ∈ N_n(a) and W ∈ N_n(b) such that V ∩ W = ∅ .

Theorem(3.32): Let (Ӿ, ţ) *tp-s* then the next two point are equivalent

1. (Ӿ, ţ) is n-T_2- space
2. Each n−convergent filter in Ӿ has only one n−limit point.

Proof:(**1** ⇒ **2**) Suppose that (Ӿ, ţ) is an n-T_2- *space* with ∑ be a filter on Ӿ so that $\Sigma \xrightarrow{n} a$ and $\Sigma \xrightarrow{n}$ b with a ≠ b. Since Ӿ is an n-T_2- *space*, then there are V ∈ N_n(a) and W ∈ N_n(b) such that V ∩ W = ∅. Since $\Sigma \xrightarrow{n} a$ then N_n(a) ⊆ Σ and $\Sigma \xrightarrow{n}$ b then N_n(b) ⊆ ∑. Since ∑ is a filter, then V ∩ W ≠ ∅. That is the opposite, and then comes the conclusion.

(**2** ⇒ **1**) On the other hand when we wont to prove that Ӿ is a n-T_2- space. Assume that not n-T_2- space , then there are a, b ∈ Ӿ with a ≠ b such that for all V ∈ N_n(a) and for all W ∈ N_n(b), V ∩ W ≠ ∅. So ∑∘ = {V ∩ W: V ∈ N_n(a) and W ∈ N_n(b)} is a filter base for some filter ∑. The resulting filter n−convergence at a and b. It is an inconsistency, because Ӿ is an n-T_2- space.

Conclusion

In this paper , we studied some of the properties of the n-open sets through their behavior on convergence and filters, which led to obtaining some new definitions and theories that are important in topological spaces. We also found several examples of reverse instances, which are usually incorrect in this study.

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