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ABSTRACT

Let N be a near –ring and α is a mapping on N .In this paper, we will introduce the notions of two sided reverse α**-**3-derivation and generalized two sided reverse α**-**3-derivation of N . Then we will study commutativity of N under some conditions determined on semigroup ideals of N .

MSC: 16A70 , 16N60 , 16W25 .

1. Introduction

N. Argac [6] studied the commutativity of prime near-ring N using the notion of two sided α-derivation . N. L. Oukhtite and A. Raji [7] continued in the same line , they generalized some known results involving semigroup ideal and generalized two sided α-derivations . M. Ashraf et al. [8,9,10,11] studied the commutativity of near-ring N using the notions of n-derivations , (σ, τ)-n-derivations and generalized n-derivations . Hence, it should be interesting to study the commutativity of a near-ring N admitting some conditions on other n-additive mappings . E. F. Adhab [5] studied the commutativity of prime near-ring N using the notions of two sided α**-**n-derivation and generalized two sided α**-**n-derivation of near-ring N .

For more information see [12, 13, 14]

Let N be a near –ring and α is a mapping on N . This paper consists of two sections . In section one , we recall some known definitions and necessary lemmas that we will use it later in this

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paper . In section two , we define the concepts of two sided reverse α-3-derivation and generalized two sided reverse α-3-derivation of N , also we determine some conditions of generalized two sided reverse α-3-derivation and semigroup ideals which make prime near-ring commutative ring .

2.Basic Concepts

Definition 2.1:[1] A right near-ring (resp. a left near-ring) is a nonempty set N equipped with two binary operations + and . such that

(i)(N, +) is a group (not necessarily abelian)

(ii)(N, .) is a semigroup .

(iii) For all x,y,z ∈N , we have

(x+y)z = xz + yz (resp. z(x+y) = zx + zy)

Example 2.2:[1] Let G be a group (not necessarily abelian) then all mapping of G into itself form a right near-ring M(G) with regard to point wise addition and multiplication by composite

Definition 2.3:[2] A near-ring N is called a prime near-ring if aNb = {0}, where a,b ∈N, implies that either a = 0 or b = 0 .

Definition 2.4:[2] Let N be a near-ring . The symbol Z will denote the multiplicative center of N , that is Z = {x∈N / xy = yx for all y ∈N } .

Definition 2.5:[2] Let R be a ring . Define a Lie product [,] on R as follows

[x,y] = xy – yx , for all x,y ∈R .

Properties 2.6:[2] Let R be a ring , then for all x,y,z ∈R, we have :

1-[x,yz] = y[x,z] + [x,y]z

2-[xy,z] = x[y,z] + [x,z]y

3-[x+y,z] = [x,z] + [y,z]

4-[x,y+z] = [x,y] + [x,z]

Definition 2.7:[2] A nonempty subset U of N will be called a semigroup right ideal (resp. semigroup left ideal) if UN ⊂ U (resp. NU⊂ U) and if U is both semigroup right ideal and semigroup left ideal , it be called a semigroup ideal .

Remark 2.8:[2] Let N be a near-ring

(i)N x N x ....x N forms a near-ring with regard to component wise addition and component wise multiplication .

(ii) If U_1 , U_2 , ...,U_n be nonzero semigroup right ideals (resp. semigroup left ideals) of N , then $U_1 \times U_2 \times \ldots \times U_n$ forms a nonzero semigroup right ideals (resp. semigroup left ideas) of $N \times N \times \ldots \times N$.

Definition 2.9:[3] Suppose that W is a near-ring . An 3-additive mapping $d : W \times W \times W \rightarrow W$ is called 3-derivation if the relations :

$d(s_1 s_1^{/}, s_2 , s_3) = d(s_1, s_2 , s_3)s_1^{/} + s_1 d(s_1^{/}, s_2 , s_3)$

$d(s_1, s_2 s_2^{/} , s_3) = d(s_1, s_2 , s_3)s_2^{/} + s_2 d(s_1, s_2^{/} , s_3)$

$d(s_1, s_2, s_3 s_3^{/}) = d(s_1, s_2 , s_3)s_3^{/} + s_3 d(s_1, s_2 , s_3^{/})$

hold for all $s_1 , s_1^{/} , s_2 , s_2^{/} , s_3 , s_3^{/} \in W$.

Example 2.10 :[4] Let S be a commutative near-ring .

Let us define

$W= \left\{ \begin{pmatrix} r & u \\ 0 & 0 \end{pmatrix} : r, u , 0 \in S \right\}$.

And $d : W x W x W \rightarrow W$

$d\left(\begin{pmatrix} r_1 & u_1 \\ 0 & 0 \end{pmatrix} , \begin{pmatrix} r_2 & u_2 \\ 0 & 0 \end{pmatrix} , \begin{pmatrix} r_3 & u_3 \\ 0 & 0 \end{pmatrix} \right) = \begin{pmatrix} 0 & r_1 r_2 r_3 \\ 0 & 0 \end{pmatrix}$

Then d is 3-derivation of W .

Definition 2.11:[4] Suppose that W is a near-ring and d be 3- derivation of W . An 3-additive mapping $f : W x W x W \rightarrow W$ is said to be generalized 3-derivation of W associated with d if the relations

$f(s_1 s_1^{/}, s_2 , s_3) = f(s_1, s_2 , s_3)s_1^{/} + s_1 d(s_1^{/}, s_2 , s_3)$

$f(s_1, s_2 s_2^{/} , s_3) = f(s_1, s_2 , s_3)s_2^{/} + s_2 d(s_1, s_2^{/} , s_3)$

$f(s_1, s_2, s_3 s_3^{/}) = f(s_1, s_2 , s_3)s_3^{/} + s_3 d(s_1, s_2 , s_3^{/})$

hold for all $s_1 , s_1^{/} , s_2 , s_2^{/} , s_3 , s_3^{/} \in W$.

Example 2.12 :[4] Let S be a commutative near-ring .

Let us define

$W = \left\{ \begin{pmatrix} 0 & r \\ 0 & u \end{pmatrix} : r, u , 0 \in S \right\}$.

And $d, f : W x W x W \rightarrow W$,

$d\left(\begin{pmatrix} 0 & r_1 \\ 0 & u_1 \end{pmatrix} , \begin{pmatrix} 0 & r_2 \\ 0 & u_2 \end{pmatrix} , \begin{pmatrix} 0 & r_3 \\ 0 & u_3 \end{pmatrix} \right) = \begin{pmatrix} 0 & r_1 r_2 r_3 \\ 0 & 0 \end{pmatrix}$

$f\left(\begin{pmatrix} 0 & r_1 \\ 0 & u_1 \end{pmatrix} , \begin{pmatrix} 0 & r_2 \\ 0 & u_2 \end{pmatrix} , \begin{pmatrix} 0 & r_3 \\ 0 & u_3 \end{pmatrix} \right) = \begin{pmatrix} 0 & 0 \\ 0 & u_1 u_2 u_3 \end{pmatrix}$

Then f is a generalized 3-derivation of W .

Lemma 2.13:[5] Let N be a prime near-ring and U be a nonzero semigroup right ideal (resp. semigroup left ideal) of N and x is an element of N such that Ux = {0} (resp. xU = {0}) , then x = 0 .

Lemma 2.14:[5] Let N be a prime near-ring and U be a nonzero semigroup ideal of N . If $x,y \in N$ and xUy = {0}, then x = 0 or y = 0 .

Lemma 2.15:[5] Let N be a prime near-ring and Z contains a nonzero semigroup left ideal or nonzero semigroup right ideal , then N is a commutative ring .

3.Main Results

First we introduce the basic definitions in this paper

Definition 3.1: Let N be a near-ring .An 3-additive mapping d :N x N x N → N is said to be two sided reverse α-3-derivation if there exists a function α: N → N such that the relations :

$d(x_1{}'x_1, x_2, x_3) = d(x_1, x_2, x_3)x_1{}' + \alpha(x_1)d(x_1{}', x_2, x_3)$
$= d(x_1, x_2, x_3)\alpha(x_1{}') + x_1d(x_1{}', x_2, x_3)$

$d(x_1, x_2{}'x_2, x_3) = d(x_1, x_2, x_3)x_2{}' + \alpha(x_2)d(x_1, x_2{}', x_3)$
$= d(x_1, x_2, x_3)\alpha(x_2{}') + x_2d(x_1, x_2{}', x_3)$

$d(x_1, x_2, x_3{}'x_3) = d(x_1, x_2, x_3)x_3{}' + \alpha(x_3)d(x_1, x_2, x_3{}')$
$= d(x_1, x_2, x_3)\alpha(x_3{}') + x_3d(x_1, x_2, x_3{}')$

hold for all $x_1, x_1{}', x_2, x_2{}', x_3, x_3{}' \in N$.

Definition 3.2: Let N be a near-ring and d :N x N x N → N be a two sided reverse α-3-derivation of N . An 3-additive mapping f : N x N x N → N is said to be generalized two sided reverse α-3-derivation associated with two sided reverse α-3-derivation d if the relations :

$f(x_1{}'x_1, x_2, x_3) = d(x_1, x_2, x_3)x_1{}' + \alpha(x_1)f(x_1{}', x_2, x_3)$
$= d(x_1, x_2, x_3)\alpha(x_1{}') + x_1f(x_1{}', x_2, x_3)$

$f(x_1, x_2{}'x_2, x_3) = d(x_1, x_2, x_3)x_2{}' + \alpha(x_2)f(x_1, x_2{}', x_3)$
$= d(x_1, x_2, x_3)\alpha(x_2{}') + x_2f(x_1, x_2{}', x_3)$

$f(x_1, x_2, x_3{}'x_3) = d(x_1, x_2, x_3)x_3{}' + \alpha(x_3)f(x_1, x_2, x_3{}')$
$= d(x_1, x_2, x_3)\alpha(x_3{}') + x_3f(x_1, x_2, x_3{}')$

hold for all $x_1, x_1{}', x_2, x_2{}', x_3, x_3{}' \in N$.

We begin with the following lemmas which are essential for developing the proofs of our main results , α will represent a homomorphism of N .

Lemma 3.3 : Let N be a near-ring and d be a two sided reverse α-3-derivation of N , then

$d(x_1{}'x_1, x_2, x_3) = x_1d(x_1{}', x_2, x_3) + d(x_1, x_2, x_3)\alpha(x_1{}')$
$= \alpha(x_1)d(x_1{}', x_2, x_3) + d(x_1, x_2, x_3)x_1{}'$

$d(x_1, x_2{}'x_2, x_3) = x_2d(x_1, x_2{}', x_3) + d(x_1, x_2, x_3)\alpha(x_2{}')$
$= \alpha(x_2)d(x_1, x_2{}', x_3) + d(x_1, x_2, x_3)x_2{}'$

$d(x_1, x_2, x_3{}'x_3) = x_3d(x_1, x_2, x_3{}') + d(x_1, x_2, x_3)\alpha(x_3{}')$
$= \alpha(x_3)d(x_1, x_2, x_3{}') + d(x_1, x_2, x_3)x_3{}'$

hold for all $x_1, x_1{}', x_2, x_2{}', x_3, x_3{}' \in N$.

Lemma 3.4 : Let N be a near-ring and d be a two sided reverse α-3-derivation of N , then

(i) $(\alpha(x_1)d(x_1{}', x_2, x_3) + d(x_1, x_2, x_3)x_1{}')y = \alpha(x_1)d(x_1{}', x_2, x_3)y + d(x_1, x_2, x_3)x_1{}'y$
$(\alpha(x_2)d(x_1, x_2{}', x_3) + d(x_1, x_2, x_3)x_2{}')y = \alpha(x_2)d(x_1, x_2{}', x_3)y + d(x_1, x_2, x_3)x_2{}'y$
$(\alpha(x_3)d(x_1, x_2, x_3{}') + d(x_1, x_2, x_3)x_3{}')y = \alpha(x_3)d(x_1, x_2, x_3{}')y + d(x_1, x_2, x_3)x_3{}'y$

for all $x_1, x_1{}', x_2, x_2{}', x_3, x_3{}', y \in N$.

(ii) $(x_1d(x_1{}', x_2, x_3) + d(x_1, x_2, x_3)\alpha(x_1{}'))y = x_1d(x_1{}', x_2, x_3)y + d(x_1, x_2, x_3)\alpha(x_1{}')y$
$(x_2d(x_1, x_2{}', x_3) + d(x_1, x_2, x_3)\alpha(x_2{}'))y = x_2d(x_1, x_2{}', x_3)y + d(x_1, x_2, x_3)\alpha(x_2{}')y$
$(x_3d(x_1, x_2, x_3{}') + d(x_1, x_2, x_3)\alpha(x_3{}'))y = x_3d(x_1, x_2, x_3{}')y + d(x_1, x_2, x_3)\alpha(x_3{}')y$

for all $x_1, x_1', x_2, x_2', x_3, x_3', y \in N$.

Lemma 3.5 : Let N be a prime near-ring and d be a nonzero two sided reverse α-3-derivation of N . Let U_1, U_2, U_3 be a nonzero semigroup ideals of N . If $d(U_1, U_2, U_3) = \{0\}$, then $d(N,N,N) = \{0\}$.

Now , we will prove the main results :

Theorem 3.6 : Let N be a prime near-ring and d be a nonzero two sided reverse α-3-derivation of N . Let U_1, U_2, U_3 be a nonzero semigroup ideals of N . If $d(U_1, U_2, U_3) \subseteq Z$, then N is a commutative ring .

Proof : We are given that

$d(u_1, u_2, u_3) \in Z$ for all $u_1 \in U_1, u_2 \in U_2, u_3 \in U_3$. (3.1)

Therefore

$t\, d(u_1'u_1, u_2, u_3) = d(u_1'u_1, u_2, u_3)\, t$ for all $u_1, u_1' \in U_1, u_2 \in U_2, u_3 \in U_3, t \in N$.

By Lemma 3.4 (ii) and defining property of d we get

$t\, u_1 d(u_1', u_2, u_3) + t\, d(u_1, u_2, u_3)\, \alpha(u_1')$

$= u_1 d(u_1', u_2, u_3)t + d(u_1, u_2, u_3)\, \alpha(u_1')t$

for all $u_1, u_1' \in U_1, u_2 \in U_2, u_3 \in U_3, t \in N$. Using (3.1) again , we obtain

$d(u_1', u_2, u_3)t\, u_1 + d(u_1, u_2, u_3)\, t\, \alpha(u_1') = d(u_1', u_2, u_3)\, u_1 t + d(u_1, u_2, u_3)\, \alpha(u_1')t$

for all $u_1, u_1' \in U_1, u_2 \in U_2, u_3 \in U_3, t \in N$. (3.2)

Replacing t by $\alpha(u_1')$ in (3.2) , we get

$d(u_1', u_2, u_3)\, \alpha(u_1')\, u_1 = d(u_1', u_2, u_3)\, u_1\, \alpha(u_1')$ for all $u_1, u_1' \in U_1, u_2 \in U_2, u_3 \in U_3$.

i.e.; $d(u_1', u_2, u_3)N\, [\alpha(u_1'), u_1] = \{0\}$ for all $u_1, u_1' \in U_1, u_2 \in U_2, u_3 \in U_3$.

Primeness of N yields that for each $u_1' \in U_1$, we get either $d(u_1', u_2, u_3) = 0$ for all $u_2 \in U_2, u_3 \in U_3$ or $[\alpha(u_1'), u_1] = 0$ for all $u_1 \in U_1$.

If $d(u_1', u_2, u_3) = 0$ for all $u_1' \in U_1, u_2 \in U_2, u_3 \in U_3$, then by Lemma 3.5 we conclude that $d(N,N,N) = \{0\}$, leading to a contradiction as d is a nonzero two sided reverse α-3-derivation of N . Therefore there exist $x_1 \in U_1, x_2 \in U_2, x_3 \in U_3$ all being nonzero such that $d(x_1, x_2, x_3) \neq 0$ and $\alpha(x_1)\, u = u\, \alpha(x_1)$ for all $u \in U_1$, replacing u by ut where $t \in N$, we get $U_1 [\alpha(x_1), t] = \{0\}$, for all $t \in N$. By Lemma 2.13 we get $\alpha(x_1) \in Z$.

Taking x_1 instead of u_1', x_2 instead of u_2, x_3 instead of u_3 in (3.2) , we obtain

$d(x_1, x_2, x_3)tu_1 = d(x_1, x_2, x_3)u_1 t$ for all $u_1 \in U_1, t \in N$.

i.e.; $d(x_1, x_2, x_3)[t, u_1] = 0$, accordingly

$d(x_1, x_2, x_3)\, N\, [t, u_1] = \{0\}$ for all $u_1 \in U_1, t \in N$.

Primeness of N and $d(x_1, x_2, x_3) \neq 0$ yield that $U_1 \subseteq Z$, by Lemma 2.15 we conclude that N is a commutative ring .

Theorem 3.7 : Let N be a prime near-ring admitting a generalized two sided reverse α-3-derivation f associated with a nonzero two sided reverse α-3-derivation d . Let U_1, U_2, U_3 be a nonzero semigroup ideals of N . If $f([u_1, u_1'], u_2, u_3) = 0$, for all $u_1, u_1' \in U_1, u_2 \in U_2, u_3 \in U_3$, then N is a commutative ring .

Proof : By our hypothesis we have

$f([u_1, u_1'], u_2, u_3) = 0$, for all $u_1, u_1' \in U_1, u_2 \in U_2, u_3 \in U_3$.

Replacing u_1' by $u_1'u_1$ in preceding equation and using it again we get

$d(u_1, u_2, u_3)\, [u_1, u_1'] = 0$, for all $u_1, u_1' \in U_1, u_2 \in U_2, u_3 \in U_3$.

i.e.; $d(u_1, u_2, u_3)\, u_1 u_1' = d(u_1, u_2, u_3)\, u_1' u_1$,

for all $u_1, u_1' \in U_1, u_2 \in U_2, u_3 \in U_3$ (3.3)

Replacing u_1' by $u_1'r$, where $r\in N$, in (3.3) and using it again we get
$d(u_1 , u_2 , u_3)\, u_1'[u_1,r] = 0$, for all $u_1 , u_1' \in U_1$, $u_2 \in U_2$, $u_3 \in U_3$, $r\in N$.
Therefore $d(u_1 , u_2 , u_3)\, U_1[u_1,r] =\{0\}$, for all $u_1 \in U_1$, $u_2 \in U_2$, $u_3 \in U_3$, $r\in N$.
By Lemma 2.14 , we conclude that for each $u_1\in U_1$ either $u_1 \in Z$ or $d(u_1 , u_2 , u_3) = 0$
for all $u_2 \in U_2$, $u_3 \in U_3$. (3.4)
Let $x_1 \in U_1\cap Z$, by Lemma 3.3 and defining property of d , we have for all $y\in N$.
$d(yx_1 , u_2 , u_3) = x_1 d(y, u_2 , u_3) + d(x_1 , u_2 , u_3)\, \alpha(y)$
$= d(x_1y , u_2 , u_3)$
$= d(y , u_2 , u_3)x_1 + \alpha(y)\, d(x_1 , u_2 , u_3)$
for all $u_2 \in U_2$, $u_3 \in U_3$, $y\in N$. Which implies that
$d(x_1 , u_2 , u_3)\, \alpha(y)= \alpha(y)\, d(x_1 , u_2 , u_3)$
for all $u_2 \in U_2$, $u_3 \in U_3$, $y\in N$.
In view of equation (3.4) we get
$d(u_1 , u_2 , u_3)\, \alpha(y)= \alpha(y)\, d(u_1 , u_2 , u_3)$
for all $u_1 \in U_1$, $u_2 \in U_2$, $u_3 \in U_3$, $y\in N$. (3.5)
On the other hand ,
$d(tx_1 , u_2 , u_3) = d(x_1 , u_2 , u_3)t + \alpha(x_1)d(t , u_2 , u_3) = d(x_1t , u_2 , u_3)$
$= td(x_1 , u_2 , u_3) + d(t , u_2 , u_3)\, \alpha(x_1)$
for all $u_2 \in U_2$, $u_3 \in U_3$, $t \in N$.
It follows that for all $u_2 \in U_2$, $u_3 \in U_3$, $t \in N$, we get
$d(x_1 , u_2 , u_3)t + \alpha(x_1)d(t , u_2 , u_3) = td(x_1 , u_2 , u_3) + d(t , u_2 , u_3)\, \alpha(x_1)$ (3.6)
In particular , taking $t \in U_1$ in (3.6) and using (3.5) , we get
$d(x_1 , u_2 , u_3)t = td(x_1 , u_2 , u_3)$ for all $t \in U_1$, $u_2 \in U_2$, $u_3 \in U_3$.
Replacing t by ty , where $y\in N$, in the preceding equation and using it again to get
$tyd(x_1 , u_2 , u_3) = d(x_1 , u_2 , u_3)ty = td(x_1 , u_2 , u_3)\, y$
for all $t \in U_1$, $u_2 \in U_2$, $u_3 \in U_3$, $y\in N$, that is
$t[d(x_1 , u_2 , u_3) , y] = 0$ for all $t \in U_1$, $u_2 \in U_2$, $u_3 \in U_3$, $y\in N$.
Therefore
$U_1\, [d(x_1 , u_2 , u_3) , y] = \{0\}$, by Lemma 2.13 we get $d(x_1 , u_2 , u_3) \in Z$. According to (3.4) we conclude that $d(u_1 , u_2 , u_3) \in Z$ for all $u_1 \in U_1$, $u_2 \in U_2$, $u_3 \in U_3$, and hence N is a commutative ring by application of Theorem 3.6 .

Corollary 3.8 : Let N be a prime near-ring admitting a nonzero two sided reverse α-3-derivation d . Let U_1 , U_2 , U_3 be a nonzero semigroup ideals of N . If $d([u_1 , u_1'] , u_2 , u_3)= 0$, for all $u_1 , u_1' \in U_1$, $u_2 \in U_2$, $u_3 \in U_3$, then N is a commutative ring .

Theorem 3.9 : Let N be a prime near-ring admitting a nonzero generalized two sided reverse α-3-derivation f associated with a nonzero two sided reverse α-3-derivation d of N . Let U_1 , U_2 , U_3 be a nonzero semigroup ideals of N.If $f([u_1 , u_1'] , u_2 , u_3)= \pm [u_1 , u_1']$, for all $u_1 , u_1' \in U_1$, $u_2 \in U_2$, $u_3 \in U_3$, then N is a commutative ring .

Proof : By our hypothesis , we have
$f([u_1 , u_1'] , u_2 , u_3)= \pm [u_1 , u_1']$, for all $u_1 , u_1' \in U_1$, $u_2 \in U_2$, $u_3 \in U_3$. (3.7)
Replacing u_1' by $u_1'u_1$ in (3.7) and using it again we get
$d(u_1 , u_2 , u_3)\, \alpha([u_1 , u_1']) = 0$, for all $u_1 , u_1' \in U_1$, $u_2 \in U_2$, $u_3 \in U_3$.
i.e.; $d(u_1 , u_2 , u_3)\, \alpha(u_1)\, \alpha(u_1') = d(u_1 , u_2 , u_3)\, \alpha(u_1')\, \alpha(u_1)$, let $\alpha(U_1) = V_1$ since α is surjective , then V_1 is a semigroup ideal of N . Now let $\alpha(u_1') = v_1'$,where $v_1' \in V_1$ so we have for all $u_1 \in U_1$, $u_2 \in U_2$, $u_3 \in U_3$, $v_1' \in V_1$.

$d(u_1, u_2, u_3)\,\alpha(u_1)\, v_1' = d(u_1, u_2, u_3)\, v_1'\, \alpha(u_1)$ (3.8)
Replacing v_1' by $v_1' r$, where $r \in N$, in (3.8) and using it again we get
$d(u_1, u_2, u_3)\, v_1' [\alpha(u_1), r] = 0$ for all $u_1 \in U_1$, $u_2 \in U_2$, $u_3 \in U_3$, $v_1' \in V_1$, $r \in N$.
i.e.; $d(u_1, u_2, u_3)\, V_1 [\alpha(u_1), r] = \{0\}$ for all $u_1 \in U_1$, $u_2 \in U_2$, $u_3 \in U_3$, $r \in N$.
By Lemma 2.14 , we get for all $u_1 \in U_1$,either $\alpha(u_1) \in Z$ or $d(u_1, u_2, u_3) = 0$
for all $u_2 \in U_2$, $u_3 \in U_3$ (3.9)
Let $u \in U_1$ such that $d(u, u_2, u_3) = 0$, for all $u_2 \in U_2$, $u_3 \in U_3$, then
$d(uu', u_2, u_3) = d(u', u_2, u_3)\, u + \alpha(u')\, d(u, u_2, u_3) = d(u', u_2, u_3)\, u$ and
$d(uu', u_2, u_3) = d(u', u_2, u_3)\alpha(u) + u'\, d(u, u_2, u_3) = d(u', u_2, u_3)\, \alpha(u)$
for all $u' \in U_1$, $u_2 \in U_2$, $u_3 \in U_3$.
Combining both expressions of $d(uu', u_2, u_3)$, we obtain
$d(u', u_2, u_3)(\alpha(u) - u) = 0$ for all $u' \in U_1$, $u_2 \in U_2$, $u_3 \in U_3$. (3.10)
Replacing u' by wu' , where $w \in U_1$, in (3.10) and using it again to get
$d(u', u_2, u_3)\, w(\alpha(u) - u) = 0$ for all $u', w \in U_1$, $u_2 \in U_2$, $u_3 \in U_3$.
i.e.; $d(u', u_2, u_3)\, U_1(\alpha(u) - u) = \{0\}$ for all $u' \in U_1$, $u_2 \in U_2$, $u_3 \in U_3$.
By Lemma 2.14 we conclude that either $d(u', u_2, u_3) = 0$ for all $u' \in U_1$, $u_2 \in U_2$, $u_3 \in U_3$ or $\alpha(u) = u$.
If $d(u', u_2, u_3) = 0$ for all $u' \in U_1$, $u_2 \in U_2$, $u_3 \in U_3$, then by Lemma 3.5 we conclude $d = 0$, which contradicts our original assumption that $d \neq 0$.
Hence , we conclude that $\alpha(u) = u$. According to (3.9) we arrive at a conclusion .
For each $u_1 \in U_1$, either $\alpha(u_1) \in Z$ or $d(\alpha(u_1), u_2, u_3) = 0$ for all $u_2 \in U_2$, $u_3 \in U_3$
It follows for all $v_1 \in V_1$, we get either $v_1 \in Z$ or $d(v_1, u_2, u_3) = 0$ for all $u_2 \in U_2$, $u_3 \in U_3$.
Which is identical with the equation (3.4) in Theorem 3.7 . Now arguing in the same way in corollary 3.8 we conclude that $d(v_1, u_2, u_3) \in Z$ for all $v_1 \in V_1$, $u_2 \in U_2$, $u_3 \in U_3$, and hence N is a commutative ring by application of Theorem 3.6

Conclusion

In present paper we introduce the notions of two sided reverse α-3-derivation and generalized two sided reverse α-3-derivation in near-ring and we see that a near-ring can be make commutative with help of generalized two sided reverse α-3-derivation and other conditions .

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