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ABSTRACT

In this paper we introduce and study on expansivity of uniform dynamical systems, we obtain various important properties and theorems about sensitivity and expansivity of uniform continuous maps.

MSC: 30C45, 30C50

1. Introduction

Dynamical systems theory is an area of mathematics used to describe the behavior of the complex dynamical systems, Dynamical systems are a portion of a lifetime. Quite often it has been studied as an abstract concept in mathematics. Chaos is one of the few concepts in mathematics which cannot usually be defined in a word or statement. Most dynamical systems are treated chaotic depending on either the topological or metric properties of the system [4] .

S. Effah-Poku et al (2018) [6] a search on chaos as a feature of nonlinear science. Systems with at minimal two of the following features are treated to be chaotic in a proven sensibility: bifurcation and interval doubling, interval three, transitivity and dense orbit, sensitive dependence to initial conditions, and expansivity. These are described as the roads to chaos.

I. J. Kadhim and A. H. Khalil (2016) [3] the random dynamical system and random sets in uniform space are known and some main features of these two thoughts are proved. Also, the expansivity of a uniform random operator is studied.

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K. Lee and C. A. Morales(2018) [5] show that a linear operator of a complex "Banach space" has a shadow able point if and only if it has the shadowing feature. furthermore, show that if a linear operator is expansive and has the shadowing feature, then the origin is the only non-wandering point.
C. A. MORALES AND V. SIRVENT (2015) [2] define positively expansive and expansive measures on uniform spaces extending the analogous concepts on metric spaces. prove that such measures can occur for measurable or bi-measurable maps on consolidated non-Hausdorff uniform spaces.
ALI BARZANOUNI AND EKTA SHAH (2010) [1] show that if there exists a topologically expansive homeomorphism on a uniform space, then the space is always regular. during paradigm, we prove that in a general compound of topologically expansive homeomorphisms need not be topological expansive and also that couple of topologically expansive homeomorphisms need not be topological expansive.
Tullio Ceccherini-Silberstein · Michel Coornaert (2011) [7] shows that a uniform version of a result of M. Gromov on the surjunctivity of maps commuting with expansive group behaviors and examine various applications. prove in particular that for any group $\boldsymbol{T}$ and any field K, the space of $\boldsymbol{T}$-marked groups G such that the group algebra K[G] is stably finite is compact.

Definition (1.1) A ***uniform action*** is a work of a uniform group on a uniform space whose fulfill the following conditions:

(1) For each $t \in \mathbb{T}$, $\theta(t,\cdot): X \longrightarrow X$ is uniformly continuous with respect to the uniformity $\mathcal{U}_X$ on X.

(2) The mapping $\mathcal{V}: G \longrightarrow G$, where $\mathcal{V}(t) = t^{-1}$ is uniform continuous

(3) For every $V \in \mathcal{U}_X$, there exists $W \in \mathcal{U}_{\mathbb{T}}$ so that $(t_1, t_2) \in W$ implies $(\theta(t_1, x), \theta(t_2, x)) \in V$ for all $x \in X$; i.e., $(t_1, t_2) \in W$ implies $(\theta(t_1,\cdot), \theta(t_2,\cdot)) \in V^X$.

Theorem (1.1) Product If $(G_1, X, \theta_1), (G_2, Y, \theta_2)$ are two uniform dynamical systems, then their product is a uniform dynamical system.

Proof. We need to show that $(G_1 \times G_2, X \times Y, \theta_1 \times \theta_2)$ is a uniform dynamical system.

(1) Let $(g_1, g_2) \in G_1 \times G_2$, then $g_1 \in G_1$, $g_2 \in G_2$ and $\theta_1(g_1,\cdot): X \longrightarrow X$,$\theta_2(g_2,\cdot): Y \longrightarrow Y$ are uniformly continuous concerning the uniformities $\mathcal{U}_{G_1}$ on X and $\mathcal{U}_{G_2}$ on Y respectively. Then

$$\theta_1 \times \theta_2((g_1, g_2),\cdot): X \times Y \longrightarrow X \times Y$$

is uniformly continuous concerning the uniformity $\mathcal{U}_{G_1 \times G_2}$.

(2) Since the mappings $g_1 \rightsquigarrow g_1^{-1}$ and $g_2 \rightsquigarrow g_2^{-1}$ are uniformly continuous, then $(g_1, g_2) \rightsquigarrow (g_1, g_2) \coloneqq (g_1^{-1}, g_2^{-1})$ is uniformly continuous.

(3) Let $V_1 \times V_2 \in \mathcal{U}_{X \times Y}$, then $V_1 \in \mathcal{U}_X$ and $V_2 \in \mathcal{U}_Y$. Then founds $W_1 \in \mathcal{U}_{G_1}$ and $W_2 \in \mathcal{U}_{G_2}$ so that

$(g_1, g_1') \in W_1$ implies $(\theta_1(g_1, x), \theta_1(g_1', x)) \in V_1$ for all $x \in X$ and

$(g_2, g_2') \in W_2$ implies $(\theta_2(g_2, y), \theta_2(g_2', y)) \in V_2$ for all $y \in Y$.

Then

$((g_1, g_1'), (g_2, g_2')) \in W_1 \times W_2$ implies

$\left(\left(\theta_1(g_1,x),\theta_1(g_1',x)\right),\left(\theta_2(g_2,y),\theta_2(g_2',y)\right)\right) \in V_1 \times V_2$ for all $x \in X$.

Theorem (1.2) Let $\theta: G \times X \to X$ be a uniform dynamical system on X, let $\alpha: G \to G$ be a uniform continuous automorphism of the additive group G, and let $h: X \to Y$ be an isomorphism. Then $\sigma: G \times Y \to Y$ defined by $\sigma \coloneqq h\theta(\alpha \times h)^{-1}$ is a uniform dynamical system on Y. We call σ the uniform dynamical system induced from θ by the pair (α, h).

Proof. To show that $\sigma \coloneqq h\theta(\alpha \times h)^{-1}$ is a group action of G on X.

$$\sigma(e,x) = h\theta(\alpha \times h)^{-1}(e,x)$$

$$= h\theta(\alpha^{-1} \times h^{-1})(e,x)$$

$$= h\theta(\alpha^{-1}(e) \times h^{-1}(x))$$

$$= h\theta(e \times h^{-1}(x))$$

$$= hh^{-1}(x) = x = id_X.$$

$$\sigma(t+s,x) = h\theta(\alpha \times h)^{-1}(t+s,x)$$

$$= h\theta[\alpha^{-1} \times h^{-1}(t+s,x)]$$

$$= h\theta(\alpha^{-1}(t+s), h^{-1}(x))$$

$$= h\theta(\alpha^{-1}(t) + \alpha^{-1}(s), h^{-1}(x))$$

$$= h[\theta\left(\alpha^{-1}(t), \theta\left(\alpha^{-1}(s), h^{-1}(x)\right)\right)]$$

$$= h[\theta\left(\alpha^{-1}(t), \theta(\alpha^{-1} \times h^{-1})(s,x)\right)]$$

$$= h[\theta\left(\alpha^{-1}(t), h^{-1}h\theta(\alpha^{-1} \times h^{-1})(s,x)\right)]$$

$$= h\theta(\alpha^{-1} \times h^{-1})\left(t, h\theta(\alpha^{-1} \times h^{-1})(s,x)\right)$$

$$= \sigma\left(t, h\theta(\alpha^{-1} \times h^{-1})(s,x)\right)$$

$$= \sigma\left(t, \sigma(s,x)\right).$$

Thus $\sigma \coloneqq h\theta(\alpha \times h)^{-1}$ is a group action of G on X. The uniformity continuity of $\sigma: G \times Y \to Y$ follows from the fact that α, h, and θ are uniform isomorphism. Now we have α is uniform is a uniform isomorphism and $\mathcal{V}$ is uniformly continuous, then $\mathcal{V}': G' \longrightarrow G'$ is uniform continuous since $\mathcal{V}' = \alpha\mathcal{V}\alpha^{-1}$.

Now, let $V' \in \boldsymbol{\mathcal{U}}_Y$, then $V \coloneqq h^{-1} \times h^{-1}(V') \in \boldsymbol{\mathcal{U}}_X$. By hypothesis, there exists $W \in \boldsymbol{\mathcal{U}}_G$ so that

$(t_1,t_2) \in W$ implies $(\theta(t_1,x),\theta(t_2,x)) \in V$ for all $x \in X$. (*)

Set $W' \coloneqq \alpha \times \alpha(W) \in \boldsymbol{\mathcal{U}}_{G'}$. Let $(s_1,s_2) \in W'$ and $y \in Y$. Then

$(\alpha^{-1}(s_1),\alpha^{-1}(s_2)) \in W$ and $h^{-1}(y) \in X$.

By (*) we have $(\alpha^{-1}(s_1),\alpha^{-1}(s_2)) \in W$ implies

$(\theta(\alpha^{-1}(s_1), h^{-1}(y)), \theta(\alpha^{-1}(s_2), h^{-1}(y))) \in V$

$\Rightarrow (\theta(\alpha^{-1} \times h^{-1})(s_1, y), \theta(\alpha^{-1} \times h^{-1})(s_2, y)) \in V$

$\Rightarrow h \times h(\theta(\alpha^{-1} \times h^{-1})(s_1, y), \theta(\alpha^{-1} \times h^{-1})(s_2, y)) \in h \times h(V)$

$\Rightarrow (h\theta(\alpha^{-1} \times h^{-1})(s_1, y), h\,\theta(\alpha^{-1} \times h^{-1})(s_2, y)) \in V'$

$\Rightarrow (\sigma(s_1, y), \sigma(s_2, y)) \in V'.$

This completes the proof.

2-Uniform Expansive Maps

Definition (2.1) a work of a group G on a uniform space $(X, \mathcal{U})$ is called expansive, if for all distinct points $x, y \in X$, there exist a point $g \in G$ and $U \in \mathcal{U}$ such that $(\theta(g, x), \theta(g, y)) \in U$. Such an entourage is then called an expansivity footnote for the dynamical system $(X, G, \mathcal{U}, \theta)$.

It is explicit that if the action of a group G on an idealistic uniform space is expansive then this action is also sensitive (any expansivity entourage is a sensitivity entourage)[7]. This Sensitivity is a weak local version of expansivity.

Definition (2.2) If $(X, \mathcal{U})$ is a uniform space and $h \in \mathcal{H}(X)$ then h is called expansive, if there exist an entourage $U \in \mathcal{U}$ such that whenever $x, y \in X, x \neq y$ then found an integer n satisfying

$$\left(h^n(x), h^n(y)\right) \notin U;$$

U is then called an **expansive entourage** for h.

Definition (2.3) Let $(X, \mathcal{U})$ be a uniform space and $h \in \mathcal{H}(X)$ then h is called **uniform G −expansive**, if there exists an entourage $U \in \mathcal{U}$ such that whenever $x, y \in X, G(x) \neq G(y)$ then there exists an integer n satisfying

$$\left(h^n(u), h^n(v)\right) \notin U, \text{ for all } u \in G(x) \text{ and } \in G(y).$$

Theorem (2.1) A homeomorphism $h \in \mathcal{H}(X)$ is Uniform G −expansive if and only if found an edge $U \in \mathcal{U}$ such as $\Gamma_\delta^{h,\theta}(G(x)) = \{G(x)\}, \forall x \in X$, where

$$\Gamma_\delta^{h,\theta}(x) := \left\{G(y) : y \in X, \left(h^i(u), h^i(v)\right) \in U, \forall i \in \mathbb{Z}, u \in G(x), v \in G(y)\right\}.$$

Proof. Suppose that $h \in \mathcal{H}(X)$ is uniform G −expansive. Assume contrary that $\Gamma_\delta^{h,\theta}(x)$ is not a singleton set. Then found $y \in X$ such as $G(x) \neq G(y)$, by hypothesis there, exists an edge $U \in \mathcal{U}$ and an integer n favorable

$$\left(h^n(u), h^n(v)\right) \notin U, \text{ for all } u \in G(x) \text{ and } v \in G(y).$$

But that is a contradiction. Thus $\Gamma_\delta^{h,\theta}(G(x)) = \{G(x)\}, \forall x \in X$.

Conversely, suppose that $\Gamma_{\delta}^{h,\theta}(G(x)) = \{G(x)\}$, $\forall x \in X$. Assume contrary that $h \in \mathcal{H}(X)$ is not uniform G −expansive, then there exist $x, y \in X$ with $G(x) \neq G(y)$ and an integer n satisfying

$$\big(h^n(u), h^n(v)\big) \notin U, \text{ for all } U \in \boldsymbol{\mathcal{U}} \text{ and all } u \in G(x), v \in G(y).$$

Thus $G(y) \in \Gamma_{\delta}^{h,\theta}(G(x))$ and this contradicts the fact that $\Gamma_{\delta}^{h,\theta}(G(x)) = \{G(x)\}$, $\forall x \in X$ .This means that h is not uniform G −expansive.

Remark down the slight work of G on X the notions of uniform expansive and uniform $G -$ expansive have synchronized. It is spotted that the notion of expansiveness and the notation of G −expansive under a non- slight action of G are independent of each other.

In the following, we shall presentation that Definition 4.1 and 4.3 have synchronized in a metrizable uniform space. To this end, we state and prove the following lemma.

Lemma (2.1) Suppose $(X, \boldsymbol{\mathcal{U}})$ be a metrizable uniform space and Suppose $x, y \in X$ with $x \neq y$, then the following statement are equivalent.

(i) found a $\delta > 0$ such that $d(x, y) \geq \delta$.

(ii) found an entourage $U \in \boldsymbol{\mathcal{U}}$ such that $(x, y) \notin U$.

Proof. "(i)⇒(ii)". Suppose (i). Define $U(\delta) \coloneqq \{(u, v): d(u, v) < \delta\}$. Then, $U(\delta) \in \boldsymbol{\mathcal{U}}$. If $d(x, y) \geq \delta$, then $(x, y) \notin U(\delta)$.

"(ii)⇒(i)". Suppose (ii). Since $\boldsymbol{\mathcal{U}}$ is metrizable uniformity, for every $U \in \boldsymbol{\mathcal{U}}$ there exists $\delta > 0$ such that $U = U(\delta) \coloneqq \{(u, v): d(u, v) < \delta\}$.If $(x, y) \notin U$, then $d(x, y) \geq \delta$

Theorem (2.2) If X is a metrizable uniform space, then (Definition 4.1 and 4.3) are equipollent.

Proof. Suppose X as a metric space. Firstly, note that if $x, y \in X$ with $x \neq y$ and $h \in \mathcal{H}(X)$, then $h^n(x) \neq h^n(y)$ and the outcome follows from Lemma 4.6.

Theorem (2.3) Let $(X, \boldsymbol{\mathcal{U}})$ be separated uniform G −space and $f \in \mathcal{H}(X)$. If f^n not constant for some $n \geq 1$, then f is uniform G −expansive on X.

Proof. Let that f^n not constant for some $n \geq 1$. Let $x, y \in X$ with $G(x) \neq G(y)$. Since f is equivariant and one-to-one, then $G\big(f(x)\big) \neq G\big(f(y)\big)$. Since $f \in \mathcal{H}(X)$, then $f^n \in \mathcal{H}(X)$ and consequently $G(f^n(x))$, $G(f^n(y))$ are disjoint sets in X. Therefore $f^n(x) \neq f^n(y)$. Since X is separated then found an edge $U \in \boldsymbol{\mathcal{U}}$ such that $(f^n(x), f^n(y)) \notin U$. Let $u \in G(x)$ and $v \in G(y)$. Then found $g, q \in G$ such that $u = \theta(g, x), v = \theta(q, y)$. Thus we have $(f^n(u), f^n(v)) \notin U$ for every $u \in G(x)$ and $v \in G(y)$.This means that f is uniform G −expansive.

Theorem (2.4) Let $(X, \boldsymbol{\mathcal{U}})$ be a separated uniform space and $f \in \mathcal{H}(X)$. If f^n not constant for some $n \geq 1$, then f is expansive on X.

Proof. Suppose $, y \in X$, with $x \neq y$. Since f^n not constant for some $n \geq 1$, then $f^n(x) \neq f^n(y)$. Since X separated, then found an entourage $U \in \boldsymbol{\mathcal{U}}$ such that $(f^n(x), f^n(y)) \notin U$. Then f is expansive on X.

Theorem (2.5) suppose $(X, \mathcal{U})$ be a uniform G_1 −space, $(Y, \mathcal{V})$ be uniform G_2 −space and $h_1 \in \mathcal{H}(X)$, $h_2 \in \mathcal{H}(Y)$ be equivariant topologically joint by

$$(\mu, \psi): (G_1, X, \theta_1) \to (G_2, Y, \theta_2).$$

h_1 is uniform G_1 −expansive, if and only if h_2 is uniform G_2 −expansive.

Proof. Let h_1 is uniform G_1 − expansive. Let $y_1, y_2 \in Y$, with $G_2(y_1) \neq G_2(y_2)$. Since φ is bijective then there exists $x_1, x_2 \in X$ such that $y_1 = \varphi(x_1), y_2 = \varphi(x_2)$ and this implies that $x_1 = \varphi^{-1}(y_1), x_2 = \varphi^{-1}(y_2)$. Since φ is equivariant, then so is φ^{-1} and so φ^{-1} is pseudoequivaria-nt. Therefore by Lemma 4.4, we have

$$\varphi^{-1}\big(G_2(y_1)\big) = G_1\big(\varphi^{-1}(y_1)\big) = G_1(x_1).$$

Similarly $\varphi^{-1}\big(G_2(y_2)\big) = G_1\big(\varphi^{-1}(y_2)\big) = G_1(x_2)$. Since $G_2(y_1) \neq G_2(y_2)$ and φ is bijective, then $G_1(x_1) \neq G_1(x_2)$. By hypothesis there exist entourage $U \in \mathcal{U}$, $n \in \mathbb{N}$, such that

$$(h_1^n(u_1), h_1^n(u_2)) \notin U \text{, for all } u_1 \in G_1(x_1) \text{ and } u_2 \in G_1(x_2). \qquad (1)$$

Since φ^{-1} is uniformly continuous, then there exists an entourage $V \in \mathcal{V}$ such that

$$(y_1, y_2) \in V \quad \text{implies} \quad (\varphi^{-1}(y_1), \varphi^{-1}(y_2)) \in U. \qquad (2)$$

By using (1) and the contrapositive of (2) we get

$$(\varphi(h_1^n(u_1)), \varphi(h_1^n(u_2))) \notin V, \text{ for all } u_1 \in G_1(x_1) \text{ and } u_2 \in G_1(x_2).$$

This implies that

$$(h_2^n(\varphi(u_1)), h_2^n(\varphi(u_2))) \notin V, \text{ for all } u_1 \in G_1(x_1) \text{ and } u_2 \in G_1(x_2).$$

Let $v_1 \in G_2(y_1), v_2 \in G_2(y_2)$, then

$$\varphi^{-1}(v_1) \in \varphi^{-1}(G_2(y_1)), \text{ and } \varphi^{-1}(v_2) \in \varphi^{-1}(G_2(y_2))$$

Thus

$$\varphi^{-1}(v_1) \in G_1(\varphi^{-1}(y_1)) = G_1(x_1),\ \varphi^{-1}(v_2) \in G_1(\varphi^{-1}(y_2)) = G_1(x_2).$$

Consequently,

$$(h_2^n(\varphi(\varphi^{-1}(v_1)), h_2^n(\varphi(\varphi^{-1}(v_2))) \notin V,$$

or equivalently, $(h_2^n(v_1), h_2^n(v_2) \notin V$.This means that h_2 is uniform G_2 − expansive. The converse also follows analogously.

Corollary (2.1). Let X, Y be two uniform spaces and $h_1 \in \mathcal{H}(X)$, $h_2 \in \mathcal{H}(Y)$ be topologically conjugate via $X \to Y$. If h_1 is uniform expansive, then so is h_2.

Theorem (2.6) suppose $(X, \mathcal{U})$ and $(Y, \mathcal{V})$ be two uniform G −spaces and $F \coloneqq \{f_k\}_{k=1}^{\infty}$, $\mathcal{G} \coloneqq \{g_k\}_{k=1}^{\infty}$ be two sequences of maps on X, Y respectively. If found an equivariant uniform isomorphism $h: X \to Y$ such that F and $\mathcal{G}$ are h −conjugate, then F is uniform G −expansive on X in the iterative (or successive) way if and only if $\mathcal{G}$ is uniform G −expansive on Y.

Proof. Suppose $F \coloneqq \{f_k\}_{k=1}^{\infty}$ has a uniform G −expansive on X in an iterative way. There exists an entourage $U \in \boldsymbol{\mathcal{U}}$ such that for any $x, y \in X$ with $G(x) \neq G(\tilde{x})$ and positive integer n, k such that

$$(F_k^n(\theta_g x), F_k^n(\theta_p \tilde{x})) \notin U \ ,$$

for all $g, p \in G$, where $F_k \coloneqq f_k \circ f_{k-1} \circ \ldots \circ f_2 \circ f_1$. Since h^{-1} is uniform continuous therefore there exists an entourage $V \in \boldsymbol{\mathcal{V}}$ such that for any $y_1, y_2 \in Y$ with

$$(y_1, y_2) \in V,\ (x_1, x_2) = (h^{-1}(y_1), h^{-1}(y_2)) \in U,$$

where $x_1 = h^{-1}(y_1), x_2 = h^{-1}(y_2)$. Hence for any $y_1 = h(x_1), y_2 = h(x_2) \in Y$, we have

$$(x_1, x_2) \notin U \ \ \Rightarrow \ \ (y_1, y_2) \notin V. \qquad (**)$$

Observe that for any $y \in Y$ and any neighbourhood M of, $h^{-1}(M)$ is a neighbourhood of $h^{-1}(y) = x$. We therefore have $\tilde{x} \in h^{-1}(M)$ with $G(x) \neq G(\tilde{x})$ and a positive integer k such that

$$(F_k^n(\theta_g x), F_k^n(\theta_p \tilde{x})) \notin U \text{ , for all } g, p \in G.$$

Now we use (**) and observe that for all $g, p \in G$

$$(hF_k^n(\theta_g x), hF_k^n(\theta_p \tilde{x})) \notin V.$$

Since $h \circ F_k^n = G_k^n \circ h$ and h is equivariant, we have

$$(G_k^n(\theta_g h(x)), G_k^n(\theta_p h(\tilde{x}))) \notin V$$

and hence

$$(G_k^n(\theta_g y), G_k^n(\theta_p \tilde{y})) \notin V$$

for all $g, p \in G$, where $\tilde{y} = h(\tilde{x}) \in M$. This establishes $\mathcal{G} \coloneqq \{g_k\}_{k=1}^{\infty}$ is uniform G −expansive in an iterative way. The converse statement can be proved similarly. The case of uniform G −expansive in a successive way also follows analogously.

Theorem (2.7) suppose $(X, \boldsymbol{\mathcal{U}})$ and $(Y, \boldsymbol{\mathcal{V}})$ be two uniform spaces and $F \coloneqq \{f_k\}_{k=1}^{\infty}$, $\mathcal{G} \coloneqq \{g_k\}_{k=1}^{\infty}$ be two series of maps on X, Y respectively. If found an equivariant uniform isomorphism $h: X \to Y$ such that F and $\mathcal{G}$ are h −conjugate, then F is uniform expansive on X in the iterative (or successive) way if and only if $\mathcal{G}$ is uniform expansive on Y.

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