On Expansivity Of Uniform Dynamical Systems

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On expansivity of Uniform Dynamical Systems

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### ABSTRACT

In this paper we introduce and study on expansivity of uniform dynamical systems, we obtain various important properties and theorems about sensitivity and expansivity of uniform continuous maps.

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1. Introduction

Dynamical systems theory is an area of mathematics used to describe the behavior of the complex dynamical systems. Dynamical systems are a portion of a lifetime. Quite often it has been studied as an abstract concept in mathematics. Chaos is one of the few concepts in mathematics which cannot usually be defined in a word or statement. Most dynamical systems are treated chaotic depending on either the topological or metric properties of the system [4].

*S. Effah-Poku et al (2018) [6] a* search on chaos as a feature of nonlinear science. Systems with at minimal two of the following features are treated to be chaotic in a proven sensibility: bifurcation and interval doubling, interval three, transitivity and dense orbit, sensitive dependence to initial conditions, and expansivity. These are described as the roads to chaos.

*I. J. Kadhim and A. H. Khalil (2016) [3] the* random dynamical system and random sets in uniform space are known and some main features of these two thoughts are proved. Also, the expansivity of a uniform random operator is studied.
K. Lee and C. A. Morales (2018) [5] show that a linear operator of a complex "Banach space" has a shadowing ability point if and only if it has the shadowing feature. Furthermore, show that if a linear operator is expansive and has the shadowing feature, then the origin is the only non-wandering point.

C. A. Morales and V. Sirvent (2015) [2] define positively expansive and expansive measures on uniform spaces extending the analogous concepts on metric spaces. Prove that such measures can occur for measurable or bi-measurable maps on consolidated non-Hausdorff uniform spaces.

Ali Barzanouni and Ekta Shah (2010) [1] show that if there exists a topologically expansive homeomorphism on a uniform space, then the space is always regular. During paradigm, we prove that in a general compound of topologically expansive homeomorphisms need not be topological expansive and also that couple of topologically expansive homeomorphisms need not be topological expansive.

Tullio Ceccherini-Silberstein · Michel Coornaert (2011) [7] shows that a uniform version of a result of M. Gromov on the surjunctivity of maps commuting with expansive group behaviors and examine various applications. Prove in particular that for any group $T$ and any field $K$, the space of $T$-marked groups $G$ such that the group algebra $K[G]$ is stably finite is compact.

**Definition (1.1)** A uniform action is a work of a uniform group on a uniform space whose fulfill the following conditions:

1. For each $t \in T$, $\theta(t, \cdot): X \to X$ is uniformly continuous with respect to the uniformity $\mathcal{U}_X$ on $X$.
2. The mapping $\mathcal{V}: G \to G$, where $\mathcal{V}(t) = t^{-1}$ is uniform continuous.
3. For every $V \in \mathcal{U}_X$, there exists $W \in \mathcal{U}_T$ so that $(t_1, t_2) \in W$ implies $(\theta(t_1, x), \theta(t_2, x)) \in V$ for all $x \in X$; i.e., $(t_1, t_2) \in W$ implies $(\theta(t_1, \cdot), \theta(t_2, \cdot)) \in V^X$.

**Theorem (1.1) Product** If $(G_1, X, \theta_1), (G_2, Y, \theta_2)$ are two uniform dynamical systems, then their product is a uniform dynamical system.

**Proof.** We need to show that $(G_1 \times G_2, X \times Y, \theta_1 \times \theta_2)$ is a uniform dynamical system.

1. Let $(g_1, g_2) \in G_1 \times G_2$, then $g_1 \in G_1$, $g_2 \in G_2$ and $\theta_1(g_1, \cdot): X \to X$, $\theta_2(g_2, \cdot): Y \to Y$ are uniformly continuous concerning the uniformities $\mathcal{U}_{G_1}$ on $X$ and $\mathcal{U}_{G_2}$ on $Y$ respectively. Then

$$\theta_1 \times \theta_2((g_1, g_2), \cdot): X \times Y \to X \times Y$$

is uniformly continuous concerning the uniformity $\mathcal{U}_{G_1 \times G_2}$.

2. Since the mappings $g_1 \mapsto g_1^{-1}$ and $g_2 \mapsto g_2^{-1}$ are uniformly continuous, then $(g_1, g_2) \mapsto (g_1^{-1}, g_2^{-1})$ is uniformly continuous.

3. Let $V_1 \times V_2 \in \mathcal{U}_{X \times Y}$, then $V_1 \in \mathcal{U}_X$ and $V_2 \in \mathcal{U}_Y$. Then founds $W_1 \in \mathcal{U}_{G_1}$ and $W_2 \in \mathcal{U}_{G_2}$ so that

$$(g_1, g_1') \in W_1 \text{ implies } (\theta_1(g_1, x), \theta_1(g_1', x)) \in V_1 \text{ for all } x \in X$$

$$(g_2, g_2') \in W_2 \text{ implies } (\theta_2(g_2, y), \theta_2(g_2', y)) \in V_2 \text{ for all } y \in Y.$$ 

Then

$$(g_1, g_1')(g_2, g_2') \in W_1 \times W_2 \text{ implies}$$
By (*) we have

\[
\left( \theta_1(g_1, x), \theta_1(g'_1, x) \right), \left( \theta_2(g_2, y), \theta_2(g'_2, y) \right) \in V_1 \times V_2 \quad \text{for all } x \in X.
\]

**Theorem (1.2)** Let \( \theta: G \times X \to X \) be a uniform dynamical system on \( X \), let \( \alpha: G \to G \) be a uniform continuous automorphism of the additive group \( G \), and let \( h: X \to Y \) be an isomorphism. Then \( \sigma: G \times Y \to Y \) defined by \( \sigma := h\theta(\alpha \times h)^{-1} \) is a uniform dynamical system on \( Y \). We call \( \sigma \) the uniform dynamical system induced from \( \theta \) by the pair \( (\alpha, h) \).

**Proof.** To show that \( \sigma := h\theta(\alpha \times h)^{-1} \) is a group action of \( G \) on \( X \).

\[
\sigma(e, x) = h\theta(\alpha \times h)^{-1}(e, x)
\]

\[
= h\theta(\alpha^{-1} \times h^{-1})(e, x)
\]

\[
= h\theta(\alpha^{-1}(e) \times h^{-1}(x))
\]

\[
= h\theta(e \times h^{-1}(x))
\]

\[
= hh^{-1}(x) = x = id_X.
\]

\[
\sigma(t + s, x) = h\theta(\alpha \times h)^{-1}(t + s, x)
\]

\[
= h\theta[\alpha^{-1} \times h^{-1}(t + s, x)]
\]

\[
= h\theta(\alpha^{-1}(t + s) , h^{-1}(x))
\]

\[
= h\theta(\alpha^{-1}(t) + \alpha^{-1}(s) , h^{-1}(x))
\]

\[
= h[\theta(\alpha^{-1}(t), \theta(\alpha^{-1}(s), h^{-1}(x))]}
\]

\[
= h[\theta(\alpha^{-1}(t), \theta(\alpha^{-1} x h^{-1})(s, x))]
\]

\[
= h[\theta(\alpha^{-1}(t), h^{-1}h\theta(\alpha^{-1} x h^{-1})(s, x))]
\]

\[
= h\theta(\alpha^{-1} x h^{-1})(t, h\theta(\alpha^{-1} x h^{-1})(s, x))
\]

\[
= \sigma(t,h\theta(\alpha^{-1} x h^{-1})(s, x))
\]

\[
= \sigma(t,\sigma(s, x)).
\]

Thus \( \sigma := h\theta(\alpha \times h)^{-1} \) is a group action of \( G \) on \( X \). The uniformity continuity of \( \sigma: G \times Y \to Y \) follows from the fact that \( \alpha, h, \) and \( \theta \) are uniform isomorphism. Now we have \( \alpha \) is uniform is a uniform isomorphism and \( V \) is uniformly continuous, then \( V': G' \to G' \) is uniform continuous since \( V' = \alpha V \alpha^{-1} \).

Now, let \( V' \in \mathcal{U}_Y \), then \( V := h^{-1} \times h^{-1}(V') \in \mathcal{U}_X \). By hypothesis, there exists \( W \in \mathcal{U}_G \) so that

\[
(t_1, t_2) \in W \quad \text{implies} \quad (\theta(t_1, x), \theta(t_2, x)) \in V \quad \text{for all } x \in X. \quad (*)
\]

Set \( W' := \alpha \times W \in \mathcal{U}_{G'} \). Let \( (s_1, s_2) \in W' \) and \( y \in Y \). Then

\[
(\alpha^{-1}(s_1), \alpha^{-1}(s_2)) \in W \quad \text{and} \quad h^{-1}(y) \in X.
\]

By (*) we have \( (\alpha^{-1}(s_1), \alpha^{-1}(s_2)) \in W \) implies...
\[(\theta(\alpha^{-1}(s_1),h^{-1}(y)),\theta(\alpha^{-1}(s_2),h^{-1}(y))) \in V\]
\[\Rightarrow (\theta(\alpha^{-1} \times h^{-1})(s_1,y),\theta(\alpha^{-1} \times h^{-1})(s_2,y)) \in V\]
\[\Rightarrow h \times h(\theta(\alpha^{-1} \times h^{-1})(s_1,y),\theta(\alpha^{-1} \times h^{-1})(s_2,y)) \in h \times h(V)\]
\[\Rightarrow (h\theta(\alpha^{-1} \times h^{-1})(s_1,y),h \theta(\alpha^{-1} \times h^{-1})(s_2,y)) \in V'\]
\[\Rightarrow (\sigma(s_1,y),\sigma(s_2,y)) \in V'.\]

This completes the proof.

2-Uniform Expansive Maps

**Definition (2.1)** a work of a group \(G\) on a uniform space \((X,\mathcal{U})\) is called expansive, if for all distinct points \(x,y \in X\), there exist a point \(g \in G\) and \(U \in \mathcal{U}\) such that \((\theta(g,x),\theta(g,y)) \in U\). Such an entourage is then called an expansivity footnote for the dynamical system \((X,G,\mathcal{U},\theta)\).

It is explicit that if the action of a group \(G\) on an idealistic uniform space is expansive then this action is also sensitive (any expansivity entourage is a sensitivity entourage)[7]. This Sensitivity is a weak local version of expansivity.

**Definition (2.2)** If \((X,\mathcal{U})\) is a uniform space and \(h \in \mathcal{H}(X)\) then \(h\) is called expansive, if there exist an entourage \(U \in \mathcal{U}\) such that whenever \(x,y \in X, x \neq y\) then found an integer \(n\) satisfying
\[(h^n(x),h^n(y)) \notin U;\]
\(U\) is then called an expansive entourage for \(h\).

**Definition (2.3)** Let \((X,\mathcal{U})\) be a uniform space and \(h \in \mathcal{H}(X)\) then \(h\) is called uniform \(G\) - expansive, if there exists an entourage \(U \in \mathcal{U}\) such that whenever \(x,y \in X, G(x) \neq G(y)\) then there exists an integer \(n\) satisfying
\[(h^n(u),h^n(v)) \notin U, \text{ for all } u \in G(x) \text{ and } v \in G(y).\]

**Theorem (2.1)** A homeomorphism \(h \in \mathcal{H}(X)\) is Uniform \(G\) - expansive if and only if found an edge \(U \in \mathcal{U}\) such as \(\Gamma_{h,\theta}^x (G(x)) = \{G(x)\}\), \(\forall x \in X\), where
\[\Gamma_{h,\theta}^x (x) := \{G(y): y \in X, (h^i(u),h^i(v)) \in U, \forall i \in \mathbb{Z}, u \in G(x), v \in G(y)\}.\]

**Proof.** Suppose that \(h \in \mathcal{H}(X)\) is uniform \(G\) - expansive. Assume contrary that \(\Gamma_{h,\theta}^x (x)\) is not a singleton set. Then found \(y \in X\) such as \(G(x) \neq G(y)\), by hypothesis there, exists an edge \(U \in \mathcal{U}\) and an integer \(n\) favorable
\[(h^n(u),h^n(v)) \notin U, \text{ for all } u \in G(x) \text{ and } v \in G(y).\]
But that is a contradiction. Thus \(\Gamma_{h,\theta}^x (G(x)) = \{G(x)\}, \forall x \in X.\)
Conversely, suppose that \( \Gamma_{\delta}^{h,\theta}(G(x)) = \{G(x)\}, \forall x \in X \). Assume contrary that \( h \in \mathcal{H}(X) \) is not uniform \( G \) -expansive, then there exist \( x, y \in X \) with \( G(x) \neq G(y) \) and an integer \( n \) satisfying

\[
\left(h^n(u), h^n(v)\right) \notin U, \text{ for all } U \in \mathcal{U} \text{ and all } u \in G(x), v \in G(y).
\]

Thus \( G(y) \in \Gamma_{\delta}^{h,\theta}(G(x)) \) and this contradicts the fact that \( \Gamma_{\delta}^{h,\theta}(G(x)) = \{G(x)\}, \forall x \in X \). This means that \( h \) is not uniform \( G \) -expansive.

**Remark** down the slight work of \( G \) on \( X \) the notions of uniform expansive and uniform \( G \) - expansive have synchronized. It is spotted that the notion of expansiveness and the notation of \( G \) -expansive under a non-slight action of \( G \) are independent of each other.

In the following, we shall presentation that Definition 4.1 and 4.3 have synchronized in a metrizable uniform space. To this end, we state and prove the following lemma.

**Lemma (2.1)** Suppose \((X, \mathcal{U})\) be a metrizable uniform space and Suppose \( x, y \in X \) with \( x \neq y \), then the following statement are equivalent.

(i) found a \( \delta > 0 \) such that \( d(x, y) \geq \delta \).

(ii) found an entourage \( U \in \mathcal{U} \) such that \( (x, y) \notin U \).

**Proof.** "(i)⇒(ii)". Suppose (i). Define \( U(\delta) \equiv \{(u, v): d(u, v) < \delta\} \). Then, \( U(\delta) \in \mathcal{U} \). If \( d(x, y) \geq \delta \), then \( (x, y) \notin U(\delta) \).

"(ii)⇒(i)". Suppose (ii). Since \( \mathcal{U} \) is metrizable uniformity, for every \( U \in \mathcal{U} \) there exists \( \delta > 0 \) such that \( U = U(\delta) \equiv \{(u, v): d(u, v) < \delta\} \). If \( (x, y) \notin U \), then \( d(x, y) \geq \delta \).

**Theorem (2.2)** If \( X \) is a metrizable uniform space, then (Definition 4.1 and 4.3) are equipollent.

**Proof.** Suppose \( X \) as a metric space. Firstly, note that if \( x, y \in X \) with \( x \neq y \) and \( h \in \mathcal{H}(X) \), then \( h^n(x) \neq h^n(y) \) and the outcome follows from Lemma 4.6.

**Theorem (2.3)** Let \((X, \mathcal{U})\) be a separated uniform \( G \) -space and \( f \in \mathcal{H}(X) \). If \( f^n \) not constant for some \( n \geq 1 \), then \( f \) is uniform \( G \) -expansive on \( X \).

**Proof.** Let that \( f^n \) not constant for some \( n \geq 1 \). Let \( x, y \in X \) with \( G(x) \neq G(y) \). Since \( f \) is equivariant and one-to-one, then \( G(f(x)) \neq G(f(y)) \). Since \( f \in \mathcal{H}(X) \), then \( f^n \in \mathcal{H}(X) \) and consequently \( G(f^n(x)), G(f^n(y)) \) are disjoint sets in \( X \). Therefore \( f^n(x) \neq f^n(y) \). Since \( X \) is separated then found an edge \( U \in \mathcal{U} \) such that \( (f^n(x), f^n(y)) \notin U \). Let \( u \in G(x) \) and \( v \in G(y) \). Then found \( g, q \in G \) such that \( u = \theta(g, x), v = \theta(q, y) \). Thus we have \( (f^n(u), f^n(v)) \notin U \) for every \( u \in G(x) \) and \( v \in G(y) \). This means that \( f \) is uniform \( G \) -expansive.

**Theorem (2.4)** Let \((X, \mathcal{U})\) be a separated uniform space and \( f \in \mathcal{H}(X) \). If \( f^n \) not constant for some \( n \geq 1 \), then \( f \) is expansive on \( X \).

**Proof.** Suppose \( x, y \in X \), with \( x \neq y \). Since \( f^n \) not constant for some \( n \geq 1 \), then \( f^n(x) \neq f^n(y) \). Since \( X \) separated, then found an entourage \( U \in \mathcal{U} \) such that \( (f^n(x), f^n(y)) \notin U \). Then \( f \) is expansive on \( X \).
Theorem (2.5) suppose $(X, \mathcal{U})$ be a uniform $G_1$ -space, $(Y, \mathcal{V})$ be uniform $G_2$ -space and $h_1 \in \mathcal{H}(X)$, $h_2 \in \mathcal{H}(Y)$ be equivariant topologically joint by

$$(\mu, \psi): (G_1, X, \theta_1) \to (G_2, Y, \theta_2).$$

$h_1$ is uniform $G_1$ -expansive, if and only if $h_2$ is uniform $G_2$ -expansive.

Proof. Let $h_1$ is uniform $G_1$ -expansive. Let $y_1, y_2 \in Y$, with $G_2(y_1) \neq G_2(y_2)$. Since $\varphi$ is bijective then there exists $x_1, x_2 \in X$ such that $y_1 = \varphi(x_1), y_2 = \varphi(x_2)$ and this implies that $x_1 = \varphi^{-1}(y_1), x_2 = \varphi^{-1}(y_2)$. Since $\varphi$ is equivariant, then so is $\varphi^{-1}$ and so $\varphi^{-1}$ is pseudoequivaria-nt. Therefore by Lemma 4.4, we have

$$\varphi^{-1}(G_2(y_1)) = G_1(\varphi^{-1}(y_1)) = G_1(x_1).$$

Similarly $\varphi^{-1}(G_2(y_2)) = G_1(\varphi^{-1}(y_2)) = G_1(x_2)$. Since $G_2(y_1) \neq G_2(y_2)$ and $\varphi$ is bijective, then $G_1(x_1) \neq G_1(x_2)$. By hypothesis there exist entourage $U \in \mathcal{U}$, $n \in \mathbb{N}$, such that

$$(h_1^n(u_1), h_1^n(u_2)) \notin U, \text{ for all } u_1 \in G_1(x_1) \text{ and } u_2 \in G_1(x_2). \quad (1)$$

Since $\varphi^{-1}$ is uniformly continuous, then there exists an entourage $V \in \mathcal{V}$ such that

$$(y_1, y_2) \in V \implies (\varphi^{-1}(y_1), \varphi^{-1}(y_2)) \in U. \quad (2)$$

By using (1) and the contrapositive of (2) we get

$$(\varphi(h_1^n(u_1)), \varphi(h_1^n(u_2))) \notin V, \text{ for all } u_1 \in G_1(x_1) \text{ and } u_2 \in G_1(x_2).$$

This implies that

$$(h_2^n(\varphi(u_1)), h_2^n(\varphi(u_2))) \notin V, \text{ for all } u_1 \in G_1(x_1) \text{ and } u_2 \in G_1(x_2).$$

Let $v_1 \in G_2(y_1), v_2 \in G_2(y_2)$, then

$$\varphi^{-1}(v_1) \in \varphi^{-1}(G_2(y_1)) \text{ and } \varphi^{-1}(v_2) \in \varphi^{-1}(G_2(y_2)).$$

Thus

$$\varphi^{-1}(v_1) \in G_1(\varphi^{-1}(y_1)) = G_1(x_1), \varphi^{-1}(v_2) \in G_1(\varphi^{-1}(y_2)) = G_1(x_2).$$

Consequently,

$$(h_2^n(\varphi(\varphi^{-1}(v_1))), h_2^n(\varphi(\varphi^{-1}(v_2))) \notin V,$$

or equivalently, $(h_2^n(v_1), h_2^n(v_2)) \notin V$. This means that $h_2$ is uniform $G_2$ -expansive. The converse also follows analogously.

Corollary (2.1). Let $X, Y$ be two uniform spaces and $h_1 \in \mathcal{H}(X), h_2 \in \mathcal{H}(Y)$ be topologically conjugate via $X \to Y$. If $h_1$ is uniform expansive, then so is $h_2$.

Theorem (2.6) suppose $(X, \mathcal{U})$ and $(Y, \mathcal{V})$ be two uniform $G$ -spaces and $F := \{f_k\}_{k=1}^{\infty}, G := \{g_k\}_{k=1}^{\infty}$ be two sequences of maps on $X, Y$ respectively. If found an equivariant uniform isomorphism $h: X \to Y$ such
that $F$ and $G$ are $h$—conjugate, then $F$ is uniform $G$—expansive on $X$ in the iterative (or successive) way if and only if $G$ is uniform $G$—expansive on $Y$.

**Proof.** Suppose $F := \{f_k\}_{k=1}^\infty$ has a uniform $G$—expansive on $X$ in an iterative way. There exists an entourage $U \in \mathcal{U}$ such that for any $x, y \in X$ with $G(x) \neq G(\bar{x})$ and positive integer $n, k$ such that

$$\left(F_k^n(\theta_g x), F_k^n(\theta_p \bar{x})\right) \notin U,$$

for all $g, p \in G$, where $F_k := f_k \circ f_{k-1} \circ \ldots \circ f_2 \circ f_1$. Since $h$ is uniform continuous, therefore there exists an entourage $V \in \mathcal{V}$ such that for any $y_1, y_2 \in Y$ with

$$(y_1, y_2) \in V, (x_1, x_2) = (h^{-1}(y_1), h^{-1}(y_2)) \in U,$$

where $x_1 = h^{-1}(y_1), x_2 = h^{-1}(y_2)$. Hence for any $y_1 = h(x_1), y_2 = h(x_2) \in Y$, we have

$$(x_1, x_2) \notin U \Rightarrow (y_1, y_2) \notin V. \quad (***)$$

Observe that for any $y \in Y$ and any neighbourhood $M$ of $h^{-1}(M)$ is a neighbourhood of $h^{-1}(y) = x$. We therefore have $\bar{x} \in h^{-1}(M)$ with $G(x) \neq G(\bar{x})$ and a positive integer $k$ such that

$$\left(F_k^n(\theta_g x), F_k^n(\theta_p \bar{x})\right) \notin U, \text{ for all } g, p \in G.$$

Now we use (***) and observe that for all $g, p \in G$

$$(h F_k^n(\theta_g x), h F_k^n(\theta_p \bar{x})) \notin V.$$

Since $h \circ F_k^n = G_k^n \circ h$ and $h$ is equivariant, we have

$$\left(G_k^n(\theta_g h(x)), G_k^n(\theta_p h(\bar{x}))\right) \notin V$$

and hence

$$\left(G_k^n(\theta_g y), G_k^n(\theta_p \bar{y})\right) \notin V$$

for all $g, p \in G$, where $\bar{y} = h(\bar{x}) \in M$. This establishes $G := \{g_k\}_{k=1}^\infty$ is uniform $G$—expansive in an iterative way. The converse statement can be proved similarly. The case of uniform $G$—expansive in a successive way also follows analogously.

**Theorem (2.7)** Suppose $(X, \mathcal{U})$ and $(Y, \mathcal{V})$ be two uniform spaces and $F := \{f_k\}_{k=1}^\infty, G := \{g_k\}_{k=1}^\infty$ be two series of maps on $X, Y$ respectively. If found an equivariant uniform isomorphism $h: X \rightarrow Y$ such that $F$ and $G$ are $h$—conjugate, then $F$ is uniform expansive on $X$ in the iterative (or successive) way if and only if $G$ is uniform expansive on $Y$. 
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