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ABSTRACT

In this paper, to circumvent the problem of multicollinearity, biased estimation method has been suggested to improve the precision of estimators. We study some types of biased estimators that can help to reduce the effect of multicollinearity on estimation for unknown parameters of regression model. The performance of these estimators have been studied by using the simulation technique that can give more area for taking different degrees of correlation among the explanatory variable. At the same time we take different estimated ridge parameter that given in the literature. Moreover, a real data set has been considered to support the simulation results based on the estimated mean square error criterion.

1. Introduction

Let

$$Y = X\beta + \varepsilon \quad (1.1)$$

be the multiple linear regression model, where Y is an (n × 1) vector of responses, X is an (n × p) design matrix of the explanatory variables, p is the number of the explanatory variables , β is a (p × 1) vector of unknown parameters of interest, ε is an (n × 1) vector of residuals that follow the standard assumptions, namely $E(\varepsilon) = 0$ and $E(\varepsilon'\varepsilon) = \sigma^2 I_n$. I_n is an identity matrix.

The ordinary least squared (OLS) estimator of β in model (1.1) is which is given by:

$$\hat{\beta}_{OLS} = (X'X)^{-1}X'y \quad (1.2)$$

This estimator is best linear unbiased (BLUE), when the explanatory variables need to be considered independent of each other in assumption of the multiple linear regression models. However, there are possibly linear dependencies between these variable values. This problem could predominantly emerge in some of econometric data. This is reasonably called Multicollinearity. Multicollinearity extremely disturbs the study of regression but it is the most obstacles that could be confronted. The existence of multicollinearity makes the correlation large and very large sampling variances of the OLS Lukman et al.[16]. Various methods have been mentioned in the literature to resolve this problem. The biased estimation is one of these methods. The ridge regression is a common biased estimation method which was proposed by Hoerl and Kennard [8] and still the researchers working in this area like Kibria, and Banik [11]. They suggested using the ordinary ridge regression (ORR) as bellow:

$$\hat{\beta}_R = (X'X + kI_p)^{-1}X'y \qquad (1.3)$$

where k is the ridge parameter and the value of $k > 0$. The ORR estimator should be estimated from the actual data due to its biased to a certain unknown k value. Multiple ways have been evolved to obtain skewed estimates of β with a smaller mean square error (MSE). Following Hoerl and Kennard model, the following methodology demonstrated the relation of prior knowledge (URR) with ridge regression that has been extended by Crouse et al [5]:

$$\hat{\beta}(kI, J) = (X'X + kI)^{-1}(X'Y + kJ), \qquad (1.4)$$

where J is a vector of random with $J \sim N(\beta, (\sigma^2/k)I)$. Moreover, a modified unbiased ridge regression estimator (MURRE) for β has been proposed by Batah and Gore [2] and still the researchers who work in this area like Lukman et al.[16]and Tarima et al. [17] which is denoted as below:

$$\hat{\beta}_J(k) = [(I - k(X'X + kI_p)^{-1}](X'X + kI_p)^{-1}(X'Y + kJ), \qquad (1.5)$$

The MURRE estimator associates the estimator ORR and unbiased ridge regression URR which is achieved from ORR by using URR rather than OLS.

Liu [14] proposed biased estimator called the Liu-type estimator (LTE). That improve the Liu estimator, since it has two parameters, by augmenting the equation $(-d/k^{1/2})\beta_{OLS} = \beta + \varepsilon^*$ to the model in (1.1) and then using the OLS method which was as follows:

$$\hat{\beta}_{LTE}(k) = (X'X + kI)^{-1}(X'Y - d\hat{\beta}_{OLS}), \qquad (1.6)$$

where d is shrinkage parameter such that $0 < d < 1$.

In the other hand Lukman, at el. [15] suggested modified ridge-type two-parameter estimator (MRTTP):

$$\hat{\beta}_{MRTTP}(k,d) = (X'X + k(1+d)I)^{-1}X'Y \ . \quad (1.7)$$

To simplify the considerations about the linear model, the canonical form is often used. Therefore, a symmetric matrix S = X′X has an eigenvalue–eigenvector decomposition of the form S = TΛT′, where T is an orthogonal matrix and Λ is a real diagonal matrix. The diagonal elements of Λ are the eigenvalues of S and the column vectors of T are the eigenvectors of S. The orthogonal version of the regression model in (1.1) is

$$y = XTT'\beta + \varepsilon = Z\gamma + \varepsilon \quad (1.8)$$

where Z = XT, $\gamma = T'\beta$ and Z′Z = Λ=$dig(\lambda_1, \lambda_2, \ldots, \lambda_p)$. The OLS estimator of γ is given by

$$\hat{\gamma}_{OLS} = (Z'Z)^{-1}Z'y = \Lambda^{-1}Z \quad (1.9)$$

The goal of this paper is to compare the different biased estimators as well as with different estimated value of k using the MSE.

In this paper, we present the methods of various estimators of k in Section 2, A simulation of Monte Carlo in Section 3, a discussion of the simulation results in Section 4 and finally, a real data set as an example of this study is given in Section 5.

2. Estimation of Ridge Parameter

The ORR properties have been shown by Hoerl and Kennard [8] in detail. They stated that when k increases, total variance decreases, and the squared bias increases as well. Thays, the variance function is decreased and the squared bias function is increased uniformly. In other words, some k can be existed such as the MSEM for ORR that is less than MSEM for the OLS.

Typically, k is unknown, and it is estimated from the study sample. Consequently, many publications suggested different ridge parameters using various techniques. Recent studies have proposed different estimates of k. Here, we review the most beneficial methods that were mentioned in literatures to estimate the value of k as follows:

- Hoerl and Kennard [8] suggested k to be (denoted here by $\hat{k}_{HK}$)

$$\hat{k}_{HK} = \frac{\hat{\sigma}^2}{\hat{\gamma}^2_{\max OLS}}, \quad (2.1)$$

where $\hat{\sigma}^2 = \frac{\sum_{i=1}^{n} \hat{e}_i^2}{n-p}$ and $\hat{\gamma}_{\max OLS}$ is the maximum element of $\hat{\gamma}_{OLS}$

- Hoerl et al. [9] proposed k to be (denoted here by $\hat{k}_{HKB}$)

$$\hat{k}_{HKB} = \frac{P\hat{\sigma}^2}{\hat{\gamma}'_{OLS}\hat{\gamma}_{OLS}}, \qquad (2.2)$$

- Lawless [13] suggested k to be (denoted here by $\hat{k}_{LW}$)

$$\hat{k}_{LW} = \frac{P\hat{\sigma}^2}{\hat{\gamma}'_{OLS}X\ 'X\ \hat{\gamma}_{OLS}}, \qquad (2.3)$$

- Khalaf [10], based on modification of $\hat{k}_{HK}$, proposed k to be (denoted by $\hat{k}_{GK}$)

$$\hat{k}_{GK} = \hat{k}_{HK} + \frac{2}{(\lambda_{\max} + \lambda_{\min})}, \qquad (2.4)$$

where $\lambda_{\max}$ and $\lambda_{\min}$ are the largest and smallest eigenvalues of the matrix X'X, respectively. Yasin et al. [18] proposed five modification of ridge parameter. They are defined as, respectively:

$$\hat{k}_{A1} = \frac{p^2}{\lambda_{\max}^2}\frac{\hat{\sigma}^2}{\sum_{i=1}^{p}\hat{\gamma}_{i\,OLS}^2} \qquad (2.5)$$

$$\hat{k}_{A2} = \frac{p^3}{\lambda_{\max}^3}\frac{\hat{\sigma}^2}{\sum_{i=1}^{p}\hat{\gamma}_{i\,OLS}^2} \qquad (2.6)$$

$$\hat{k}_{A3} = \frac{p}{(\lambda_{\max})^{1/3}}\frac{\hat{\sigma}^2}{\sum_{i=1}^{p}\hat{\gamma}_{i\,OLS}^2} \qquad (2.7)$$

$$\hat{k}_{A4} = \frac{p}{\left(\sum_{i=1}^{p}\sqrt{\lambda_i}\right)^{1\backslash 3}}\frac{\hat{\sigma}^2}{\sum_{i=1}^{p}\hat{\gamma}_{i\,OLS}^2} \qquad (2.8)$$

$$\hat{k}_{A5} = \frac{2p}{\sqrt{\lambda_{\max}}}\frac{\hat{\sigma}^2}{\sum_{i=1}^{p}\hat{\gamma}_{i\,OLS}^2} \qquad (2.9)$$

- Estimator from Göktaş and Volkan [6] denoted by $\hat{k}_{AV}$:

$$\hat{k}_{AV1} = \frac{\hat{\sigma}^2}{\left(Median(\hat{\gamma}_{i\,OLS}^2)\right)^2} \qquad (2.10)$$

$$\hat{k}_{AV2} = \sqrt{Median(\hat{\gamma}_{i\,OLS}^2)} \qquad (2.11)$$

- Bhat [4] proposed two modification of $\hat{k}_{HKB}$. They are defined as follows:

$$\hat{k}_{B1} = \frac{P\hat{\sigma}^2}{\hat{\gamma}'_{OLS}\hat{\gamma}_{OLS}} + \frac{1}{\lambda_{\max}\hat{\gamma}'_{OLS}\hat{\gamma}_{OLS}} \qquad (2.12)$$

$$\hat{k}_{B2} = \frac{P\hat{\sigma}^2}{\hat{\gamma}'_{OLS}\hat{\gamma}_{OLS}} + \frac{1}{2\left(\sqrt{\lambda_{\max}/\lambda_{\min}}\right)} \qquad (2.13)$$

- Following Batah and Alheety [3] suggested k to be(denoted by $\hat{k}_{BA}$):

$$\hat{k}_{BA1} = max\left[\frac{2\hat{\sigma}^2}{\lambda_{\max}(\hat{\gamma}^2_{i\,OLS})}\right] \quad i=1,2,\ldots,p \tag{2.14}$$

$$\hat{k}_{BA2} = max\left[\frac{\lambda_{\max}(\hat{\gamma}^2_{i\,OLS})}{2\hat{\sigma}^2}\right] \quad i=1,2,\ldots,p \tag{2.15}$$

$$\hat{k}_{BA3} = \prod_{i=1}^{p}\left[\frac{2\hat{\sigma}^2}{\lambda_{\max}(\hat{\gamma}^2_{i\,OLS})}\right]^{\frac{1}{p}} \quad i=1,2,\ldots,p \tag{2.16}$$

$$\hat{k}_{BA4} = \prod_{i=1}^{p}\left[\frac{\lambda_{\max}(\hat{\gamma}^2_{i\,OLS})}{2\hat{\sigma}^2}\right]^{\frac{1}{p}} \quad i=1,2,\ldots,p \tag{2.17}$$

3. A study via simulation

The current study aims to compare various biased estimators for different estimates of ridge parameter given in (2.1-2.17) and to recognize some suitable estimators for practitioners. Typically, MATLAB software is used in a simulation analysis. The simulation is set to rely on different variables that are supposed to affect the properties of estimators which are subjected to a statistical investigation. Since the degree of collinearity between several explanatory variables (Xs) is very fundamental, the following equation was followed by Kibria[12] to generate X's:

$$X_{ij} = \left(1-\varphi^2\right)^{\frac{1}{2}} z_{ij} + \varphi z_{ip}, \; \text{i=1,2,...,n,j=1,2,...,p,} \tag{3.1}$$

where the z_{ij} independent typical regular pseudo-random numbers and φ denotes the association between any two X's. These diverse are standardized so that X'X is being in association forms. The response adjustable y is considered by

$$y_i = \beta_1 X_{i1} + \beta_2 X_{i2} + \ldots + \beta_p X_{ip} + e_i, \quad \text{i=1,2,...,n} \; , \tag{3.2}$$

where the e_i is i.i.d. N(0, σ^2). Consequently, zero intercept for (3.2) will be presumed. Also the number of explanatory variables $p=5$, while the values of σ are chosen as (0.1, 1, 5, 10). The shrinkage parameter d will be chosen as 0.1, The correlation φ will be chosen as (0.75, 0.85, 0.95), sample size n=(50, 100, 150) and the value of ridge parameter given in (2.1-2.17).The coefficients β_1, β_2, …, β_p are designated as the eigenvectors related to the greatest eigenvalue of the matrix X'X subject to constraint $\beta'\beta = 1$. Therefore, for n, p, β, λ, φ, and σ, sets of Xs are produced. Subsequently, the experiment was repeated 10000 times by generating new error terms. The estimated mean squared error (EMSE) for each estimator is estimated as follows:

$$EMSE(\beta^*) = \frac{1}{10000}\sum_{i=1}^{10000}(\beta^* - \beta)'(\beta^* - \beta), \qquad (3.3)$$

where β^* can be one of the following estimators:

OLS: Ordinary least squared

ORR: Ordinary ridge regression

MURRE: Modified unbiased ridge regression estimator

LTE: Liu-type estimator

MRTTP: Modified ridge-type two-parameter estimator

4. The discussion of simulation results

In this section, the results are discussed of our Monte Carlo experiment on the properties of the several approaches that were used to select the ridge parameter k, when the multicollinearity among the columns of the descriptive variables design matrix is existed. The results of the simulation are seen in Tables (1-12). the results were analyzed by splitting the results into two sections as follow:

4-1 The simulation results according to the different estimators

Table (4.1) shows a description of the estimators' preferences detailed in this paper, in which we can obtain the following estimations:

1. The MURRE estimator is the best estimator with the lowest estimated mean squared errors for all correlations relatives to the other estimators in different sample sizes and $\sigma = 5{,}10$ that is noted also in Tables (1-12).
2. The LTE estimator has shown better performance than others in the case (n=50, $\sigma = 1$, φ =0.75,0.85, 0.95) , (n=100, 150 , $\sigma = 1$, φ =0.85,0.95) and (n=100, 150 , $\sigma = 5$, φ =0.75), which can be clearly used instead of MURRE in case we need it.

Table 4.1: The simulation results according to the best estimators in each case

Table	n	σ	φ	Best estimator	Table	n	σ	φ	Best estimator
1-4	50	0.1	0.75	OLS	8-12	150	0.1	0.75	OLS
			0.85	OLS				0.85	OLS
			0.95	OLS				0.95	OLS
		1	0.75	LTE			1	0.75	OLS
			0.85	LTE				0.85	LTE
			0.95	LTE				0.95	LTE
		5	0.75	MURRE			5	0.75	LTE
			0.85	MURRE				0.85	MURRE
			0.95	MURRE				0.95	MURRE
		10	0.75	MURRE			10	0.75	MURRE
			0.85	MURRE				0.85	MURRE
			0.95	MURRE				0.95	MURRE
		0.1	0.75	OLS					

4-8	100		0.85	OLS	
			0.95	OLS	
		1	0.75	MURRE	
			0.85	LTE	
			0.95	LTE	
		5	0.75	LTE	
			0.85	MURRE	
			0.95	MURRE	
		10	0.75	MURRE	
			0.85	MURRE	
			0.95	MURRE	

4.2 The simulation results according to the different estimated ridge parameter

The comparison of the performance of estimated ridge parameter with other estimators is provided in Tables (4.2 to 4.5) that describe the following points:

1- From Table (4.2) showing the best of estimated ridge parameter k for each of the biased estimator for different correlation. With the sample size increases, we observe others estimated of ridge parameter which give minimum EMSE and still AB1, B1, B2 and A2 show well performance

2- From Tables (4.3 to 4. 5) showing the best biased estimator of each ridge parameter k at different level of correlation. We generally observe that all estimated ridge parameter well-behaved with MURRE and LTE estimators which can be said it is the best estimators according to results.

Table 4.2 The simulation results according to the different estimated ridge parameter

Table	n	σ	φ	Best estimator of k	Table	n	σ	φ	Best estimator of k
1-4	50	0.1	0.75	--	8-12	150	0.1	0.75	--
			0.85	--				0.85	--
			0.95	--				0.95	--
		1	0.75	AB1			1	0.75	--
			0.85	AB1				0.85	AB1
			0.95	A1				0.95	AB1
		5	0.75	AB2			5	0.75	A2
			0.85	AV2				0.85	LW
			0.95	GK				0.95	LW
		10	0.75	A5			10	0.75	A5
			0.85	B1				0.85	GK
			0.95	AB3				0.95	B2
		0.1	0.75	--					
			0.85	--					

4-8	100		0.95	--	
		1	0.75	A2	
			0.85	A2	
			0.95	HKB	
		5	0.75	HK	
			0.85	B1	
			0.95	B2	
		10	0.75	A5	
			0.85	B1	
			0.95	AB3	

Table 4.3: The simulation results according to the best estimated ridge parameter when n=50

n	σ	φ	HK	HKB	LW	GK	A1	A2	A3	A4	A5	AV1	AV2	B1	B2	AB1	AB2	AB3	AB4
50	0.1	0.75	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--
		0.85	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--
		0.95	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--
	1		HK	HKB	LW	GK	A1	A2	A3	A4	A5	AV1	AV2	B1	B2	AB1	AB2	AB3	AB4
		0.75	LTE	MURRE	LTE	--	LTE	MURRE	MURRE	MURRE	MURRE	--	ORR	--	--	LTE	--	ORR	--
		0.85	LTE	LTE	LTE	--	LTE	LTE	LTE	LTE	LTE	--	LTE	--	ORR	LTE	--	ORR	--
		0.95	LTE	LTE	LTE	ORR	LTE	LTE	LTE	LTE	LTE	--	LTE	ORR	LTE	LTE	--	LTE	--
	5		HK	HKB	LW	GK	A1	A2	A3	A4	A5	AV1	AV2	B1	B2	AB1	AB2	AB3	AB4
		0.75	MURRE	MURRE	MURRE	MURRE	MURRE	MURRE	MURRE	MURRE	MURRE	MURRE	MURRE	MURRE	MURRE	MURRE	LTE	MURRE	LTE
		0.85	LTE	LTE	MURRE	LTE	LTE	LTE	LTE	LTE	LTE	ORR	MURRE	LTE	MURRE	LTE	ORR	LTE	ORR
		0.95	LTE	LTE	MURRE	MURRE	LTE	LTE	LTE	LTE	LTE	MURRE	MURRE	MURRE	LTE	LTE	MURRE	LTE	MURRE
	10		HK	HKB	LW	GK	A1	A2	A3	A4	A5	AV1	AV2	B1	B2	AB1	AB2	AB3	AB4
		0.75	MURRE	MURRE	LTE	LTE	MURRE	MURRE	MURRE	MURRE	MURRE	ORR	MURRE	LTE	MURRE	LTE	ORR	MURRE	ORR
		0.85	MURRE	MURRE	MURRE	MURRE	MURRE	MURRE	MURRE	MURRE	MURRE	MURRE	LTE	MURRE	MURRE	MURRE	MURRE	MURRE	MURRE
		0.95	LTE	MURRE	MURRE	MURRE	LTE	LTE	LTE	LTE	MURRE	MURRE	MURRE	MURRE	MURRE	LTE	MURRE	MURRE	MURRE

Table 4.4: The simulation results according to the best estimated ridge parameter when n=100

n	σ	φ	HK	HKB	LW	GK	A1	A2	A3	A4	A5	AV1	AV2	B1	B2	AB1	AB2	AB3	AB4
100	0.1	0.75	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--
		0.85	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--
		0.95	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--
	1		HK	HKB	LW	GK	A1	A2	A3	A4	A5	AV1	AV2	B1	B2	AB1	AB2	AB3	AB4
		0.75	MURRE	MURRE	MURRE	--	MURRE	MURRE	MURRE	MURRE	MURRE	--	--	--	--	LTE	--	--	--
		0.85	LTE	LTE	LTE	--	LTE	LTE	LTE	LTE	LTE	--	LTE	--	ORR	LTE	--	LTE	--
		0.95	LTE	LTE	LTE	ORR	LTE	LTE	LTE	LTE	LTE	ORR	LTE	ORR	LTE	LTE	--	LTE	--
			HK	HKB	LW	GK	A1	A2	A3	A4	A5	AV1	AV2	B1	B2	AB1	AB2	AB3	AB4
	5	0.75	LTE	LTE	LTE	ORR	LTE	LTE	LTE	LTE	LTE	--	LTE	--	LTE	LTE	--	LTE	--
		0.85	LTE	LTE	MURRE	MURRE	LTE	LTE	LTE	LTE	LTE	MURRE	LTE	MURRE	LTE	LTE	MURRE	LTE	MURRE
		0.95	LTE	LTE	MURRE	MURRE	LTE	LTE	LTE	LTE	LTE	MURRE	MURRE	MURRE	MURRE	LTE	MURRE	MURRE	MURRE
			HK	HKB	LW	GK	A1	A2	A3	A4	A5	AV1	AV2	B1	B2	AB1	AB2	AB3	AB4
	10	0.75	MURRE	MURRE	MURRE	MURRE	MURRE	MURRE	MURRE	MURRE	MURRE	--	MURRE	ORR	MURRE	MURRE	ORR	MURRE	ORR
		0.85	MURRE	MURRE	MURRE	MURRE	MURRE	MURRE	MURRE	MURRE	MURRE	MURRE	LTE	MURRE	MURRE	LTE	MURRE	MURRE	MURRE
		0.95	LTE	LTE	MURRE	MURRE	LTE	LTE	LTE	LTE	LTE	MURRE	LTE	MURRE	LTE	LTE	MURRE	LTE	MURRE

Table 4.5: The simulation results according to the best estimated ridge parameter when n=150

n	σ	φ	HK	HKB	LW	GK	A1	A2	A3	A4	A5	AV1	AV2	B1	B2	AB1	AB2	AB3	AB4
150	0.1	0.75	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--
		0.85	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--
		0.95	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--
	1		HK	HKB	LW	GK	A1	A2	A3	A4	A5	AV1	AV2	B1	B2	AB1	AB2	AB3	AB4
		0.75	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--
		0.85	LTE	LTE	LTE	--	LTE	LTE	LTE	LTE	LTE	--	LTE	--	LTE	LTE	--	LTE	--
		0.95	LTE	LTE	LTE	--	LTE	LTE	LTE	LTE	LTE	--	ORR	--	ORR	LTE	--	LTE	--
	5		HK	HKB	LW	GK	A1	A2	A3	A4	A5	AV1	AV2	B1	B2	AB1	AB2	AB3	AB4
		0.75	LTE	LTE	LTE	ORR	LTE	LTE	LTE	LTE	LTE	--	LTE	--	LTE	LTE	--	--	LTE
		0.85	LTE	LTE	MURRE	LTE	LTE	LTE	LTE	LTE	LTE	ORR	LTE	ORR	LTE	LTE	--	LTE	ORR
		0.95	LTE	LTE	MURRE	MURRE	LTE	LTE	LTE	LTE	LTE	MURRE	LTE	MURRE	MURRE	LTE	MURRE	LTE	MURRE
	10		HK	HKB	LW	GK	A1	A2	A3	A4	A5	AV1	AV2	B1	B2	AB1	AB2	AB3	AB4
		0.75	LTE	MURRE	MURRE	MURRE	MURRE	MURRE	MURRE	MURRE	MURRE	--	MURRE	ORR	MURRE	LTE	--	MURRE	ORR
		0.85	LTE	MURRE	MURRE	MURRE	LTE	LTE	LTE	LTE	MURRE	MURRE	LTE	MURRE	MURRE	LTE	MURRE	MURRE	MURRE
		0.95	LTE	LTE	MURRE	MURRE	LTE	LTE	LTE	LTE	LTE	MURRE	MURRE	MURRE	MURRE	LTE	MURRE	LTE	MURRE

5. A numerical example

To give further clarification for the analysis, we find, among others, the data set on total national expenditure for research and development as a percent of gross national product originally according to Gruber [7] and later by Akdeniz and Erol [1].

The aim is to perform a comparison of the traces of the estimated mean square error matrices of (ORR), (MURRE) ,(TPE) and (MRTTP). The trace of the mean square error matrix of the (ORR) is known by

$$mse(\hat{\beta}_R)=tr(EMSE(\hat{\beta}_R,\beta))=\sum_{i=1}^{p}\frac{\lambda_i\sigma^2+k^2\beta_i^2}{(\lambda_i+k)^2}, \tag{5.1}$$

the trace of the mean square error matrix of the (MURRE) is given by

$$mse(\hat{\beta}_J(k))=tr(EMSE(\hat{\beta}_J(k),\beta))=\sum_{i=1}^{p}\frac{\lambda_i\sigma^2+k^2(\lambda_i+k)\beta_i^2}{(\lambda_i+k)^3}, \tag{5.2}$$

the trace of the mean square error matrix of the (TPE) is given by

$$mse(\hat{\beta}(k,d))=tr(EMSE(\hat{\beta}(k,d),\beta))=\sum_{i=1}^{p}\frac{\lambda_i\sigma^2(\lambda_i+d)^2+((k+1-d)\lambda_i+k)^2\beta_i^2}{(\lambda_i+1)^2(\lambda_i+k)^2}, \tag{5.3}$$

and trace of the mean square error matrix of the (MRTTP) is given by

$$mse(\hat{\beta}_{MRTTP}(k,d))=tr(MSEM(\hat{\beta}_{MRTTP}(k,d),\beta))=$$

$$\sum_{i=1}^{p}\frac{\sigma^2\lambda_i+k^2(1+d)^2\beta_i^2}{(\lambda_i+k(1+d))^2}, \tag{4.1}$$

we have substituted β and σ^2 by their OLS estimates $\hat{\beta}$ and $\hat{\sigma}^2$ respectively. For the standardized data since there are ten observations and four parameters, we obtain $\hat{\sigma}^2 = 0.003932$. The four eigenvalues of X' X are 2.95743, 0.91272, 0.10984, and 0.02021. The factors will define a 4-dimensional space and the $X'X$ matrix will be as follows:

$$X'X=\begin{bmatrix}1.000 & 0.888 & .925 & 0.309\\ 0.888 & 1.000 & 0.962 & 0.157\\ 0.925 & 0.962 & 1.000 & 0.328\\ 0.309 & 0.157 & 0.328 & 1.000\end{bmatrix}$$

We can detect that the variables in $X'X$ matrix undergo for high correlations among them and this is the one benefit of standardizing the X matrix where it can be seen which variables are greatly associated. The condition index is another method for diagnosing multicollinearity in linear regression Condition Index (C.I.) which is defined as follows:

$$C.I.=\sqrt{\frac{\lambda_{max}}{\lambda_{min}}},$$

where λ_{max} and λ_{min} are the greatest and the lowest eigenvalues of X' X , if C.I. ≤ 10 , therefore , no multicollinearity among the explanatory variables is existed , if 10 < C.I. < 30, then the multicollinearity is moderate, but if C.I .≥ 30, then it means that there is a severe multicollinearity that must be corrected. So in this example $,10 < C.I. = \sqrt{\frac{2.95743}{0.02021}} = 12.1 < 30$, which suggests that there is a moderate multicollinearity and may be modified with a correction.

Table 5.1: The scaler mean squares error for different estimators and different estimated ridge parameter

	HK	HKB	LW	GK	A1	A2	A3	A4	A5
OLS	0.2361	0.2361	0.2361	0.2361	0.2361	0.2361	0.2361	0.2361	0.2361
ORR	0.1166	0.1140	0.1213	0.2825	0.1338	0.1233	0.1200	0.1205	0.1135
MURRE	0.0880	0.0876	0.0910	0.2810	0.1023	0.0926	0.0901	0.0904	0.0890
TPE	2.0861	1.7965	2.4163	0.3704	3.0426	2.5322	2.3379	2.3714	1.5975
MRTTP	0.1151	0.1135	0.1189	0.2885	0.1302	0.1206	0.1178	0.1183	0.1137
	AV1	AV2	B1	B2	AB1	AB2	AB3	AB4	
OLS	0.2361	0.2361	0.2361	0.2361	0.2361	0.2361	0.2361	0.2361	
ORR	0.1786	0.2477	0.2675	0.1321	0.1462	0.2043	0.1211	0.2299	
MURRE	0.1719	0.2452	0.2656	0.1184	0.1156	0.1899	0.1035	0.2268	
TPE	0.5086	0.3872	0.3751	0.8245	3.5433	5.4289	1.0462	6.1569	
MRTTP	0.1851	0.2540	0.2736	0.1364	0.1421	0.2016	0.1239	0.2293	

The lowest mse for the ORR estimator can be clearly seen from Table (5.1) when the ridge parameter estimated by A5 is shown, the smallest mse for the MURRE and MRTTP estimators is also given by estimating the ridge parameter by HKB and by estimating the ridge parameter AB1 for the TPE estimator.

To sum up, from Table (5-1) and Figure 1 , The performance of the estimated k given in this study shows that the most estimated k gives the lowest mse under a moderate level of multicollinearity when it is used in the MURRE estimator except (GK, AV2,B1,AB2 and AB4). However, the OLS estimator is better than the MURRE estimator. Therefore, we found that not all proposed ridge parameter can be used to get lower mse if the degree of multicollinearity is high. This analysis provide us a clear view of the estimators' actions. Mre importantly, estimators' can be used to produce a successful result compared with the other estimators suggested.

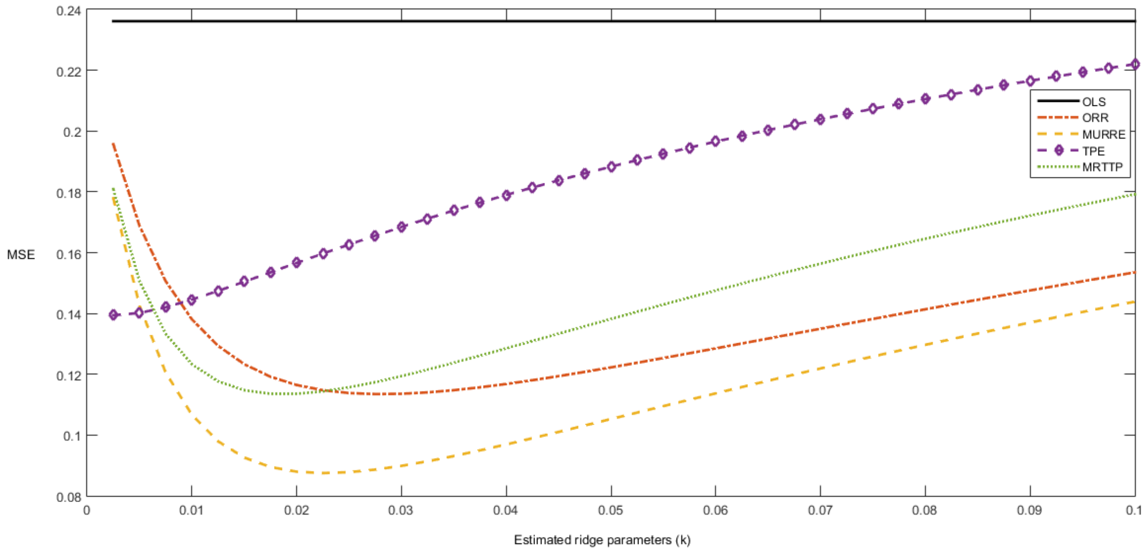

Figure 1 MSE of ridge estimators of (ORR, MURRE and TPE)

6. Conclusions and Recommendations

By comparing the performance of biased estimators, we found that, the MURRE doing will for different sample size, some levels of correlation and σ. When the sample size and σ increasing and for different levels of correlation, the best performance for estimating ridge parameter was by still AB1, B1, B2 and A2.

We recommend the use of biased methods to solve the multicollinearity and when there is very high Multicollinearity it is suggested to use the MURRE method. Also As well as the use of the ridge parameter AB1, B1, B2 and A2.

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Appendix

Table1: The stimated MSE when n=50 σ =0.1

φ	0.75					0..85					0.95				
	OLS	ORR	MURRE	LTE	MRTTP	0.2572	0.2572	0.2573	0.2693	0.2572	OLS	ORR	MURRE	LTE	MRTTP
HK	0.1636	0.1636	0.1636	0.1807	0.1636	0.2572	0.2573	0.2574	0.2694	0.2573	0.2893	0.2893	0.2893	0.3726	0.2893
HKB	0.1636	0.1637	0.1639	0.1809	0.1637	0.2572	0.2572	0.2573	0.2693	0.2572	0.2893	0.2893	0.2894	0.3726	0.2893
LW	0.1636	0.1636	0.1637	0.1808	0.1636	0.2572	0.3106	0.3541	0.3227	0.3156	0.2893	0.2893	0.2893	0.3726	0.2893
GK	0.1636	0.2631	0.3445	0.2812	0.2722	0.2572	0.2572	0.2573	0.2693	0.2573	0.2893	0.3311	0.3649	0.3726	0.3352
A1	0.1636	0.1637	0.1638	0.1808	0.1637	0.2572	0.2573	0.2573	0.2693	0.2573	0.2893	0.2893	0.2893	0.3726	0.2893
A2	0.1636	0.1637	0.1639	0.1809	0.1638	0.2572	0.2573	0.2573	0.2694	0.2573	0.2893	0.2893	0.2893	0.3726	0.2893
A3	0.1636	0.1637	0.1638	0.1808	0.1637	0.2572	0.2573	0.2573	0.2694	0.2573	0.2893	0.2893	0.2893	0.3726	0.2893
A4	0.1636	0.1637	0.1638	0.1808	0.1637	0.2572	0.2573	0.2574	0.2694	0.2573	0.2893	0.2893	0.2893	0.3726	0.2893
A5	0.1636	0.1638	0.1639	0.1809	0.1638	0.2572	0.9995	0.9999	0.9996	0.9996	0.2893	0.2893	0.2894	0.3726	0.2893
AV1	0.1636	0.9908	0.9979	0.9911	0.9916	0.2572	0.2582	0.2589	0.2703	0.2583	0.2893	0.9672	0.9920	0.3726	0.9701
AV2	0.1636	0.1669	0.1701	0.1841	0.1672	0.2572	0.3643	0.4441	0.3760	0.3735	0.2893	0.2911	0.2944	0.3726	0.2913
B1	0.1636	0.3138	0.4273	0.3317	0.3266	0.2572	0.2695	0.2812	0.2817	0.2707	0.2893	0.3823	0.4507	0.3726	0.3905
B2	0.1636	0.1943	0.2227	0.2121	0.1974	0.2572	0.2572	0.2572	0.2693	0.2572	0.2893	0.2943	0.3014	0.3726	0.2948
AB1	0.1636	0.1636	0.1636	0.1807	0.1636	0.2572	0.9997	0.9999	0.9997	0.9997	0.2893	0.2893	0.2893	0.3726	0.2893
AB2	0.1636	0.9996	0.9999	0.9996	0.9997	0.2572	0.2705	0.2831	0.2827	0.2718	0.2893	0.9997	0.9999	0.3726	0.9998
AB3	0.1636	0.1681	0.1724	0.1853	0.1685	0.2572	0.7132	0.8643	0.7196	0.7301	0.2893	0.2922	0.2971	0.3726	0.2925
AB4	0.1636	0.9180	0.9773	0.9209	0.9249	0.2572	0.2572	0.2573	0.2693	0.2572	0.2893	0.8768	0.9594	0.3726	0.8863

Table2: The stimated MSE when n=50 σ =1

φ	0.75					0..85					0.95				
	OLS	ORR	MURRE	LTE	MRTTP	OLS	ORR	MURRE	LTE	MRTTP	OLS	ORR	MURRE	LTE	MRTTP
HK	0.3115	0.3100	0.3077	**0.3046**	0.3098	0.3215	0.3181	0.3147	**0.3046**	0.3178	0.4213	0.3971	0.3800	**0.3516**	0.3954
HKB	0.3115	0.3079	**0.3059**	0.3074	0.3079	0.3215	0.3125	0.3084	**0.3064**	0.3121	0.4213	0.3737	0.3614	**0.3545**	0.3721
LW	0.3115	0.3095	0.3068	**0.3049**	0.3093	0.3215	0.3171	0.3130	**0.3047**	0.3167	0.4213	0.3959	0.3785	**0.3516**	0.3941
GK	**0.3115**	0.3456	0.3971	0.3564	0.3513	**0.3215**	0.3456	0.3961	0.3559	0.3512	0.4213	**0.3827**	0.4157	0.3897	0.3864
A1	0.3115	0.3092	0.3063	**0.3051**	0.3090	0.3215	0.3165	0.3121	**0.3048**	0.3161	0.4213	0.4010	0.3852	**0.3515**	0.3994
A2	0.3115	0.3086	**0.3055**	0.3056	0.3085	0.3215	0.3151	0.3101	**0.3051**	0.3146	0.4213	0.3996	0.3833	**0.3515**	0.3980
A3	0.3115	0.3083	**0.3053**	0.3060	0.3082	0.3215	0.3143	0.3094	**0.3053**	0.3139	0.4213	0.3835	0.3663	**0.3525**	0.3816
A4	0.3115	0.3084	**0.3053**	0.3058	0.3083	0.3215	0.3146	0.3096	**0.3052**	0.3142	0.4213	0.3815	0.3648	**0.3528**	0.3796
A5	0.3115	0.3079	**0.3061**	0.3077	0.3079	0.3215	0.3122	0.3084	**0.3066**	0.3119	0.4213	0.3750	0.3617	**0.3541**	0.3733
AV1	**0.3115**	0.9155	0.9796	0.9178	0.9224	**0.3215**	0.8373	0.9429	0.8416	0.8491	**0.4213**	0.8302	0.9400	0.8337	0.8421
AV2	0.3115	**0.3082**	0.3091	0.3099	0.3085	0.3215	0.3112	0.3132	**0.3106**	0.3114	0.4213	0.3668	0.3638	**0.3588**	0.3662
B1	**0.3115**	0.4032	0.4991	0.4153	0.4137	**0.3215**	0.3985	0.4881	0.4105	0.4088	0.4213	**0.4193**	0.4824	0.4282	0.4264
B2	**0.3115**	0.3139	0.3279	0.3201	0.3155	0.3215	**0.3140**	0.3260	0.3177	0.3152	0.4213	0.3663	0.3649	**0.3597**	0.3658
AB1	0.3115	0.3106	0.3091	**0.3044**	0.3105	0.3215	0.3195	0.3173	**0.3045**	0.3193	0.4213	0.4089	0.3974	**0.3515**	0.4079
AB2	**0.3115**	0.9706	0.9944	0.9714	0.9731	**0.3215**	0.9708	0.9934	0.9716	0.9734	**0.4213**	0.9657	0.9930	0.9664	0.9686
AB3	0.3115	**0.3101**	0.3165	0.3142	0.3109	0.3215	**0.3136**	0.3249	0.3171	0.3148	0.4213	0.3753	0.3617	**0.3541**	0.3736
AB4	**0.3115**	0.6961	0.8562	0.7038	0.7132	**0.3215**	0.6490	0.8069	0.6577	0.6667	**0.4213**	0.8134	0.9303	0.8173	0.8261

Table3: The stimated MSE when n=50 σ =5

φ	0.75					0..85					0.95				
	OLS	ORR	MURRE	LTE	MRTTP	OLS	ORR	MURRE	LTE	MRTTP	OLS	ORR	MURRE	LTE	MRTTP
HK	1.0009	0.9775	**0.9602**	0.9771	1.0009	1.1690	0.9384	0.8160	**0.7701**	0.9256	1.9599	1.4174	1.1353	**0.8919**	1.3873
HKB	1.0009	0.9834	**0.9606**	0.9837	1.0009	1.1690	0.8488	0.7489	**0.7487**	0.8375	1.9599	1.2073	0.9711	**0.8829**	1.1796
LW	1.0009	0.9894	**0.9694**	0.9896	1.0009	1.1690	0.7730	**0.7318**	0.7356	0.7680	1.9599	0.9005	**0.8692**	0.8801	0.8983
GK	1.0009	0.9789	**0.9587**	0.9789	1.0009	1.1690	0.7576	0.7425	**0.7382**	0.7559	1.9599	0.9128	**0.8687**	0.8770	0.9082
A1	1.0009	0.9785	**0.9589**	0.9784	1.0009	1.1690	0.9976	0.8820	**0.7848**	0.9861	1.9599	1.6385	1.3969	**0.9015**	1.6146
A2	1.0009	0.9805	**0.9587**	0.9807	1.0009	1.1690	0.9693	0.8483	**0.7777**	0.9570	1.9599	1.6181	1.3687	**0.9006**	1.5933
A3	1.0009	0.9806	**0.9587**	0.9808	1.0009	1.1690	0.9082	0.7889	**0.7627**	0.8954	1.9599	1.3660	1.0878	**0.8897**	1.3356
A4	1.0009	0.9801	**0.9586**	0.9803	1.0009	1.1690	0.9085	0.7891	**0.7628**	0.8957	1.9599	1.3334	1.0602	**0.8883**	1.3031
A5	1.0009	0.9841	**0.9613**	0.9844	1.0009	1.1690	0.8484	0.7487	**0.7486**	0.8372	1.9599	1.2285	0.9841	**0.8838**	1.2001
AV1	1.0009	0.9999	**0.9998**	0.9999	1.0009	1.1690	**0.8163**	0.8602	0.8193	0.8234	1.9599	0.9131	**0.8687**	0.8770	0.9084
AV2	1.0009	0.9938	0.9895	**0.9838**	1.0009	1.1690	0.7970	**0.7311**	0.7383	0.7891	1.9599	0.9156	**0.8687**	0.8767	0.9105
B1	1.0009	0.9927	**0.9769**	0.9929	1.0009	1.1690	0.7553	0.7486	**0.7408**	0.7548	1.9599	0.9558	**0.8734**	0.8750	0.9452
B2	1.0009	0.9838	**0.9610**	0.9841	1.0009	1.1690	0.7989	**0.7314**	0.7386	0.7908	1.9599	1.0658	0.9031	**0.8774**	1.0453
AB1	1.0009	0.9778	**0.9639**	0.9762	1.0009	1.1690	1.0229	0.9152	**0.7910**	1.0123	1.9599	1.6529	1.4173	**0.9021**	1.6297
AB2	1.0009	0.9793	**0.9586**	0.9794	1.0009	1.1690	**0.9508**	0.9755	0.9520	0.9546	1.9599	0.9883	**0.9821**	0.9885	0.9893
AB3	1.0009	0.9918	**0.9749**	0.9921	1.0009	1.1690	0.8678	0.7597	**0.7531**	0.8559	1.9599	1.2086	0.9719	**0.8830**	1.1809
AB4	1.0009	0.9904	0.9845	**0.9820**	1.0009	1.1690	**0.8787**	0.9231	0.8814	0.8857	1.9599	0.9594	**0.9426**	0.9601	0.9620

Table4: The stimated MSE when n=50 σ =10

φ	0.75					0..85					0.95				
	OLS	ORR	MURRE	LTE	MRTTP	OLS	ORR	MURRE	LTE	MRTTP	OLS	ORR	MURRE	LTE	MRTTP
HK	1.0852	0.9606	**0.9013**	0.9022	0.9541	1.1642	1.1039	**1.0751**	1.0981	1.1021	1.5421	1.1452	0.8826	**0.7776**	1.1237
HKB	1.0852	0.9027	**0.8691**	0.8785	0.8987	1.1642	1.0754	**1.0216**	1.0731	1.0728	1.5421	0.9372	**0.7746**	0.7768	0.9212
LW	1.0852	0.8826	0.8829	**0.8764**	0.8826	1.1642	1.0536	**0.9940**	1.0521	1.0509	1.5421	0.8057	**0.7677**	0.7941	0.8055
GK	1.0852	0.8851	0.8754	**0.8746**	0.8838	1.1642	1.0913	**1.0507**	1.0877	1.0892	1.5421	0.8086	**0.7675**	0.7891	0.8070
A1	1.0852	0.9586	**0.8995**	0.9013	0.9521	1.1642	1.1016	**1.0709**	1.0964	1.0998	1.5421	1.2331	0.9745	**0.7786**	1.2130
A2	1.0852	0.9359	**0.8822**	0.8915	0.9298	1.1642	1.0973	**1.0626**	1.0929	1.0954	1.5421	1.2098	0.9473	**0.7783**	1.1892
A3	1.0852	0.9254	**0.8763**	0.8872	0.9197	1.1642	1.0868	**1.0420**	1.0837	1.0846	1.5421	1.0360	0.8079	**0.7767**	1.0156
A4	1.0852	0.9291	**0.8783**	0.8887	0.9233	1.1642	1.0867	**1.0419**	1.0837	1.0845	1.5421	1.0193	0.8001	**0.7767**	0.9994
A5	1.0852	0.9002	**0.8690**	0.8777	0.8964	1.1642	1.0754	**1.0217**	1.0731	1.0729	1.5421	0.9472	**0.7766**	0.7767	0.9306
AV1	1.0852	**0.9652**	0.9851	0.9661	0.9677	1.1642	1.0004	**0.9995**	1.0004	1.0004	1.5421	0.8522	**0.7930**	0.8544	0.8574
AV2	1.0852	0.9409	**0.8856**	0.8937	0.9347	1.1642	1.1424	1.1314	**1.1193**	1.1409	1.5421	0.8313	**0.7673**	0.7816	0.8253
B1	1.0852	0.8871	0.9030	**0.8859**	0.8891	1.1642	1.0422	**0.9863**	1.0411	1.0397	1.5421	0.8139	**0.7675**	0.7859	0.8109
B2	1.0852	0.8909	**0.8706**	0.8751	0.8883	1.1642	1.0739	**1.0193**	1.0717	1.0713	1.5421	0.8695	**0.7675**	0.7784	0.8589
AB1	1.0852	0.9983	0.9420	**0.9189**	0.9924	1.1642	1.1171	**1.0970**	1.1069	1.1153	1.5421	1.3108	1.0805	**0.7796**	1.2937
AB2	1.0852	**0.9610**	0.9826	0.9620	0.9638	1.1642	1.0582	**0.9985**	1.0566	1.0555	1.5421	0.9795	**0.9596**	0.9800	0.9813
AB3	1.0852	0.9005	**0.8690**	0.8778	0.8967	1.1642	1.0681	**1.0107**	1.0662	1.0655	1.5421	0.8424	**0.7672**	0.7803	0.8349
AB4	1.0852	**0.9027**	0.9288	0.9037	0.9061	1.1642	1.1104	**1.0865**	1.1027	1.1086	1.5421	0.8718	**0.8094**	0.8741	0.8773

Table5: The stimated MSE when n=100 σ =0.1

φ	0.75					0..85					0.95				
	OLS	ORR	MURRE	LTE	MRTTP	OLS	ORR	MURRE	LTE	MRTTP	OLS	ORR	MURRE	LTE	MRTTP
HK	0.2012	0.2012	0.2012	0.2164	0.2012	0.2325	0.2325	0.2325	0.2454	0.2325	0.2854	0.2854	0.2854	0.2937	0.2854
HKB	0.2012	0.2012	0.2013	0.2164	0.2012	0.2325	0.2325	0.2326	0.2454	0.2325	0.2854	0.2854	0.2855	0.2938	0.2854
LW	0.2012	0.2012	0.2012	0.2164	0.2012	0.2325	0.2325	0.2325	0.2454	0.2325	0.2854	0.2854	0.2854	0.2937	0.2854
GK	0.2012	0.2787	0.3423	0.2941	0.2859	0.2325	0.2954	0.3482	0.3089	0.3013	0.2854	0.3275	0.3620	0.3382	0.3317
A1	0.2012	0.2012	0.2012	0.2164	0.2012	0.2325	0.2325	0.2325	0.2454	0.2325	0.2854	0.2854	0.2854	0.2937	0.2854
A2	0.2012	0.2012	0.2013	0.2164	0.2012	0.2325	0.2325	0.2325	0.2454	0.2325	0.2854	0.2854	0.2854	0.2937	0.2854
A3	0.2012	0.2012	0.2013	0.2164	0.2012	0.2325	0.2325	0.2325	0.2454	0.2325	0.2854	0.2854	0.2854	0.2938	0.2854
A4	0.2012	0.2012	0.2012	0.2164	0.2012	0.2325	0.2325	0.2325	0.2454	0.2325	0.2854	0.2854	0.2855	0.2938	0.2854
A5	0.2012	0.2012	0.2013	0.2164	0.2013	0.2325	0.2325	0.2326	0.2454	0.2325	0.2854	0.2854	0.2855	0.2938	0.2854
AV1	0.2012	0.9975	0.9995	0.9976	0.9977	0.2325	0.9710	0.9943	0.9718	0.9735	0.2854	0.9186	0.9747	0.9204	0.9253
AV2	0.2012	0.2030	0.2045	0.2182	0.2032	0.2325	0.2353	0.2370	0.2483	0.2355	0.2854	0.2873	0.2921	0.2966	0.2875
B1	0.2012	0.3335	0.4331	0.3485	0.3447	0.2325	0.3495	0.4407	0.3627	0.3597	0.2854	0.3787	0.4464	0.3892	0.3871
B2	0.2012	0.2250	0.2463	0.2404	0.2273	0.2325	0.2491	0.2631	0.2624	0.2507	0.2854	0.2915	0.3014	0.3015	0.2922
AB1	0.2012	0.2012	0.2012	0.2164	0.2012	0.2325	0.2325	0.2325	0.2454	0.2325	0.2854	0.2854	0.2854	0.2937	0.2854
AB2	0.2012	0.9998	1.0000	0.9998	0.9998	0.2325	0.9998	1.0000	0.9998	0.9998	0.2854	0.9998	1.0000	0.9998	0.9998
AB3	0.2012	0.2162	0.2296	0.2315	0.2176	0.2325	0.2347	0.2360	0.2476	0.2349	0.2854	0.2859	0.2875	0.2946	0.2859
AB4	0.2012	0.7436	0.8923	0.7508	0.7602	0.2325	0.9378	0.9859	0.9394	0.9431	0.2854	0.9678	0.9914	0.9685	0.9706

Table6: The stimated MSE when n=100 σ =1

φ	0.75					0..85					0.95				
	OLS	ORR	MURRE	LTE	MRTTP	OLS	ORR	MURRE	LTE	MRTTP	OLS	ORR	MURRE	LTE	MRTTP
HK	0.3014	0.3011	**0.3002**	0.3005	0.3011	0.3784	0.3759	0.3737	**0.3648**	0.3757	0.5546	0.5250	0.5280	**0.3466**	0.5224
HKB	0.3014	0.3008	**0.2994**	0.3024	0.3008	0.3784	0.3715	0.3683	**0.3651**	0.3711	0.5546	0.4963	0.5013	**0.3453**	0.4918
LW	0.3014	0.3010	**0.2999**	0.3007	0.3009	0.3784	0.3752	0.3726	**0.3647**	0.3749	0.5546	0.5238	0.5270	**0.3466**	0.5211
GK	**0.3014**	0.3485	0.4113	0.3607	0.3548	**0.3784**	0.3958	0.4498	0.4041	0.4006	0.5546	**0.3741**	0.3899	0.3776	0.3771
A1	0.3014	0.3009	**0.2996**	0.3010	0.3008	0.3784	0.3750	0.3722	**0.3647**	0.3747	0.5546	0.5387	0.5404	**0.3473**	0.5372
A2	0.3014	0.3008	**0.2993**	0.3016	0.3007	0.3784	0.3739	0.3707	**0.3647**	0.3736	0.5546	0.5376	0.5395	**0.3473**	0.5360
A3	0.3014	0.3008	**0.2993**	0.3015	0.3008	0.3784	0.3732	0.3697	**0.3648**	0.3728	0.5546	0.5165	0.5202	**0.3462**	0.5132
A4	0.3014	0.3008	**0.2993**	0.3014	0.3008	0.3784	0.3734	0.3700	**0.3648**	0.3730	0.5546	0.5125	0.5165	**0.3460**	0.5090
A5	0.3014	0.3008	**0.2996**	0.3027	0.3009	0.3784	0.3713	0.3682	**0.3652**	0.3709	0.5546	0.4998	0.5046	**0.3455**	0.4955
AV1	**0.3014**	0.9339	0.9907	0.9359	0.9395	**0.3784**	0.8728	0.9800	0.8762	0.8824	0.5546	**0.3700**	0.3837	0.3714	0.3721
AV2	**0.3014**	0.3019	0.3033	0.3060	0.3023	0.3784	0.3696	0.3699	**0.3671**	0.3695	0.5546	0.3662	0.3761	**0.3600**	0.3663
B1	**0.3014**	0.4126	0.5284	0.4257	0.4240	**0.3784**	0.4528	0.5627	0.4633	0.4628	0.5546	**0.3813**	0.4001	0.3868	0.3854
B2	**0.3014**	0.3099	0.3260	0.3180	0.3117	0.3784	**0.3704**	0.3795	0.3718	0.3711	0.5546	0.4042	0.4108	**0.3454**	0.3990
AB1	0.3014	0.3012	0.3006	**0.3003**	0.3012	0.3784	0.3770	0.3756	**0.3649**	0.3768	0.5546	0.5411	0.5427	**0.3475**	0.5399
AB2	**0.3014**	0.9822	0.9984	0.9828	0.9838	**0.3784**	0.9733	0.9991	0.9741	0.9757	**0.5546**	0.9890	0.9931	0.9893	0.9900
AB3	**0.3014**	0.3154	0.3397	0.3247	0.3180	0.3784	0.3699	0.3688	**0.3663**	0.3696	0.5546	0.5170	0.5207	**0.3462**	0.5138
AB4	**0.3014**	0.6033	0.7887	0.6138	0.6212	**0.3784**	0.7889	0.9432	0.7944	0.8027	**0.5546**	0.9673	0.9792	0.9680	0.9702

Table7: The stimated MSE when n=100 σ =5

φ	0.75					0..85					0.95				
	OLS	ORR	MURRE	LTE	MRTTP	OLS	ORR	MURRE	LTE	MRTTP	OLS	ORR	MURRE	LTE	MRTTP
HK	0.7371	0.6866	0.6725	**0.6682**	0.6848	1.0864	0.9956	0.9772	**0.9126**	0.9897	1.1528	0.9599	0.8315	**0.7617**	0.9489
HKB	0.7371	0.6786	0.6781	**0.6727**	0.6786	1.0864	0.9344	0.9105	**0.8918**	0.9290	1.1528	0.8555	0.7610	**0.7571**	0.8463
LW	0.7371	0.6794	0.6746	**0.6706**	0.6788	1.0864	0.8939	**0.8634**	0.8827	0.8926	1.1528	0.7825	**0.7540**	0.7615	0.7803
GK	0.7371	**0.6915**	0.7140	0.6945	0.6949	1.0864	0.8941	**0.8638**	0.8826	0.8927	1.1528	0.7771	**0.7594**	0.7671	0.7769
A1	0.7371	0.6911	0.6748	**0.6683**	0.6891	1.0864	1.0049	0.9879	**0.9159**	0.9994	1.1528	1.0114	0.8916	**0.7645**	1.0017
A2	0.7371	0.6845	0.6721	**0.6684**	0.6830	1.0864	0.9907	0.9718	**0.9109**	0.9848	1.1528	1.0009	0.8778	**0.7639**	0.9908
A3	0.7371	0.6819	0.6723	**0.6690**	0.6807	1.0864	0.9628	0.9410	**0.9012**	0.9567	1.1528	0.9103	0.7897	**0.7592**	0.8994
A4	0.7371	0.6828	0.6721	**0.6687**	0.6815	1.0864	0.9634	0.9416	**0.9014**	0.9572	1.1528	0.9007	0.7833	**0.7588**	0.8899
A5	0.7371	0.6785	0.6795	**0.6735**	0.6787	1.0864	0.9340	0.9100	**0.8917**	0.9286	1.1528	0.8616	0.7633	**0.7573**	0.8520
AV1	**0.7371**	0.8583	0.9127	0.8621	0.8666	1.0864	0.9417	**0.8873**	0.9429	0.9449	1.1528	0.9539	**0.9485**	0.9549	0.9574
AV2	0.7371	0.6817	0.6724	**0.6690**	0.6806	1.0864	0.9350	0.9112	**0.8920**	0.9296	1.1528	0.8279	**0.7535**	0.7567	0.8203
B1	**0.7371**	0.7611	0.8129	0.7666	0.7691	1.0864	0.8940	**0.8511**	0.8925	0.8956	1.1528	0.7776	**0.7643**	0.7726	0.7786
B2	0.7371	0.6804	0.6892	**0.6792**	0.6817	1.0864	0.9114	0.8855	**0.8851**	0.9075	1.1528	0.8130	**0.7514**	0.7570	0.8067
AB1	0.7371	0.6988	0.6810	**0.6692**	0.6967	1.0864	1.0313	1.0186	**0.9252**	1.0270	1.1528	1.0429	0.9371	**0.7663**	1.0345
AB2	**0.7371**	0.9106	0.9526	0.9130	0.9169	1.0864	0.9799	**0.9531**	0.9804	0.9815	1.1528	0.9786	**0.9758**	0.9791	0.9804
AB3	0.7371	0.6789	0.6761	**0.6715**	0.6786	1.0864	0.9457	0.9226	**0.8955**	0.9399	1.1528	0.7838	**0.7535**	0.7609	0.7813
AB4	**0.7371**	0.8114	0.8689	0.8162	0.8202	1.0864	0.9405	**0.8858**	0.9418	0.9438	1.1528	0.8419	**0.8324**	0.8447	0.8478

Table8: The stimated MSE when n=100 σ =10

φ	0.75					0..85					0.95				
	OLS	ORR	MURRE	LTE	MRTTP	OLS	ORR	MURRE	LTE	MRTTP	OLS	ORR	MURRE	LTE	MRTTP
HK	1.0852	0.9606	**0.9013**	0.9022	0.9541	1.1642	1.1039	**1.0751**	1.0981	1.1021	1.5421	1.1452	0.8826	**0.7776**	1.1237
HKB	1.0852	0.9027	**0.8691**	0.8785	0.8987	1.1642	1.0754	**1.0216**	1.0731	1.0728	1.5421	0.9372	**0.7746**	0.7768	0.9212
LW	1.0852	0.8826	0.8829	**0.8764**	0.8826	1.1642	1.0536	**0.9940**	1.0521	1.0509	1.5421	0.8057	**0.7677**	0.7941	0.8055
GK	1.0852	0.8851	0.8754	**0.8746**	0.8838	1.1642	1.0913	**1.0507**	1.0877	1.0892	1.5421	0.8086	**0.7675**	0.7891	0.8070
A1	1.0852	0.9586	**0.8995**	0.9013	0.9521	1.1642	1.1016	**1.0709**	1.0964	1.0998	1.5421	1.2331	0.9745	**0.7786**	1.2130
A2	1.0852	0.9359	**0.8822**	0.8915	0.9298	1.1642	1.0973	**1.0626**	1.0929	1.0954	1.5421	1.2098	0.9473	**0.7783**	1.1892
A3	1.0852	0.9254	**0.8763**	0.8872	0.9197	1.1642	1.0868	**1.0420**	1.0837	1.0846	1.5421	1.0360	0.8079	**0.7767**	1.0156
A4	1.0852	0.9291	**0.8783**	0.8887	0.9233	1.1642	1.0867	**1.0419**	1.0837	1.0845	1.5421	1.0193	0.8001	**0.7767**	0.9994
A5	1.0852	0.9002	**0.8690**	0.8777	0.8964	1.1642	1.0754	**1.0217**	1.0731	1.0729	1.5421	0.9472	**0.7766**	0.7767	0.9306
AV1	1.0852	**0.9652**	0.9851	0.9661	0.9677	1.1642	1.0004	**0.9995**	1.0004	1.0004	1.5421	0.8522	**0.7930**	0.8544	0.8574
AV2	1.0852	0.9409	**0.8856**	0.8937	0.9347	1.1642	1.1424	1.1314	**1.1193**	1.1409	1.5421	0.8313	**0.7673**	0.7816	0.8253
B1	1.0852	0.8871	0.9030	**0.8859**	0.8891	1.1642	1.0422	**0.9863**	1.0411	1.0397	1.5421	0.8139	**0.7675**	0.7859	0.8109
B2	1.0852	0.8909	**0.8706**	0.8751	0.8883	1.1642	1.0739	**1.0193**	1.0717	1.0713	1.5421	0.8695	**0.7675**	0.7784	0.8589
AB1	1.0852	0.9983	0.9420	**0.9189**	0.9924	1.1642	1.1171	**1.0970**	1.1069	1.1153	1.5421	1.3108	1.0805	**0.7796**	1.2937
AB2	1.0852	**0.9610**	0.9826	0.9620	0.9638	1.1642	1.0582	**0.9985**	1.0566	1.0555	1.5421	0.9795	**0.9596**	0.9800	0.9813
AB3	1.0852	0.9005	**0.8690**	0.8778	0.8967	1.1642	1.0681	**1.0107**	1.0662	1.0655	1.5421	0.8424	**0.7672**	0.7803	0.8349
AB4	1.0852	**0.9027**	0.9288	0.9037	0.9061	1.1642	1.1104	**1.0865**	1.1027	1.1086	1.5421	0.8718	**0.8094**	0.8741	0.8773

Table9: The stimated MSE when n=150 σ =0.1

φ	0.75					0..85					0.95				
	OLS	ORR	MURRE	LTE	MRTTP	OLS	ORR	MURRE	LTE	MRTTP	OLS	ORR	MURRE	LTE	MRTTP
HK	0.2048	0.2048	0.2048	0.2195	0.2048	0.2531	0.2531	0.2531	0.2652	0.2531	0.2881	0.2881	0.2881	0.2976	0.2881
HKB	0.2048	0.2048	0.2048	0.2195	0.2048	0.2531	0.2531	0.2531	0.2653	0.2531	0.2881	0.2881	0.2881	0.2976	0.2881
LW	0.2048	0.2048	0.2048	0.2195	0.2048	0.2531	0.2531	0.2531	0.2652	0.2531	0.2881	0.2881	0.2881	0.2976	0.2881
GK	0.2048	0.2799	0.3419	0.2950	0.2868	0.2531	0.3080	0.3526	0.3203	0.3131	0.2881	0.3300	0.3659	0.3407	0.3342
A1	0.2048	0.2048	0.2048	0.2195	0.2048	0.2531	0.2531	0.2531	0.2652	0.2531	0.2881	0.2881	0.2881	0.2976	0.2881
A2	0.2048	0.2048	0.2048	0.2195	0.2048	0.2531	0.2531	0.2531	0.2652	0.2531	0.2881	0.2881	0.2881	0.2976	0.2881
A3	0.2048	0.2048	0.2048	0.2195	0.2048	0.2531	0.2531	0.2531	0.2653	0.2531	0.2881	0.2881	0.2881	0.2976	0.2881
A4	0.2048	0.2048	0.2048	0.2195	0.2048	0.2531	0.2531	0.2531	0.2653	0.2531	0.2881	0.2881	0.2881	0.2976	0.2881
A5	0.2048	0.2048	0.2048	0.2195	0.2048	0.2531	0.2531	0.2531	0.2653	0.2531	0.2881	0.2881	0.2881	0.2976	0.2881
AV1	0.2048	0.9978	0.9996	0.9979	0.9980	0.2531	0.9995	0.9999	0.9995	0.9996	0.2881	0.9771	0.9956	0.9776	0.9791
AV2	0.2048	0.2062	0.2072	0.2210	0.2063	0.2531	0.2538	0.2546	0.2660	0.2539	0.2881	0.2893	0.2898	0.2992	0.2894
B1	0.2048	0.3348	0.4337	0.3496	0.3459	0.2531	0.3617	0.4420	0.3737	0.3711	0.2881	0.3813	0.4542	0.3916	0.3895
B2	0.2048	0.2283	0.2490	0.2434	0.2306	0.2531	0.2658	0.2785	0.2782	0.2671	0.2881	0.2932	0.2985	0.3036	0.2938
AB1	0.2048	0.2048	0.2048	0.2195	0.2048	0.2531	0.2531	0.2531	0.2652	0.2531	0.2881	0.2881	0.2881	0.2976	0.2881
AB2	0.2048	0.9999	1.0000	0.9999	0.9999	0.2531	0.9999	1.0000	0.9999	0.9999	0.2881	0.9999	1.0000	0.9999	0.9999
AB3	0.2048	0.2101	0.2145	0.2250	0.2107	0.2531	0.2571	0.2613	0.2694	0.2575	0.2881	0.2885	0.2883	0.2982	0.2886
AB4	0.2048	0.8827	0.9659	0.8861	0.8920	0.2531	0.8839	0.9638	0.8867	0.8930	0.2881	0.9710	0.9943	0.9716	0.9735

Table10: The stimated MSE when n=150 σ =1

φ	0.75					0..85					0.95				
	OLS	ORR	MURRE	LTE	MRTTP	OLS	ORR	MURRE	LTE	MRTTP	OLS	ORR	MURRE	LTE	MRTTP
HK	**0.2981**	0.2982	0.2984	0.3007	0.2982	0.3400	0.3391	0.3380	**0.3294**	0.3391	0.3662	0.3627	0.3589	**0.3458**	0.3624
HKB	**0.2981**	0.2986	0.3000	0.3026	0.2987	0.3400	0.3369	0.3338	**0.3299**	0.3367	0.3662	0.3558	0.3492	**0.3476**	0.3552
LW	**0.2981**	0.2982	0.2985	0.3010	0.2982	0.3400	0.3390	0.3377	**0.3294**	0.3389	0.3662	0.3625	0.3585	**0.3458**	0.3621
GK	**0.2981**	0.3533	0.4108	0.3663	0.3600	**0.3400**	0.3613	0.4008	0.3703	0.3657	**0.3662**	0.3771	0.4148	0.3864	0.3810
A1	**0.2981**	0.2982	0.2988	0.3013	0.2983	0.3400	0.3389	0.3376	**0.3294**	0.3388	0.3662	0.3628	0.3590	**0.3457**	0.3625
A2	**0.2981**	0.2984	0.2994	0.3020	0.2985	0.3400	0.3387	0.3371	**0.3294**	0.3386	0.3662	0.3625	0.3586	**0.3458**	0.3622
A3	**0.2981**	0.2983	0.2992	0.3018	0.2984	0.3400	0.3379	0.3355	**0.3296**	0.3377	0.3662	0.3587	0.3525	**0.3466**	0.3582
A4	**0.2981**	0.2983	0.2990	0.3016	0.2983	0.3400	0.3379	0.3355	**0.3296**	0.3377	0.3662	0.3581	0.3517	**0.3468**	0.3576
A5	**0.2981**	0.2987	0.3004	0.3029	0.2988	0.3400	0.3370	0.3339	**0.3299**	0.3367	0.3662	0.3562	0.3496	**0.3474**	0.3557
AV1	**0.2981**	0.8007	0.9183	0.8066	0.8144	**0.3400**	0.9867	0.9977	0.9870	0.9879	**0.3662**	0.6895	0.8572	0.6956	0.7055
AV2	**0.2981**	0.3029	0.3115	0.3103	0.3038	0.3400	0.3353	0.3319	**0.3309**	0.3351	0.3662	**0.3518**	0.3570	0.3559	0.3523
B1	**0.2981**	0.4239	0.5259	0.4374	0.4359	**0.3400**	0.4140	0.4952	0.4245	0.4231	**0.3662**	0.4294	0.5108	0.4389	0.4375
B2	**0.2981**	0.3106	0.3290	0.3202	0.3126	0.3400	**0.3350**	0.3386	0.3370	0.3355	0.3662	**0.3512**	0.3530	0.3536	0.3514
AB1	**0.2981**	0.2981	0.2983	0.3005	0.2981	0.3400	0.3396	0.3390	**0.3293**	0.3395	0.3662	0.3646	0.3627	**0.3455**	0.3644
AB2	**0.2981**	0.9865	0.9968	0.9869	0.9877	**0.3400**	0.9900	0.9983	0.9903	0.9909	**0.3662**	0.9893	0.9992	0.9895	0.9902
AB3	**0.2981**	0.4039	0.4950	0.4175	0.4147	0.3400	**0.3344**	0.3353	0.3350	0.3346	0.3662	0.3512	0.3513	0.3525	**0.3512**
AB4	**0.2981**	0.2982	0.2984	0.3007	0.2982	**0.3400**	0.7478	0.8910	0.7535	0.7631	**0.3662**	0.7858	0.9304	0.7901	0.7995

Table11: The stimated MSE when n=150 σ =5

φ	0.75					0..85					0.95				
	OLS	ORR	MURRE	LTE	MRTTP	OLS	ORR	MURRE	LTE	MRTTP	OLS	ORR	MURRE	LTE	MRTTP
HK	0.7362	0.7123	0.7052	**0.6911**	0.7109	0.9760	0.9177	0.8772	**0.8343**	0.9134	1.0468	0.9360	0.9154	**0.8560**	0.9305
HKB	0.7362	0.6980	0.6976	**0.6921**	0.6978	0.9760	0.8658	0.8216	**0.8173**	0.8608	1.0468	0.8761	0.8631	**0.8564**	0.8737
LW	0.7362	0.6996	0.6966	**0.6906**	0.6989	0.9760	0.8346	**0.8069**	0.8092	0.8312	1.0468	0.8693	**0.8499**	0.8699	0.8706
GK	0.7362	**0.7081**	0.7218	0.7108	0.7110	0.9760	0.8191	0.8155	**0.8100**	0.8185	1.0468	0.8655	**0.8506**	0.8640	0.8660
A1	0.7362	0.7080	0.7012	**0.6903**	0.7065	0.9760	0.9236	0.8856	**0.8363**	0.9197	1.0468	0.9440	0.9232	**0.8562**	0.9385
A2	0.7362	0.7027	0.6975	**0.6900**	0.7015	0.9760	0.9138	0.8720	**0.8329**	0.9093	1.0468	0.9395	0.9188	**0.8561**	0.9340
A3	0.7362	0.7015	0.6970	**0.6901**	0.7005	0.9760	0.8906	0.8442	**0.8252**	0.8856	1.0468	0.8939	0.8777	**0.8555**	0.8899
A4	0.7362	0.7024	0.6974	**0.6900**	0.7013	0.9760	0.8908	0.8443	**0.8252**	0.8857	1.0468	0.8896	0.8741	**0.8555**	0.8859
A5	0.7362	0.6978	0.6983	**0.6927**	0.6977	0.9760	0.8657	0.8215	**0.8173**	0.8607	1.0468	0.8781	0.8648	**0.8562**	0.8755
AV1	**0.7362**	0.9911	0.9946	0.9914	0.9919	0.9760	**0.9696**	0.9886	0.9703	0.9720	1.0468	0.9665	**0.9381**	0.9672	0.9689
AV2	0.7362	0.7114	0.7043	**0.6909**	0.7100	0.9760	0.8729	0.8272	**0.8195**	0.8678	1.0468	0.8799	0.8662	**0.8560**	0.8771
B1	**0.7362**	0.7928	0.8272	0.7980	0.8007	0.9760	**0.8300**	0.8539	0.8306	0.8331	1.0468	0.8788	**0.8520**	0.8806	0.8811
B2	0.7362	0.6993	0.7055	**0.6986**	0.7004	0.9760	0.8373	**0.8073**	0.8097	0.8336	1.0468	0.8691	**0.8571**	0.8578	0.8678
AB1	0.7362	0.7200	0.7138	**0.6931**	0.7188	0.9760	0.9426	0.9150	**0.8429**	0.9398	1.0468	0.9816	0.9636	**0.8576**	0.9769
AB2	**0.7362**	0.9504	0.9678	0.9518	0.9544	**0.9760**	0.9778	0.9922	0.9783	0.9796	1.0468	0.9875	**0.9749**	0.9877	0.9885
AB3	**0.7362**	0.7885	0.8225	0.7937	0.7963	0.9760	0.8678	0.8231	**0.8179**	0.8627	1.0468	0.8773	0.8641	**0.8563**	0.8748
AB4	0.7362	0.6999	0.7069	**0.6996**	0.7012	0.9760	**0.9172**	0.9554	0.9191	0.9221	1.0468	0.9353	**0.8950**	0.9366	0.9388

Table12: The stimated MSE when n=150 σ =10

φ	0.75					0..85					0.95				
	OLS	ORR	MURRE	LTE	MRTTP	OLS	ORR	MURRE	LTE	MRTTP	OLS	ORR	MURRE	LTE	MRTTP
HK	0.9565	0.9290	0.9047	**0.9013**	0.9270	1.0658	1.0270	1.0027	**0.9578**	1.0239	1.1218	1.0051	0.8920	**0.7736**	0.9967
HKB	0.9565	0.8995	**0.8746**	0.8870	0.8976	1.0658	0.9664	**0.9311**	0.9315	0.9620	1.1218	0.8878	0.7841	**0.7730**	0.8786
LW	0.9565	0.8910	**0.8751**	0.8846	0.8902	1.0658	0.9305	**0.9049**	0.9185	0.9286	1.1218	0.8061	**0.7700**	0.7785	0.8031
GK	0.9565	0.8892	**0.8801**	0.8858	0.8892	1.0658	0.9279	**0.9036**	0.9181	0.9264	1.1218	0.7956	**0.7747**	0.7858	0.7954
A1	0.9565	0.9167	**0.8884**	0.8950	0.9144	1.0658	1.0127	0.9828	**0.9515**	1.0089	1.1218	1.0255	0.9228	**0.7739**	1.0182
A2	0.9565	0.9057	**0.8782**	0.8897	0.9035	1.0658	1.0004	0.9672	**0.9461**	0.9961	1.1218	1.0190	0.9125	**0.7738**	1.0113
A3	0.9565	0.9085	**0.8803**	0.8910	0.9062	1.0658	0.9872	0.9520	**0.9404**	0.9827	1.1218	0.9398	0.8182	**0.7729**	0.9299
A4	0.9565	0.9112	**0.8828**	0.8923	0.9089	1.0658	0.9890	0.9540	**0.9411**	0.9845	1.1218	0.9295	0.8098	**0.7729**	0.9197
A5	0.9565	0.8972	**0.8739**	0.8861	0.8955	1.0658	0.9648	**0.9296**	0.9309	0.9604	1.1218	0.8947	0.7875	**0.7729**	0.8853
AV1	**0.9565**	0.9918	0.9943	0.9920	0.9925	1.0658	0.9994	**0.9989**	0.9994	0.9994	1.1218	0.9480	**0.9314**	0.9491	0.9518
AV2	0.9565	0.9232	**0.8964**	0.8983	0.9209	1.0658	1.0185	0.9907	**0.9540**	1.0149	1.1218	0.8420	**0.7704**	0.7741	0.8352
B1	0.9565	**0.9201**	0.9311	0.9219	0.9235	1.0658	0.9287	**0.9083**	0.9284	0.9303	1.1218	0.7969	**0.7799**	0.7935	0.7982
B2	0.9565	0.8917	**0.8746**	0.8846	0.8907	1.0658	0.9446	**0.9135**	0.9229	0.9412	1.1218	0.8359	**0.7696**	0.7745	0.8295
AB1	0.9565	0.9376	0.9187	**0.9059**	0.9360	1.0658	1.0430	1.0271	**0.9650**	1.0410	1.1218	1.0622	0.9877	**0.7744**	1.0571
AB2	**0.9565**	0.9785	0.9846	0.9791	0.9802	1.0658	0.9910	**0.9849**	0.9913	0.9918	1.1218	0.9880	**0.9834**	0.9883	0.9891
AB3	0.9565	0.8930	**0.8739**	0.8848	0.8917	1.0658	0.9368	**0.9084**	0.9202	0.9341	1.1218	0.8989	0.7897	**0.7729**	0.8894
AB4	0.9565	**0.9149**	0.9252	0.9166	0.9182	1.0658	0.9374	**0.9163**	0.9381	0.9396	1.1218	0.9373	**0.9185**	0.9387	0.9417