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ABSTRACT

In this research, we presented a new Style of sets named semi-feebly open sets that were studied and identified some of its properties and found relationships with other sets where we obtained some results that show the relationship between sets through theories that were obtained using the set from Style (sf-open).

1. Introduction

The topological idea from study this set is generalization the properties and using it's to prove the theorems. In [7] N. Leven (1963) gives the definition of semi open(s-open)set, semi closed (s-closed) set and studies the properties of it . He defined a set A named (s-open) set in topological space Ӿ if find an open set : $O \subseteq A \subseteq \overline{O}$ where $\overline{O}$ denoted by the closure of O in Ӿ, the complement semi-open (s-open) set called semi-closed (s-closed) set. In (1971) S. G. Crossiey and S. K. Hildebrand defined the concept semi closure and they defined it , the semi closure of a set A in topological space Ӿ is the smallest semi-closed (s-closed) set contained A [2] and shortened by Scl(A) or $\overline{A}^{s}$. The truth $\overline{A}^{s}$ is the intersection of all semi closed sets contained A, $\overline{A}^{s} \subseteq \overline{A}$ and $\overline{\overline{A}^{s}}^{s} = \overline{A}^{s}$. Maheswari and Tapi (1978) in [3] defined feebly closed (f-closed), feebly open (f-open) set. A set A in a topological space Ӿ named feebly open (f-open) set in Ӿ if find an open set V such that $V \subseteq A \subseteq \overline{V}^{s}$. A set A in a topological space Ӿ is feebly closed

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if it is complement is feebly open. Every open is (f- open)set, but the converse may be not true. Every closed is (f- closed) set, but the converse may be not true.

We will use the(tp-s)symbol to denote the topological space, (s-open) to semi open set,(s-closed) to semi closed set,(f-open) to feebly open set and (f-closed) to feebly closed set . wherever it is found in this paper.

2. Preliminaries

Definition(2.1)[7]: Assume that $(\mathfrak{X}, \mathfrak{t})$ is a tp-s & $A \subseteq \mathfrak{X}$. Then A is named s-open in $\mathfrak{X}$ if there exists $O \in \mathfrak{t} : O \subseteq A \subseteq \overline{O}$. Or equivalent [5], A called s-open in $\mathfrak{X} \Leftrightarrow A \subseteq \overline{A^{\circ}}$, equivalent $\overline{A} = \overline{A^{\circ}}$, the complement of s-open is named s-closed.

Definition(2.2)[7]: Let $(\mathfrak{X}, \mathfrak{t})$ be tp-s & $A \subseteq \mathfrak{X}$ then A called s-closed in $\mathfrak{X}$ if there exists a closed set F such that $F^{\circ} \subseteq A \subseteq F$, or equivalent[5], A is s-closed in $\mathfrak{X} \Leftrightarrow \overline{A}^{\circ} \subseteq A$, equivalent $A^{\circ} = \overline{A}^{\circ}$.

Definition(2.3)[5]: Let $(\mathfrak{X}, \mathfrak{t})$be tp-s& $A \subseteq \mathfrak{X}$, then the intersection of all s-closed subset of $\mathfrak{X}$ contained A named (s-closure) of A and the union of all s-open subset of $\mathfrak{X}$ contained A named (s-interior) of A and are shortened by $\overline{A}^{s}$, $A^{\circ s}$ respectively.

Proposition (2.4)[7]: Let $\{A_{\lambda}\}_{\lambda \in \Lambda}$ be a family of s-open in a tp-s $\mathfrak{X}$ then $\bigcup_{\lambda \in \Lambda} A_{\lambda}$ is s-open .

Proposition (2.5)[7]: Let $\mathfrak{X}$ be a tp-s then the intersection of two s-open in $\mathfrak{X}$ does not need to be s-open.

Example (2.6): Let $\mathfrak{X} = \{k, v, h\}, \mathfrak{t} = \{\{k\}, \{v\}, \{k, v\}, \mathfrak{X}, \emptyset\}$ then each of $\{k, h\}, \{v, h\}$ are s-open, but $\{k, h\} \cap \{v, h\} = \{h\}$ not s-open.

Definition(2.7)[4]: The intersection of every semi open subset of tp-s $\mathfrak{X}$ contained a set A is named Semi kernel of A and shortened by (S ker(A)).

Means that : S ker (A) = ∩ {U : U s-open and A ⊆ U}.

Definition(2.8)[8]: A set A in a tp-s $\mathfrak{X}$ called f-open in $\mathfrak{X}$ if there exists an open set V such that $V \subseteq A \subseteq \overline{V}^{s}$, or equivalent, A set A called f-open in $\mathfrak{X}$ if and only if $A \subseteq \overline{A}^{\circ}$, the complement of f-open is called f-closed that $\overline{A^{\circ}} \subseteq A$.

Remark(2.9)[6]: Let $(\mathfrak{X}, \mathfrak{t})$be tp-s & $A \subseteq \mathfrak{t}$ then A is f-open and A^{c} is f- closed .

But the converse is not true in general as in the next example.

Example (2.10): Assume that $\mathfrak{X} = \{1,2,3,4,5\}$ and $\mathfrak{t} = \{\emptyset, X, \{1\}, \{2,4\}, \{1,2,4\}\}$ then, f-open sets are $\{\emptyset, X, \{1\}, \{2,4\}, \{1,2,4\}, \{1,2,3,4\}\}$, f-closed sets are $\{\emptyset, X, \{2,3\}, \{1,3,5\}, \{3,5\}, \{5\}\}$.

Take $A = \{1,2,3,4\}$ is f-open , but it not open set & $A^{c} = \{5\}$ f-closed , but it is not closed.

Proposition (2.11)[9]: Assume that $(\mathfrak{X}, \mathfrak{t})$is a tp-s & A, B subsets of $\mathfrak{X}$ then :

1. $\overline{A}^{f}$ f-closed .
2. $A \subseteq \overline{A}^{f}$.
3. A is (f-closed) $\Leftrightarrow$ $A = \overline{A}^{f}$.
4. $A \subseteq B \implies \overline{A}^{f} \subseteq \overline{B}^{f}$.
5. If $\{A_\lambda\}_{\lambda\in\Lambda}$ be a collection of subset of Ꭓ then $\overline{\cup_{\lambda\in\Lambda} A_\lambda}^{f} = \cup_{\lambda\in\Lambda} \overline{A_\lambda}^{f}$.
6. If $\{A_\lambda\}_{\lambda\in\Lambda}$ be a collection of subset of Ꭓ then $\overline{\cap_{\lambda\in\Lambda} A_\lambda}^{f} \subseteq \cap_{\lambda\in\Lambda} \overline{A_\lambda}^{f}$.
7. $\overline{\overline{A}^{f}}^{f} = \overline{A}^{f}$.
8. $\overline{A}^{f} \subseteq \overline{A}$
9. $\overline{\overline{A}}^{f} = \overline{\overline{A}^{f}} = \overline{A}$.
10. $\overline{A}^{f} = A \cup A'^{f}$.
11. $\overline{A}^{f} = A \cup \overline{\overline{A}^{\circ}}$.
12. $x \in \overline{A}^{f}$ $\Leftrightarrow$ any f-open G contained x, $A \cap G \neq \emptyset$.

Proposition(2.12)[9]: Let Ꭓ be a tp-s & A, B subset of Ꭓ where B f-open, If $x \in B$ and $A \cap B = \emptyset$ then $x \notin \overline{A}^{f}$.

Definition(2.13)[10]: Let Ꭓ be a tp-s a subset A of Ꭓ is said to be

i. Dense or (every dense) in Ꭓ $\Leftrightarrow$ $\overline{A} =$ Ꭓ.

ii. Nowhere dense or (non-dense) in Ꭓ iff $(\overline{A})^{\circ} = \emptyset$.

Definition(2.14)[5]: Let (Ꭓ, ţ)be a tp-s and $A \subseteq$ Ꭓ, A is named preopen (p-open) if $A \subseteq \overline{A}^{\circ}$ and A^c is named pre closed (p-closed) that $\overline{A^{\circ}} \subseteq A$.

Lemma(2.15)[4]: Every singleton $\{x\}$ in a tp-s Ꭓ is either nowhere dense or preopen.

3. The Main Results

Definition(3.1): Assume that (Ꭓ, ţ) is a tp-s then a subset A in a space Ꭓ is named semi feebly open (sf-open) set in a space Ꭓ if $A \subseteq U$ where U semi open set in Ꭓ then $\overline{A}^{f} \subseteq U$. The complement of semi feebly open is called semi feebly closed (sf-closed) it is as follows $U \subseteq A^{\circ f}$ where U semi closed set in Ꭓ.

Example (3.2): Let $X = \{k, v, h\}$, $\tau = \{X, \emptyset, \{k\}\}$ then

open set : $\{X, \emptyset, \{k\}\}$, closed set : $\{\emptyset, X, \{v, h\}\}$

s-open : $\{\emptyset, X, \{k\}, , \{k, v\}, \{k, h\}\}$, s-closed : $\{\emptyset, X, \{v, h\}, \{h\}, \{v\}\}$

f-open : $\{\emptyset, X, \{k\}, , \{k, v\}, \{k, h\}\}$, f-closed : $\{\emptyset, X, \{v, h\}, \{h\}, \{v\}\}$

sf-open : $\{\emptyset, X, \{v\}, \{h\}, \{v, h\}\}$

we notes that $\{\{k\}, \{k, v\}, \{k, h\}\}$ not sf-open because $\{k\} \subseteq \{k\}$ where $\{k\}$ s-open, but $\overline{\{k\}}^f = X \not\subset \{k\}$, $\{k\}$ is not sf-open.

$\{k, v\} \subseteq \{k, v\}$ where $\{k, v\}$ s-open, but $\overline{\{k, v\}}^f = X \not\subset \{k, v\}$, $\{k, v\}$ is not sf-open.

$\{k, h\} \subseteq \{k, h\}$ where $\{k, h\}$ s-open, but $\overline{\{k, h\}}^f = X \not\subset \{k, h\}$, $\{k, h\}$ is not sf-open.

Remark(3.3): Each f-closed is sf-open.

Proof : let A be f-closed set in a tp-s X. $A \subseteq U$, U s-open, A is (f-closed) set then $A = \overline{A}^f$ and $A = \overline{A}^f \subseteq U \Rightarrow A$ is (sf-open) set.

The converse of (Remark(3-3)) is not true in general, as in the next example shows:

Exampl(3.4): Let $X = \{1,2,3,4,5\}$, $\tau = \{X, \emptyset, \{1\}, \{3,4\}, \{1,3,4\}, \{2,3,4,5\}\}$ and $A = \{1,2,4,5\}$ then A is sf-open not f-closd.

Proof: The open sets are $\{X, \emptyset, \{1\}, \{3,4\}, \{1,3,4\}, \{2,3,4,5\}\}$,

the closed sets are $\{\emptyset, X, \{2,3,4,5\}, \{1,2,5\}, \{2,5\}, \{1\}\}$ and

s-open sets are $\{X, \emptyset, \{1\}, \{3,4\}, \{1,3,4\}, \{2,3,4,5\}, \{2,3,4\}, \{3,4,5\}, \{1,2,3,4\}, \{1,3,4,5\}\}$

Let $A = \{1,2,4,5\}$ and $U = X$ which is s-open, then $A \subseteq U$

$$\overline{A}^f = \{1,2,4,5\} \cup \overline{\overline{\{1,2,4,5\}}}^{\circ} \quad \text{[Proposition (2.11)(11)]}$$

$\overline{A}^f = \{1,2,4,5\} \cup X = X \subseteq U = X \quad \Rightarrow A$ sf –open set, but A not f-closed because $\overline{\overline{\{1,2,4,5\}}}^{\circ} = X \not\subset \{1,2,4,5\}$.

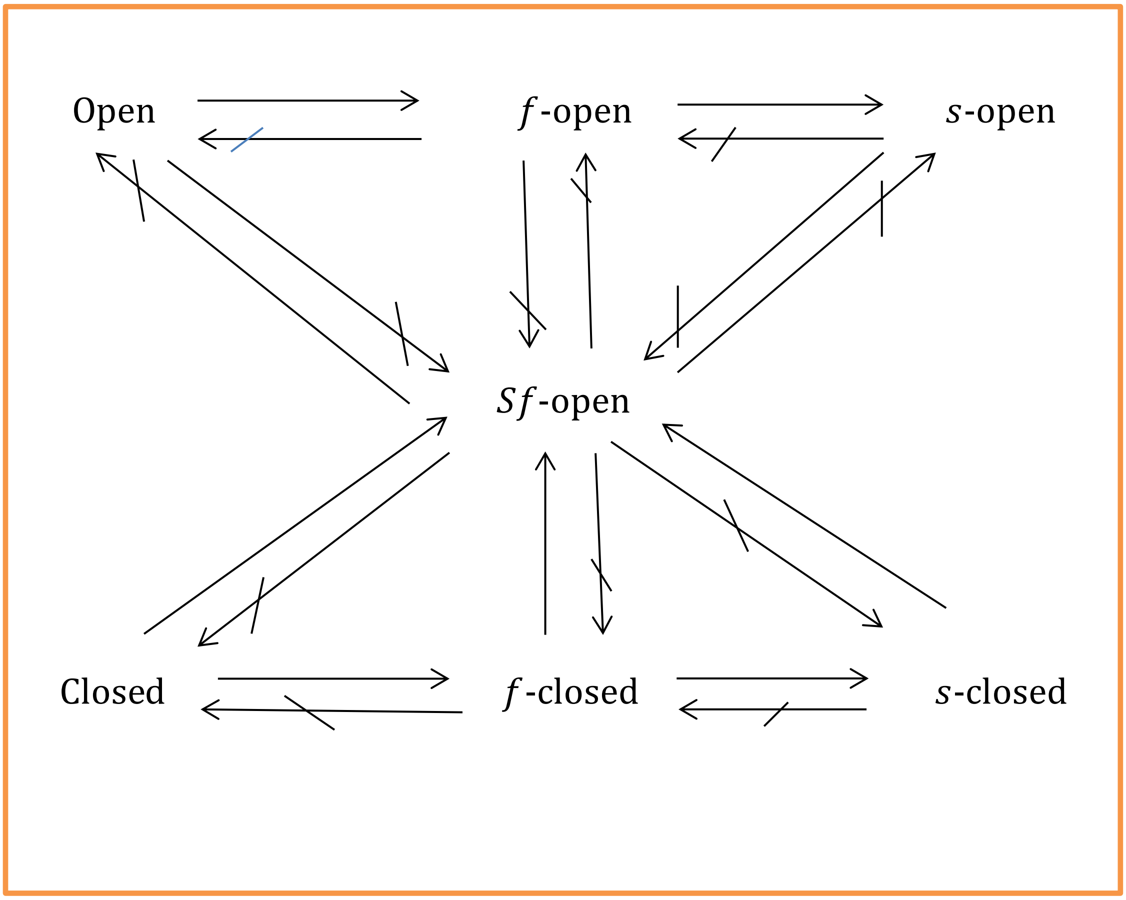
Remark(3.5): Every closed set is sf-open set.

Proof: Let A closed set $\Rightarrow A$ is f-closed by [Remark (1.15)] A is sf-open set.

But the converse of (Remark(3-5)) in general is not true as in our [Example (3.4)] shows :

$A = \{1,2,4,5\}$ sf-open, $\overline{A} = X \neq A \Rightarrow \overline{A} \neq A$ then A is not closed.

The next diagram explains the relationship these types of sets.



Notes(3.6)

For each tp-s $\emptyset$, Ӿ are sf-open.

Every subset of discreet or indiscreet tp-s is sf-open.

Every closed interval in (R, U) where U is usual topology is sf-open.

Proposition(3.7): Let Ӿ be tp-s then the union of all sf-open sets in Ӿ is also sf- open set.

Proof: Let $\bigcup_{\lambda\in\Lambda} A_\lambda \subseteq U, U$ semi-open in a topological space Ӿ then

$A_\lambda \subseteq \bigcup_{\lambda\in\Lambda} A_\lambda \subseteq U \Rightarrow A_\lambda \subseteq U \quad \Rightarrow \overline{A_\lambda}^f \subseteq U \Rightarrow \bigcup_{\lambda\in\Lambda} \overline{A_\lambda}^f \subseteq U$

since $\{A_\lambda\}_{\lambda\in\Lambda}$ be a collection of all subset of Ӿ then $\overline{\bigcup_{\lambda\in\Lambda} A_\lambda}^f = \bigcup_{\lambda\in\Lambda} \overline{A_\lambda}^f$

$\Rightarrow \overline{\bigcup_{\lambda\in\Lambda} A_\lambda}^f = \bigcup_{\lambda\in\Lambda} \overline{A_\lambda}^f \subseteq U \quad \Rightarrow \quad \overline{\bigcup_{\lambda\in\Lambda} A_\lambda}^f \subseteq U$ is $\bigcup_{\lambda\in\Lambda} A_\lambda$ sf-Open.

Proposition(3.8): Let A be nowhere dense in a tp-s Ӿ then A is f-closed.

Proof: Assume that A is a subset of Ӽ so that A is nowhere dense then $(\overline{A})^{\circ} = \emptyset$ and $\overline{(\overline{A})^{\circ}} = \emptyset$, but $\emptyset \subseteq A \Rightarrow A$ is f-closed set.

Lemma(3.9): A subset A of a tp-s Ӽ is sf-open iff $\overline{A}^{f} \subseteq Sker\ (A)$.

Proof:($\Rightarrow$) Let A be sf-open in Ӽ, then $\overline{A}^{f} \subseteq U$ when $A \subseteq U$ and U is sf-open in Ӽ, this emplace $\overline{A}^{f} \subseteq \cap$ {U : A ⊆ U and U∈ s-open(X)} = $S\ ker\ (A)$. ($\Longleftarrow$)Conversely,

assume that $\overline{A}^{f} \subseteq S\ ker\ (A) \implies \overline{A}^{f} \subseteq \cap$ {U : A ⊆ U and U∈ s-open(X)}$\Rightarrow \overline{A}^{f} \subseteq U$ for all s-open set U in Ӽ.

Proposition(3.10): Let Ӽ be tp-s , then the arbitrary intersection of sf-open sets in Ӽ is sf-open set.

Proof: Let $\{A_{\lambda}: \lambda \epsilon\ \Lambda\}$ be arbitrary collection of sf-open sets in a space Ӽ, and let $A = \cap A_{\lambda_{\lambda \in \Lambda}}$, let x∈ $\overline{A}^{f}$ then by [Lemma(2.15)] we consider the following two cases.

Case 1: $\{x\}$ is nowhere dense

If $x \notin A$ then for some $\alpha \epsilon\ \Lambda$ we have $x \notin A$, as nowhere dense subset are feebly closed [Proposition(3.8)] there fore $x \notin S\ ker\ (A)$.On the other hand by [Lemma(3.9)] A_{α} is sf-open, then, $x \in \overline{A}^{f} \subseteq \overline{A_{\alpha}}^{f} \subseteq S\ ker\ (A)$

paradoxically $x \in A$ and hence $x \in S\ ker\ (A) \implies \overline{A}^{f} \subseteq S\ ker\ (A), A\ sf$-open set.

Case 2: $\{x\}$ is preopen, Let $F = \overline{\{x\}}^{\circ}$ and $x \notin S\ ker\ (A)$, ∃ semi closed set C containing X, so that $C \cap A = \emptyset$, $x \in F = \overline{\{x\}}^{\circ} \subseteq \overline{C}^{\circ} \subseteq C$.As F is an open set containing x and $x \in \overline{A}^{f}$ therefore, $F \cap A \neq \emptyset$ as $F \subseteq C \implies C \cap A = \emptyset$ paradoxically $x \epsilon\ S\ ker\ (A) \Rightarrow A\ sf$-open set.

Proposition(3.11): If A is sf-open and B f-closed in a tp-s Ӽ then $A \cap B$ is sf-open in Ӽ .

Proof: Assume that $A \cap B \subseteq U$ where U is s-open set then $A \cap B \cap U^{c} = \emptyset$

$\implies A \cap (B \cap U^{c}) = \emptyset \implies A \subseteq B^{c} \cup U$,but $B^{c} \cup U$ s-open $\overline{A}^{f} \subseteq B^{c} \cup U$

$\implies \overline{A}^{f} \cap (B^{c} \cup U)^{c} = \emptyset \implies \overline{A}^{f} \cap B \subseteq U \implies \overline{A \cap B}^{f} \subseteq U \implies A \cap B$ sf-open.

Proposition(3.12): Assume that Ӽ is a tp-s & $A \subseteq X$ then $\overline{A}^{f}$ is sf-open set.

Proof: Let $\overline{A}^{f} \subseteq G$ where G is s-open set, since $\overline{\overline{A}^{f}}^{f} = \overline{A}^{f} \subseteq U$

$\Rightarrow \overline{\overline{A}^{f}}^{f} \subseteq G \Rightarrow \overline{A}^{f}$ sf-open set.

Proposition(3.13): Assume that Ӽ is a tp-s& $A \subseteq X$ then $\overline{A}$ is sf-open set.

Proof: Let $\overline{A} \subseteq U$ where U is s-open set, since $\overline{\overline{A}}^{f} = \overline{\overline{\overline{A}}^{f}} = \overline{A}$ [Proposition(2.11)(9)]. Then $\overline{\overline{A}}^{f} = \overline{A} \Rightarrow \overline{\overline{A}}^{f} \subseteq U$ where U is s-open $\Rightarrow \overline{A}$ is sf-open set.

Proposition(3.14): Assume that Ꭓ is an tp-s & $A \subseteq$ Ꭓ, if A is s-closed and pre closed then A is sf-open set.

Proof: Let A is s-closed then $A^{\circ} = \overline{A}^{\circ}$, since A pre closed then $\overline{A^{\circ}} \subseteq A$, but $A^{\circ} = \overline{A}^{\circ}$ then $\overline{\overline{A}^{\circ}} \subseteq A$ $\Rightarrow A$ f-closed by using [Remark(3.3)] A is sf-open set.

Definition(3.15): Assume that Ꭓ is a tp-s & $A \subseteq$ Ꭓ .Then the intersection of all sf-closed of X which containing A is named sf-closure of A and shortened by $\overline{A}^{sf}$, that means $\overline{A}^{sf} = \cap \{F: F \text{ is } sf\text{-closed in } X\}$.

Lemma(3.16): Assume that Ꭓ is a tp-s & $A \subseteq X$. Then $x \in \overline{A}^{sf}$ iff for all sf-open set G and $x \in G$, $G \cap A \neq \emptyset$.

Proof:($\Rightarrow$) Assume that $x \notin \overline{A}^{sf}$ then $x \notin \cap \{F: F \text{ is } sf\text{-closed in } Ꭓ\}$ and $A \subseteq F$, then $x \in [\cap F]^c$, $[\cap F]^c$ sf-open containing x .Hence $[\cap F]^c \cap A \subseteq [\cap F]^c \cap [\cap F] = \emptyset$.($\Leftarrow$) Conversely, Suppose that $\exists$ sf-open set G so that x $\in G$, $G \cap A = \emptyset$ then $A \subseteq G^c$, G^c is sf-closed hence $x \notin \overline{A}^{sf}$.

Definition(3.17): Let Ꭓ be a tp-s , $x \in$ Ꭓ & $A \in$ Ꭓ .The point x is called sf-limit point of A if each sf-open set containing U, contains a point of A distinct from x .We shall call the set of all sf-limit point of A the sf- derivative set of A and denoted by A'^{sf}. Therefore $x \in A'^{sf}$ if for every sf-open set U in Ꭓ such $x \in V$ implies that $\cap(A - \{x\}) \neq \emptyset$.

Proposition(3.18): Let Ꭓ be a tp-s and $A \subseteq B \subseteq$ Ꭓ. Then:

1. $\overline{A}^{sf} = A \cup A'^{sf}$.
2. A is an sf-closed set iff $A'^{sf} \subseteq A$.
3. $A'^{sf} \subseteq B'^{sf}$.

Proof: 1- By definition $A \subseteq \overline{A}^{sf}$ ......(1) . Let $x \in A'^{sf} \Rightarrow x \notin A$. Then $\forall$ sf-open set U contained x ,then $(U \cap A) - \{x\} \neq \emptyset$. Then $\forall$ sf-open set in U contained x ,then $U \cap A \neq \emptyset$ by[Lemma(3.16)]. Then $x \in \overline{A}^{sf} \Rightarrow A'^{sf} \subseteq \overline{A}^{sf}$ ......(2).From (1) and (2) $A \cup A'^{sf} \subseteq \overline{A}^{sf}$.

Let $x \in \overline{A}^{sf}$. Since $A \subseteq \overline{A}^{sf}$ by definition and $\because x \in \overline{A}^{sf}$ Then either $x \in A$ or $x \notin A$. If $x \in A \Rightarrow x \in A \cup A'^{sf}$ and if $x \notin A$. Since $x \in \overline{A}^{sf} \Rightarrow \forall$ sf-open set U contained x ,then $U \cap A \neq \emptyset$,Since $x \notin A$ then $(U \cap A) - \{x\} \neq \emptyset$. Then $x \in A'^{sf} \Rightarrow x \in A \cup A'^{sf}$ then $\overline{A}^{sf} \subseteq A \cup A'^{sf}$ then $\overline{A}^{sf} = A \cup A'^{sf}$.

2- ($\Rightarrow$) Let $A'^{sf} \subseteq A$. $\overline{A}^{sf} = A \cup A'^{sf} \subseteq A$, Since $A \subseteq \overline{A}^{sf}$ *then* $A = \overline{A}^{sf}$, then A is an sf-closed set .

($\Leftarrow$) Let A be sf-closed set . Thus $A = \overline{A}^{sf}$ from [proposition (3.18)(1)]. $A = A \cup A'^{sf}$ then $A'^{sf} \subseteq A$.

3- Let $A \subseteq B$ and let $x \in A'^{sf}$, $\forall\, U$ is sf-open set contained x then $(U \cap A) - \{x\} \neq \emptyset$ .Since $A \subseteq B \Rightarrow (U \cap B) - \{x\} \neq \emptyset$. Then $x \in B'^{sf}$ *then* $A'^{sf} \subseteq B'^{sf}$.

Remark(3.19): Assume that Ӿ is a tp-s & $A \subseteq$ Ӿ , then $\overline{A}^{sf}$ is smallest sf-closed set containing A.

proof : Suppose that B is sf-closed set contend such that $A \subseteq B$ since $\overline{A}^{sf} = A \cup A'^{sf}$. And $\overline{A}^{sf} \subseteq \overline{B}^{sf}$, $A \subseteq B$,then $\overline{A}^{sf} = A \cup A'^{sf} \subseteq A \cup A'^{sf} \subseteq B$, then $\overline{A}^{sf} \subseteq B$ therefore $\overline{A}^{sf}$ is smallest sf-closed set contained A.

Proposition(3.20): Let Ӿ be a tp-s & A, B are subset of Ӿ with B sf-open set . If $x \in B$ and $B \cap A = \emptyset$ then $x \notin \overline{A}^{sf}$.

proof : Suppose $x \in \overline{A}^{sf}$, then either $x \in A$ or $x \in A'^{sf}$. If $x \in A$,then $B \cap A \neq \emptyset$ which contradicts the assumption and if $x \in A'^{sf}$ and $x \notin A$, then then $(B \cap A) - \{x\} \neq \emptyset$ for every sf-open G in Ӿ containing x and hence $G \cap A \neq \emptyset$ which is a contradiction since B is sf –open set containing x and $B \cap A = \emptyset$ and hence $x \notin \overline{A}^{sf}$.

Definition(3.21): Assume that Ӿ is a tp-s& $B \subseteq X$ .An sf-neighborhood of B is any subset of Ӿ which contains an sf-open set containing B .The sf-neighborhood of a subset $\{x\}$ is also called sf-neighborhood of the point x.

Definition(3.22): Assume that A is a subset of a tp-s Ӿ . For each $x \in$ Ӿ, then x is said to be sf-boundary point of A if each sf-neighborhood U_x of x, we have $U_x \cap A \neq \emptyset$ and $U_x \cap A^c \neq \emptyset$. The set of all sf-boundary point of A is denoted by $b_{sf}(A)$.

Proposition(3.23): Assume that Ӿ is a tp-s and $A, B \subseteq$ Ӿ, then

1. A is an sf-closed set $\Leftrightarrow A = \overline{A}^{sf}$.
2. $\overline{A}^{sf} \subseteq \overline{A}$.
3. $\overline{A}^{sf} = \overline{\overline{A}^{sf}}^{sf}$.
4. If $A \subseteq B$ then $\overline{A}^{sf} \subseteq \overline{B}^{sf}$.

proof :1- ($\Rightarrow$) Let A is an sf-closed set. Since A $\subseteq \overline{\mathrm{A}}^{sf}$. Then $\overline{A}^{sf} \subseteq A$ (since $\overline{A}^{sf}$ is the smallest sf-closed set containing A), then $A = \overline{A}^{sf}$.

($\Leftarrow$) Let $\overline{A}^{sf} = A$. Then $\overline{A}^{sf}$ is an sf-closed set. as $A = \overline{A}^{sf} \Longrightarrow A$ is a sf-closed set.

2- Let $x \in \overline{A}^{sf}$ and A is a sf-closed set, then $A = \overline{A}^{sf} \Rightarrow x \in A \subseteq \overline{A}$. Then $x \in \overline{A}$. Therefor $\overline{A}^{sf} \subseteq \overline{A}$.

3- Since $\overline{A}^{sf}$ is sf-closed set, then $\overline{A}^{sf} = \overline{\overline{A}^{sf}}^{sf}$ by (2) .

4- Let $A \subseteq B$ and $B \subseteq \overline{B}^{sf}$, then $A \subseteq \overline{B}^{sf} \Rightarrow \overline{B}^{sf}$ is a sf-closed set containing A. Since $\overline{A}^{sf}$ is smallest sf-closed set containing A. Then $\overline{A}^{sf} \subseteq \overline{B}^{sf}$.

Definition(3.24): Assume that Ӽ is tp-s and $A \subseteq$ Ӽ. The union of all sf-open sets of Ӽ contained in A is named sf-Interior of A, shortened by $A^{\circ sf}$ or $sf\text{-}In_{\tau}(A)$, that means $sf\text{-}In_{\tau}(A) = \cup\{B\text{:}B$ is sf-open in X and $B \subseteq A\}$.

Proposition(3.25): Assume that Ӽ is tp-s & $A \subseteq X$. Then $\overline{A}^{sf} = \left(A^{c^{\circ sf}}\right)^{c}$.

Proof: Since $A \subseteq \overline{A}^{sf} \Rightarrow \overline{A}^{sf^{c}} \subseteq A^{c} \Rightarrow \overline{A}^{sf^{c^{\circ sf}}} \subseteq A^{c^{\circ sf}} \Rightarrow \overline{A}^{sf^{c}} \subseteq A^{c^{\circ sf}} \Rightarrow A^{c^{\circ sf^{c}}} \subseteq \overline{A}^{sf}$ … … . (1). Since $A^{c^{\circ sf}} \subseteq A^{c} \Rightarrow A \subseteq A^{c^{\circ sf^{c}}} \Rightarrow \overline{A}^{sf} \subseteq \overline{A^{c^{\circ sf^{c}}}}^{sf} = A^{c^{\circ sf^{c}}}$ … … . . (2). From (1) and (2) we get $\overline{A}^{sf} = \left(A^{c^{\circ sf}}\right)^{c}$.

Proposition(3.26): Assume that Ӽ is tp-s & $A \subseteq$ Ӽ. Then $x \in A^{\circ sf}$ iff there is an sf-open set U containing x so that $x \in U \subseteq A$.

Proof: Assume that $x \in A^{\circ sf} \Leftrightarrow x \in \cup\{U\text{:}\, U \subseteq A \textit{ such that } U \textit{ is } sf - open \textit{ in } X\} \Leftrightarrow \exists\, U$ *is* sf-*open* in X so that $x \in U \subseteq A$.

Proposition(3.27): Assume that Ӽ tp-s & $A \subseteq B \subseteq$ Ӽ, then:

1. $A^{\circ sf}$ is an sf-open set .
2. A is an sf-open set iff $A = A^{\circ sf}$.
3. $A^{\circ sf} = A^{\circ sf^{\circ sf}}$.
4. If $A \subseteq B$ then $A^{\circ sf} \subseteq B^{\circ sf}$.

Proof: 1- $A^{\circ sf} = \cup\{B\text{:}\, B$ is sf-open and $B \subseteq A\}$,by [proposition(3. 7)]. Then $A^{\circ sf}$ is an sf-open set.

2- ($\Rightarrow$) Let A be an sf-open set from definition $A^{\circ sf} \subseteq A$, $A^{\circ sf} = \cup\,\{U\text{:}\, U \subseteq A\,, U \textit{ is an } sf - \textit{open set in } X\}$. Since A is sf-open set in X. Then $A \subseteq A^{\circ sf} \Rightarrow A = A^{\circ sf}$.

($\Leftarrow$) Let $A = A^{\circ sf}$, since $A^{\circ sf}$ is the union sf-open sets and since $A^{\circ sf} = A \Rightarrow A$ is a sf-open set.

3- Let $A^{\circ sf}$ $=\cup\{B: B$ is an sf-open set in X and $B\subseteq A\}$. $A^{\circ sf^{\circ sf}}$ $=\cup\{B: B$ is an sf-open set in X and $B\subseteq A\}$ By (2) $A = A^{\circ sf}$. Then $A^{\circ sf} = A^{\circ sf^{\circ sf}}$.

4- Let $A \subseteq B$ & $x \in A^{\circ sf}$. Then $\exists$ sf-open U in X such that $x \in U\subseteq A$. Since $A \subseteq B$. Then $\exists$ sf-open U in X such that $x \in U\subseteq A \subseteq B$. $x \in U\subseteq B \Rightarrow x \in B^{\circ sf}$. Then $A^{\circ sf}\subseteq B^{\circ sf}$.

<u>Proposition(3.28):</u> Assume that Ӽ is a tp-s & $A \subseteq$ Ӽ.Then:

1. $b_{sf}(A) = \overline{A}^{sf} \cap \overline{A^c}^{sf}$.
2. $A^{\circ sf} = A - b_{sf}(A)$.
3. $\overline{A}^{sf} = A \cup b_{sf}(A)$.

Proof: Clear

<u>Proposition(3.29):</u> Assume that Ӽ is a tp-s & $A \subseteq$ Ӽ.Then:

1- $\overline{A}^{sf} = A^{\circ sf} \cup b_{sf}(A)$.

2- A is an sf-open set$\Leftrightarrow$ $b_{sf}(A) \subseteq A^c$.

3- $(A)^{\circ sf} = \left(\overline{A^c}^{sf}\right)^c$.

Proof: Clear

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