Al-Qadisiyah Journal of Pure Science

Volume 25 | Number 3 Article 4

7-7-2020

Some Properties of Fuzzy Soft Metric Space

Jehad Rmadhan Kider
Department of Mathematics and Computer Applications, School of Applied Sciences, University of Technology, Baghdad, Iraq, 10046@uoteghnology.edu.iq

Al-Qadisiyah Journal of Pure Science Vol.(25) Issue (3) (2020) pp. Math. 46-61

Al-Qadisiyah Journal of Pure Science

ISSN(Printed): 1997-2490 ISSN(Online): 2411-3514

DOI: /10.29350/jops.

http://qu.edu.iq/journalsc/index.php/JOPS

Some Properties of Fuzzy Soft Metric Space

Authors Names

a. jehad Rmadhan Kider

Article History

Received on: 16/5/2020
Revised on: 10 /6/2020
Accepted on: 14/6/2020

Keywords:

Fuzzy soft metric space, Fuzzy soft bounded set, Fuzzy soft convergence of sequences, Fuzzy soft continuous

DOI: https://doi.org/10.29350/jops.2020.25. 3.1149

ABSTRACT

In this paper first we introduce the notion fuzzy soft metric space after that we define open fuzzy soft ball, fuzzy soft bounded set, fuzzy soft convergence of sequences, fuzzy soft continuous function from a fuzzy soft metric space to another fuzzy soft metric space. The main goal of the present paper is to study this space and investigated some basic properties of the fuzzy soft metric space..

MSC: 30C45, 30C50

1. Introduction

Molodtsov [1] in 1999 introduced the concept of a soft set and develop the basics corresponding theory which is in that time a new approach for modeling uncertainties also introduced many directions for the applications of soft set. Majlet al.[2] in 2001used the notions of fuzzy set and soft set to introduced the notion fuzzy soft set. Ahmad and Kharal [3] in 2009 introduced some properties of fuzzy soft sets. In 2011 [4] Varol, B. P. ,Shostak Varol and Aygun introduced the structure of fuzzy soft topology. In [5] Varol and Aygun introduced basic properties of of fuzzy soft topology.

[a] Department of Mathematics and Computer Applications, School of Applied Sciences, University of Technology, Baghdad, Iraq.
Email.10046@uoteghnology.edu.iq

The fuzzy topological structure of a fuzzy normed space was studied by Sadeqi and Kia in 2009 [6]. Kider introduced a fuzzy normed space in 2011 [7]. Also he proved this fuzzy normed space has a completion in [8]. Again Kider introduced a new fuzzy normed space in 2012 [9]. The properties of fuzzy continuous mapping which is defined on a fuzzy normed spaces was studied by Nadaban in 2015 [10].

Kider and Kadhum in 2017 [11] introduce the fuzzy norm for a fuzzy bounded operator on a fuzzy normed space and proved its basic properties then other properties was proved by Kadhum in 2017 [12]. Ali in 2018 [13] proved basic properties of complete fuzzy normed algebra. Kider and Ali in 2018 [14] introduce the notion of fuzzy absolute value and study properties of finite dimensional fuzzy normed space.

The concept of general fuzzy normed space were presented by Kider and Gheeab in 2019 [15] [16] also they proved basic properties of this space and the general fuzzy normed space GFB(V, U). Kider and Kadhum in 2019 [17] introduce the notion fuzzy compact linear operator and proved its basic properties.

In the present research we introduce the structure of fuzzy soft metric space which very different from the notion of fuzzy soft metric space that introduced by the authors in [18] and [19] after that we introduce preliminaries then we proved basic properties of this space.

2. Properties of Fuzzy Soft Set

Definition (2.1):[6]

A fuzzy set D on a universe set S is a mapping D:S→[0, 1] . The value D(s) represent the degree of membership of s ∈S in the fuzzy set, for all s ∈S. Let I=[0, 1] and I^S denotes the family of all fuzzy set on S.

Definition (2.2):[5]

A pair (f, A) is called a fuzzy soft set over S, where f :A→ I^S, that is for each a ∈ A, f(a)= f_a ∈ I^S.Put f_A={f_a: a ∈ A}.

Definition (2.3):[5]

A fuzzy soft set f_A on the universe set S is a mapping from the parameter set E to I^S, that is f_A:E $\to I^S$, where $f_A(d) \neq 0_X$ if d ∈A⊆E and $f_A(d) = 0_X$ if d ∉A, where 0_X is the empty fuzzy set on S. Let F(S, E) denotes the family of all fuzzy soft sets over S.

Definition (2.4):[5]

Let $f_A \in F(S, E)$ and let $g_B \in F(U, K)$ then the fuzzy product $f_A \times g_B$ is defined by $(f \times g)_{A\times B}$ where $(f \times g)_{A\times B}(a,b) = f_A(a) \times g_B(b) \in I^{S\times U}$, for all (a, b) ∈ A×B. Also for all (x, y) ∈ S×U where $f_A(a) \times g_B(b)](x, y) = f_A(a)(x) \wedge g_B(b)(y)$. According to this definition the fuzzy soft set $f_A \times g_B$ is a fuzzy soft set over S×Uand its parameter universe is E×K.

Definition (2.5) : [20]

A binary operation ⊛:I ×I→I is a continuous **triangular norm** (or simply **t-norm**) if for all a, b, c, e∈ I have the following conditions

(1)a⊛b = b⊛a (commutative)

(2)a⊛1= a

(3) [a ⊛b] ⊛c=a⊛[b⊛c]

(4)If a ≤c and b≤e then a⊛b ≤ c⊛e

Examples (2.6) : [20]

(1)Define a⊛b=ab for all a. b∈ I where ab usual multiplication in I. Then ⊛ is continuous t-norm

(2)Define a ⊛b=a∧b for all a, b∈ I then ⊛ is continuous t-norm

Remark (2.7):[20]

(1)For any a, b ∈I with a>b we can find c ∈I such that a⊛c ≥b.

(2)We can find q ∈I such that q⊛q ≥d for some d ∈ I.

Theorem (2.8):[20]

If ⊛ is a continuous t-norm then

(1)1⊛1=1

(2)0⊛1=0=1⊛0

(3)0⊛0=0

(4)a⊛a ≤a for all a ∈I.

3. Properties of Fuzzy Soft Metric Space

First we define a fuzzy soft metric space

Definition (3.1):

Let S be a universe set and let F(S, E) be the family of all soft fuzzy set over S. Let $f_A, g_B \in$ F(S, E) and put $h_{A\times B}$= $(f \times g)_{A\times B}$= $f_A \times g_B$. Let ⊛be a continuous t-norm then $h_{A\times B}$ is a fuzzy soft metric on S×S if for all a ∈A, b ∈B

(S1) $h_{A\times B}$(a, b)[s, u] >0 for all s, u ∈S.

(S2) $h_{A\times B}$(a, b)[s, u]=1 ⟺ s=u

(S3) $h_{A\times B}$(a, b)[s, u] = $h_{A\times B}$(a, b)[u, s] for all s, u ∈S.

(S4) $h_{A\times B}$(a, b)[s, w] ≥ $h_{A\times B}$(a, b)[s, u] ⊛ $h_{A\times B}$(a,b)[u, w] for all s, u, w ∈S.

(S5) $h_{A\times B}$(a, b)[s, u] is a continuous function for all s, u ∈S.

Then the triple (S, $h_{A\times B}$(a, b), ⊛) is called a fuzzy soft metric space. In particular (S, h_{A^2}(a, b), ⊛) is a fuzzy soft metric space.

Proposition (3.2):

Let (S, d) be a metric space and let $\alpha \circledast \beta=\alpha\beta$ for all $\alpha, \beta \in$ I. Put $h_{A\times B}$(a, b)[s, u]= $\frac{1}{\exp[d(s,u)]}$ then (X, $h_{A\times B}$(a, b), ⊛) is a fuzzy soft metric space for all a ∈A, b ∈B.

Proof:

(S1) It is clear that $h_{A\times B}$(a, b)[s, u] >0 for all s, u ∈S.

(S2) $h_{A\times B}$(a, b)[s, u]=1 ⟺ $\frac{1}{\exp[d(s,u)]}$=1⟺exp[d(s, u)]=1⟺d(s, u)=0 ⟺ s=u

(S3) $h_{A\times B}$(a, b)[s, u] = $\frac{1}{\exp[d(s,u)]}$= $\frac{1}{\exp[d(u,s)]}$ = $h_{A\times B}$(a, b)[u, s] for all s, u ∈S.

(S4)d(s, w) ≤ d(s, u) + d(u, w) or exp[d(s, w)] ≤ exp[d(s, u)].exp[d(u, w)] then

$h_{A\times B}$(a, b)[s, w]= $\frac{1}{\exp[d(s,w)]} \geq \frac{1}{\exp[d(s,u)]} \circledast \frac{1}{\exp[d(u,w)]}$

=$h_{A\times B}$(a, b)[s, u] ⊛ $h_{A\times B}$(a,b)[u, w]

for all s, u, w ∈S

(S5) It is clear that $h_{A\times B}$(a, b)[s, u] is a continuous function for all s, u ∈S.

Hence (S, $h_{A\times B}$(a, b), ⊛) is a fuzzy soft metric space

The proof of the next result is similar to example 3.2 and hence is omitted

Example (3.3):

Let S=ℝ and let $\alpha \circledast \beta=\alpha\beta$ for all α, β ∈I. Put $h_{A\times B}(a, b)[x, y]=\frac{1}{\exp[|x-y|]}$ then (X, $h_{A\times B}$(a, b), ⊛) is a fuzzy soft metric space for all a ∈A, b ∈B.

Definition (3.4):

Let (S, $h_{A\times B}$(a, b), ⊛) be a fuzzy soft metric space for all a ∈A, b ∈B. We define the **soft fuzzy open ball** SB(s, r) with center s ∈S and radius r∈J=(0, 1) by: SB(s, r)= {u ∈S : $h_{A\times B}$(a, b)[u, s] >(1−r) }. Also the **soft fuzzy closed ball** is defined by: SB[s, r]= {u ∈S : $h_{A\times B}$(a, b)[u, s] ≥(1−r)}.

The proof of the next result is clear and hence is deleted.

Proposition (3.5):

Let (S, $h_{A\times B}$(a, b), ⊛) be a fuzzy soft metric space and let SB(x, r) and SB(x, q) where s ∈ S and r, q ∈J=(0, 1). Then either SB(s, r) ⊆SB(s, q) or SB(s, q) ⊆SB(s, r)

Definition (3.6):

A subset D of a fuzzy soft metric space (S, $h_{A\times B}$(a, b), ⊛) is said to be **fuzzy soft open** if for any d ∈ D there exists r∈J such that SB(d, r) ⊆ A. Also a subset G ⊆ S is said to be **fuzzy soft closed** if G^C is a fuzzy soft open.

Theorem (3.7):

In a fuzzy soft metric space every soft fuzzy open ball is a fuzzy soft open set

Proof:

Let (S, $h_{A\times B}$(a, b), ⊛) be a fuzzy soft metric space and let SB(s, r) be a soft fuzzy open ball let y∈ SB(s, r) then $h_{A\times B}(a, b)[s, y] > (1-r)$. Since $h_{A\times B}(a, b)[s, y] > (1-r)$. Put $h_{A\times B}(a, b)[s, y]=(1-q)$.

Now for (1−q) and (1−r) we can find $(1-p)$ for some p ∈ J such that

$(1-q) > (1-p) > (1-r)$ we can find t with t ∈ J such that $(1-q) \circledast (1-t) \geq (1-p)$ by Remark 2.3. We will show SB(y, t) is a subset of SB(s, r). Let w ∈ SB(y, t) so

$h_{A\times B}(a, b)[y, w] > (1-t)$.

Now $h_{A\times B}(a, b)[s, w] \geq h_{A\times B}(a, b)[s, y] \circledast h_{A\times B}(a, b)[y, w]$

$$\geq (1-q) \circledast (1-t) \geq (1-p) > (1-r)$$

So w∈SB(s, r). Hence SB(y, t) ⊆ SB(s, r). Thus SB(s, r) is a fuzzy soft open set.

Proposition (3.8):

Every fuzzy soft metric space is a fuzzy topological space.

Proof:

Let $(S, h_{A\times B}(a, b), \circledast)$ be a fuzzy soft metric space, put τ_h= {D ⊆S:d ∈D ⟺ there is 0< r <1 such that SB(d, r) ⊆D}. We will prove that τ_h is a fuzzy topology.

1-since S and ϕ are fuzzy soft open set so S, ϕ belongs to τ_h

2-Let D, G ∈ τ_h ,put E = D ∩ G. Let e ∈ D ∩ G then e ∈ D and e ∈ G so there is r_1 ∈ Jand 0< r_2 <1 such that SB(e, r_1) ⊆ D and SB(e, r_2) ⊆ G. Let $r = r_1 \wedge r_2$, $r \leq r_1$ implies $(1-r) \geq (1-r_1)$, also $r \leq r_2$ implies $(1-r) \geq (1-r_2)$, therefore SB(e, r) ⊆ D ∩ G = E Hence D ∩ G = E ∈ τ_h.

3-Let $G_1, G_2, \dots, G_J \in \tau_h$ put G=$\cup_{j\in J} G_j$ suppose that d ∈ G this implies that d ∈ G_j for some j ∈ J then there is r∈I such that SB(d, r) ⊆ G_j ⊆ G=$\cup_{j\in J} G_j$. This shows that G ∈ τ_h. Hence (S, τ_h) is a fuzzy topological space.

Theorem (3.9):

Every fuzzy soft metric space is Hausdorff space.

Proof:

Let $(S, h_{A\times B}(a, b), \circledast)$ be a fuzzy soft metric space and let s, u ∈S with s≠u. Then $0 < h_{A\times B}(a,b)[s,u] < 1$, put $h_{A\times B}(a,b)[s,u] = (1-r)$ for some 0< r <1 then by Remark 2.7, for each $(1-r) < (1-q) < 1$ we can find 0< $(1-\varepsilon)$ <1 such that $(1-\varepsilon) \circledast (1-\varepsilon) \geq (1-q)$.

Now consider SB(s, ε) and SB(u, ε) , we claim that SB(s, ε) ∩ SB(u, ε) =ϕ if there is w ∈ SB(s, ε) ∩ SB(u, ε) then

$$(1-r)=h_{A\times B}(a,b)[s,u] \geq h_{A\times B}(a,b)[s,w] \circledast h_{A\times B}(a,b)[w,u]$$

$$\geq (1-\varepsilon) \circledast (1-\varepsilon) \geq (1-q) > (1-r).$$

Which is impossible. Hence $(S, h_{A\times B}(a, b), \circledast)$ is a Hausdorff space.

Definition (3.10):

Let $(S, h_{A\times B}(a, b), \circledast)$ be a fuzzy soft metric space. A subset D of S is said to be **fuzzy soft bounded** if there exists 0< $(1-r)$ <1 such that $h_{A\times B}(a,b)[s,u] > (1-r)$ for each s, u ∈D.

Definition (3.11) :

A sequence (s_n) in a fuzzy soft metric space (S, $h_{A\times B}$(a,b), ⊛) is said to be **fuzzy soft converges to** s ∈ S if for each $\varepsilon > 0$ there exists N such that $h_{A\times B}(a,b)[s_n, s] > (1-\varepsilon)$ for each $n \geq N$. This is written $\lim_{n\to\infty} s_n = s$ or simply written $s_n \to^s s$. s is called the **fuzzy soft limit** of (s_n). Or a sequence (s_n) in a fuzzy soft metric space (X, $h_{A\times B}$(a, b), ⊛) is **fuzzy soft converges to** s∈S if and only if $\lim_{n\to\infty} h_{A\times B}(a,b)[s_n, s]=1$.

Definition (3.12) :

A sequence (s_n) in a fuzzy soft metric space (S, $h_{A\times B}$(a, b), ⊛) is said to be **fuzzy soft Cauchy sequence** if for each $0 < (1-r) < 1$ there exists a positive number N such that $h_{A\times B}(a,b)[s_n, s_m] > (1-r)$ for all m, n ≥N.

The proof of the following lemma is clear hence is omitted.

Lemma (3.13):

Every fuzzy soft convergent sequence in a fuzzy soft metric space is a fuzzy soft Cauchy sequence.

Definition (3.6):

Let (S, $h_{A\times B}$(a, b), ⊛) be a fuzzy soft metric space and let D ⊆ S. Then the **fuzzy soft closure of D** is denoted by FSC[D] and is defined to be the smallest fuzzy soft closed set contains D.

Definition (3.7):

A subset D of a fuzzy soft metric space (S, $h_{A\times B}$(a, b), ⊛) is said to be **fuzzy soft dense** in S if FSC[D]=S.

Lemma (3.8):

Let (S, $h_{A\times B}$(a, b), ⊛) be a fuzzy soft metric space and let D ⊆ S then d ∈ FSC[D] if and only if there is a sequence (d_n) in D such that $d_n \to^s d$.

Proof:

Let d ∈ FSC[D] if d ∈ D then we take a sequence of the type (d, d, … , d, …) if d ∉ D then it is a fuzzy soft limit point of D. Hence we construct the sequence (d_n) ∈ D by $h_{A\times B}(a,b)[d_n, d] > \left(1-\frac{1}{n}\right)$ for each n=1, 2, … then the open fuzzy soft ball SB$\left(d, \frac{1}{n}\right)$ contains d_n which is fuzzy soft converges to d because $\lim_{n\to\infty} h_{A\times B}(a,b)[d_n, d] = 1$.

Conversely if (d_n) in D and $d_n \to^s d$. Then $d \in D$ or every open fuzzy soft ball of d contains $d_n \neq d$. That is d is a fuzzy soft limit point of D hence $d \in FSC[D]$.

The proof of the following result is easy hence is deleted.

Proposition (3.9):

Let (s_n) be a sequences in a fuzzy soft metric space $(S, h_{A\times B}(a, b), \circledast)$ fuzzy soft converges to $s \in S$then every subsequence (s_{n_k}) of (s_n) fuzzy soft converges to s.

Proposition (3.10):

If a Cauchy sequence (s_n) in a fuzzy soft metric space $(S, h_{A\times B}(a, b), \circledast)$ contains a fuzzy soft convergent subsequence (s_{n_k}) then (s_n) fuzzy soft converges to the same fuzzy soft limit as the subsequence converges.

Proof:

Since (s_n) is a fuzzy soft Cauchy then given $0< \varepsilon < 1$ there is a positive number N such that $h_{A\times B}(a,b)[s_j, s_m] > (1-\varepsilon)$ for all $m, j \geq N$. Denote by (s_{n_j}) a fuzzy soft convergent subsequence of (s_n) and its fuzzy soft limit by s. It follows that $\lim_{n\to\infty} h_{A\times B}(a,b)\left[s_{n_j}, s\right] = 1$ since (n_j) is strictly increasing sequence of positive numbers, we have

$$h_{A\times B}(a,b)[s_n, s] = h_{A\times B}(a,b)[s_n, s_{n_m}] \circledast h_{A\times B}(a,b)[s_{n_m}, s]$$

$$> h_{A\times B}(a,b)[s_n, s_{n_m}] \circledast (1-\varepsilon)$$

Letting $n \to \infty$ we have $h_{A\times B}(a,b)[s_n, s] \geq 1 \circledast (1-\varepsilon) = (1-\varepsilon)$.

Hence (s_n) fuzzy soft converges to s.

Lemma (3.11):

In a fuzzy soft metric space $(S, h_{A\times B}(a, b), \circledast)$ if $s_n \to^s s$ then (s_n) is fuzzy soft bounded.

Proof:

Since (s_n) is fuzzy soft converges to $s \in S$ then for any $0 < r < 1$ we can find a positive number N such that $h_{A\times B}(a,b)[s_n, s] > (1-r)$ for all $n \geq N$. Put

$(1-s) = \min\{h_{A\times B}(a,b)[s_1, s], h_{A\times B}(a,b)[s_2, s], \ldots, h_{A\times B}(a,b)[s_N, s]\}$.

Then by Remark 2.7 there is $0 < (1-\varepsilon) < 1$ such that

$(1-s) \circledast (1-r) > (1-\varepsilon)$. Since

$$h_{A\times B}(a,b)[s_n, s] \geq h_{A\times B}(a,b)[s_n, s_N] \circledast h_{A\times B}(a,b)[s_N, s]$$

$$\geq (1-s) \circledast (1-r) > (1-\varepsilon)$$

Hence (s_n) is fuzzy soft bounded.

Lemma (3.12):

In a fuzzy soft metric space $(S, h_{A\times B}(a,b), \circledast)$ if $s_n \to^{s} s$ and $s_n \to^{s} u$ then s=u.

Proof:

Assume that $s_n \to^{s} s$ and $s_n \to^{s} u$ so $\lim_{n\to\infty} h_{A\times B}(a,b)[s_n, s] = 1$ and $\lim_{n\to\infty} h_{A\times B}(a,b)[s_n, u] = 1$. But

$h_{A\times B}(a,b)[s,u] \geq h_{A\times B}(a,b)[s,s_n] \circledast h_{A\times B}(a,b)[s_n,u]$.

Taking limit to both sides as n tends to ∞ we obtain

$h_{A\times B}(a,b)[s,u] \geq 1 \circledast 1 = 1$ which implies that $h_{A\times B}(a,b)[s,u] = 1$ hence u=s.

Proposition (3.13):

In a fuzzy soft metric space $(S, h_{A\times B}(a,b), \circledast)$ every closed fuzzy soft ball is a fuzzy soft closed set.

Proof:

Assume that $SB[s,r]$ is closed fuzzy soft ball in S and let $y \in FSC[SB[s,r]]$ then by lemma 3.8

there is sequence (y_n) in $SB[s,r]$ such that $y_n \to^{s} y$ so $\lim_{n\to\infty} h_{A\times B}(a,b)[y_n, y] = 1$. Now

$h_{A\times B}(a,b)[s,y] \geq h_{A\times B}(a,b)[s,y_n] \circledast h_{A\times B}(a,b)[y_n,y]$. Taking limit to both sides as $n \to \infty$ we

obtain $h_{A\times B}(a,b)[s,y] \geq (1-r) \circledast 1 = (1-r)$. So $y \in SB[s,r]$. Therefore $SB[s,r]$ is a fuzzy

closed soft set .

Theorem (3.14):

Let $(S, h_{A\times B}(a,b), \circledast)$ be a fuzzy soft metric space and let $D \subseteq S$ then D is fuzzy soft dense in S if

and only if for every $s \in S$ there is $d \in D$ such that $h_{A\times B}(a,b)[x,d] > (1-\varepsilon)$ for some

$0 < (1-\varepsilon) < 1$.

Proof

Suppose that D is a fuzzy soft dense in S and $s \in S$ so $s \in FSC[D]$ then by lemma 3.8 there is a sequence $(d_n) \in$ D such that $d_n \to^s$s that is for any $0 < (1-\varepsilon) < 1$ and we can find N with

$h_{A\times B}(a,b)[d_n,s] > (1-\varepsilon)$ for all $n \geq N$. Take d= d_N so $h_{A\times B}(a,b)[d,s] > (1-\varepsilon)$.

Conversely to prove D is a fuzzy soft dense in S we have to show that S⊆ FSC[D] let s ∈S then

there is $d_j \in$ D such that $h_{A\times B}(a,b)[d_j,s] > \left(1-\frac{1}{j}\right)$. Now take $0 < \varepsilon < 1$ such that $\frac{1}{j} < \varepsilon$ for each j ≥N for some N. Hence we have a sequence $(d_j) \in$ D such that $h_{A\times B}(a,b)[d_j,s] > \left(1-\frac{1}{j}\right) > (1-\varepsilon)$ for all j≥N that is $d_j \to^s$ s so s∈ FSC[D].

4.Fuzzy soft continuous functions

Definition (4.1):

Let (S, $h_{A\times B}(a,b)$, ⊛) and (M, $v_{U\times W}(u,w)$, ⊛) be two fuzzy soft metric space. The function T: X →Y is known as **fuzzy soft continuous at s_0** ∈ X if for every and for every $0 < (1-\alpha) < 1$ there exists 0< $(1-\beta)$ <1 [depends on $(1-\alpha)$ and s_0] such that for all s∈S

$h_{A\times B}(a,b)[s,s_0] > (1-\beta)$ we have $v_{U\times W}(u,w)[T(s),T(s_0)] > (1-\alpha)$

Then T is known as **fuzzy soft continuous** if T is fuzzy soft continuous at every point s_0 ∈S

Theorem (4.2):

Let (S, $h_{A\times B}(a,b)$, ⊛) and (M, $v_{U\times W}(u,w)$, ⊛) be two fuzzy soft metric spaces. The function T:S→M is fuzzy soft continuous at s_0 ∈S if and only if whenever $s_n \to^s s_0$ then $T(s_n) \to^s T(s_0)$.

Proof :

Assume that T is fuzzy soft continuous at s_0 ∈S and let (s_n) ∈S with $s_n \to^s s_0$. Let 0< $(1-\varepsilon)$ <1 be taken then there is $0 < (1-\delta) < 1$ with for all s ∈S $h_{A\times B}(a,b)[s,s_0] > (1-\delta)$ implies $v_{U\times W}(u,w)[T(s),T(s_0)] > (1-\varepsilon)$ since $s_n \to^s s_0$ then we can find N ∈ ℕ such that for all n≥N we have $h_{A\times B}(a,b)[s_n,s_0] > (1-\delta)$.Therefore for all n≥N we $v_{U\times W}(u,w)[T(s_n),T(s_0)] > (1-\varepsilon)$ thus $T(s_n) \to^s T(s_0)$.

Conversely assume that for every sequences (s_n) in S fuzzy soft converges to s_0 implies $(T(s_n)) \to^s T(s_0)$ suppose that T is not fuzzy soft continuous at s_0 . There must exists

$0 < (1-\varepsilon) < 1$ and for all $0 < (1-\delta) < 1$ there exists s $\in$S such that $h_{A\times B}(a,b)[s, s_0] > (1-\delta)$ but $v_{U\times W}(u,w)[T(s), T(s_0)] \leq (1-\varepsilon)$. Now for any $n \in N$ we can find $s_n \in$S such that

$h_{A\times B}(a,b)[s_n, s_0] > \left(1-\frac{1}{n}\right)$ but $v_{U\times W}(u,w)[T(s_n), T(s_0)] \leq (1-\varepsilon)$.

Then the sequence $(T(s_n))$ does not fuzzy soft converges to $T(s_0)$.This contradicts our assumption that $T(s_n)$ fuzzy soft converges to $T(s_0)$. Therefore T is fuzzy soft continuous at s_0.

Proposition (4.3):

Let $(S, h_{A\times B}(a,b), \circledast)$ and $(M, v_{U\times W}(u,w), \circledast)$ be two fuzzy soft metric spaces. .The function T:S $\to$M is fuzzy soft continuous at $s_0 \in$ S if and only if for every $0 < \varepsilon < 1$ there exists $0 < \delta < 1$such that $SB(s_0, \delta) \subseteq T^{-1}[SB(T(s_0), \varepsilon)]$

Proof:

The function T is a fuzzy soft continuous at $s_0 \in$S if and only if for every $0 < (1-\varepsilon) < 1$ there exists $0 < (1-\delta) < 1$ such that $v_{U\times W}(u,w)[T(s), T(s_0)] > (1-\varepsilon)$ for all s satisfying $h_{A\times B}(a,b)[s, s_0] > (1-\delta)$ that is $s \in SB(s_0,\delta)$ implies $T(s) \in SB\big(T(s_0), \varepsilon\big)$ or $T[SB(s_0,\delta)] \subseteq SB(T(s_0), \varepsilon)$. This is equivalent to $SB(s_0,\delta) \subseteq T^{-1}[SB(T(s_0), \varepsilon)]$.

Theorem (4.4) :

Let $(S, h_{A\times B}(a,b), \circledast)$ and $(M, v_{U\times W}(u,w), \circledast)$ be two fuzzy soft metric spaces. The function T:S$\to$M is fuzzy soft continuous if and only if $T^{-1}(G)$ is fuzzy soft open set in S for all fuzzy soft open subset G of M.

Proof:

Suppose that T is fuzzy soft continuous on S and let G be a fuzzy soft open subset of M. If

$T^{-1}(G) = \phi$ or $T^{-1}(G) = S$ then the proof is done. we may suppose that $T^{-1}(G) \neq \phi$ and $T^{-1}(G) \neq S$. Let $x \in T^{-1}(G)$ then $T(x) \in G$ but G is a fuzzy soft open then there exists $0 < \varepsilon < 1$ such that $SB(T(x), \varepsilon) \subseteq G$. But T is fuzzy soft continuous at x so by proposition 4.3 for this ε there exists $0 < \delta < 1$ and such that $SB(x, \delta) \subseteq T^{-1}[SB(T(x), \varepsilon)] \subseteq T^{-1}(G)$. Hence $T^{-1}(G)$ is a fuzzy soft open in S.

Conversely, suppose that $T^{-1}(G)$ is open for all open subset G of Y. Let $x \in S$ for each $0 < \varepsilon < 1$ the open fuzzy soft ball $SB(T(x), \varepsilon)$ is a fuzzy soft open set by Theorem 3.16 so $T^{-1}[SB(T(x), \varepsilon)]$ is a fuzzy soft open in S. Since $x \in T^{-1}[SB(T(x), \varepsilon)]$ it follows that there exists $0 < \delta < 1$ such that $SB(x, \delta) \subseteq T^{-1}[SB(T(x), \varepsilon)]$, Hence T is fuzzy soft continuous by Proposition 4.3.

Theorem (4.5):

Let $(S, h_{A\times B}(a, b), \circledast)$ and $(M, v_{U\times W}(u, w), \circledast)$ be two fuzzy soft metric spaces. The function $T: S \to M$ is fuzzy soft continuous if and only if $T^{-1}(G)$ is fuzzy soft closed set in S for all fuzzy soft closed subset G of M.

Proof:

Suppose that G be a fuzzy soft closed subset of M. Then $M-G$ is a fuzzy soft open in M so $T^{-1}(M-G)$ is a fuzzy soft open in S by Theorem 4.4. But $T^{-1}(M-G) = S - T^{-1}(G)$ so $T^{-1}(G)$ is a fuzzy soft closed in S.

Conversely suppose that $T^{-1}(G)$ is a fuzzy soft closed in S for all closed subset G of Y. But ϕ and S are fuzzy soft closed .Then $S - T^{-1}(G)$ is a fuzzy soft open in S and $T^{-1}(M-G) = S - T^{-1}(G)$ is a fuzzy soft open in S.

Theorem (4.6):

Let (S, $h_{A\times B}$(a, b), ⊛), (Y, $v_{U\times W}$(u, w), ⊛) and (Z, $l_{M\times Q}$(m, q), ⊛) be three fuzzy soft metric spaces. Let T: S →Y and H: Y →Z be fuzzy soft continuous functions. Then H∘T is a fuzzy soft continuous function from X to Z.

Proof:

Let G be fuzzy soft open set in Z then by Theorem 4.6 H^{-1}(G) is fuzzy soft open set in Y. Again by Theorem 4.4 $T^{-1}[H^{-1}(G)] = (H \circ T)^{-1}(G)$ is fuzzy soft open set in S. Hence H∘Tis fuzzy soft continuous .

Theorem (4.7):

Let (S, $h_{A\times B}$(a, b), ⊛) and (M, $v_{U\times W}$(u, w), ⊛) be two fuzzy soft metric spaces. Let T: S →M be a function then (1)⟺(2)⟺(3).

1- T is fuzzy soft continuous
2- For all subset D of M we have $FSC[T^{-1}(D)] \subseteq T^{-1}(FSC[D])$.
3- for all subset G of S we have $T(FSC[G]) \subseteq FSC[T(G)]$.

Proof : (1)⟹(2)

Le D be a subset of M Since FSC[D] is a fuzzy soft closed subset of M then $T^{-1}(FSC[D])$ is a fuzzy soft closed in S. Moreover $T^{-1}(D) \subseteq T^{-1}(FSC[D])$ and so $FSC[T^{-1}(D)] \subseteq T^{-1}(FSC[D])$. [since $FSC[T^{-1}(D)]$ is the smallest fuzzy soft closed set containing $T^{-1}(D)$].

(2)⟹(3):

Let G be a subset of S, Put $D = T(G)$, we have $G \subseteq T^{-1}(D)$ and $FSC[G] \subseteq FSC[T^{-1}(D)] \subseteq T^{-1}(FSC[D])$. Thus $T(FSC[G] \subseteq T\left(T^{-1}(FSC[D])\right) = FSC[D] = FSC[T(G)]$.

(3)⟹(1):

Let D be a fuzzy soft closed set in M and put $T^{-1}(D) = D_1$. By Theorem 4.5 it is sufficient to show that D_1 is a fuzzy soft closed in S that is $D_1 = FSC[D_1]$. Now $T(FSC[D]) \subseteq T\left(T^{-1}(FSC[D])\right) \subseteq FSC[D] = D$. $FSC[D_1] \subseteq T^{-1}(T(FSC[D_1])) \subseteq T^{-1}(D) = D_1$.

Theorem (4.8):

Let $(S, h_{A\times B}(a,b), \circledast)$ and $(M, v_{U\times W}(u,w), \circledast)$ be two fuzzy soft metric spaces. If T: S→M and H:S→ M be fuzzy soft continuous functions then the set $\{ s\in S: v_{U\times W}(u,w)[T(s),H(s)]=1\}$ is a fuzzy soft closed subset of S.

Proof:

Put $G=\{ s\in S: v_{U\times W}(u,w)[T(s),H(s)]=1\}$ then $S-G=\{ s\in S: 0< v_{U\times W}(u,w)[T(s),H(s)] <1\}$. We show that S−G is fuzzy soft open. If $S-G=\phi$ then there is nothing to prove. So let $S-G \neq \phi$ and let $d \in S-G$, then $v_{U\times W}(u,w)[T(s),H(s)] < 1$ let $v_{U\times W}(u,w)[T(s),H(s)]= (1-\varepsilon)$ for some $0<\varepsilon<1$. Then by fuzzy soft continuity of T and H there is $0<\delta<1$ and such that $h_{A\times B}(a,b)[s,d] > (1-\delta)$ implies $v_{U\times W}(u,w)[T(s),T(d)] > (1-\varepsilon)$ and $v_{U\times W}(u,w)[H(s),H(d)] > (1-\varepsilon)$. Hence by Remark 2.7 There exist $(1-r)$ for some r, $0<r<1$ such that $(1-\varepsilon)\circledast(1-\varepsilon)\circledast(1-\varepsilon) > (1-r)$. Now

$$v_{U\times W}(u,w)[T(d),H(d)] \geq v_{U\times W}(u,w)[T(d),T(s)] \circledast$$

$$v_{U\times W}(u,w)[T(s),H(s)] \circledast v_{U\times W}(u,w)[H(s),H(d)]$$

$$\geq (1-\varepsilon)\circledast(1-\varepsilon)\circledast(1-\varepsilon) > (1-r).$$

For all $s\in S$ satisfying $h_{A\times B}(a,b)[s,d] > (1-\delta)$. Thus for each $d \in SB(s,\delta,)$ we have $v_{U\times W}(u,w)[T(d),S(d)] <1$. That is $T(d) \neq S(d)$ so $SB(d,\delta) \subseteq S-G$. Hence S−G is fuzzy soft open so G is a fuzzy soft closed.

Corollary (4.9):

Suppose that $(S, h_{A\times B}(a,b), \circledast)$ and $(M, v_{U\times W}(u,w), \circledast)$ be two fuzzy soft metric spaces and assume that T: S→M and H:S→ M be fuzzy soft continuous functions. If the set $\{ s\in S: v_{U\times W}(u,w)[T(s),H(s)]=1\}$ is a fuzzy soft dense in S. Then T=H.

Proof:

By Theorem 4.8 the set G is a fuzzy soft closed but G is assumed to be fuzzy soft dense in S so we have S= FSC[G]=G that is T(s)=H(s) for all $s\in S$ Hence T=H.

Conclusion

In this paper, a new structure called a fuzzy soft metric space is introduced and studied which very different from the notion of fuzzy soft metric space that introduced by the authors in [18] and [19]. The notions of open fuzzy soft ball, fuzzy soft open set, fuzzy soft bounded set, fuzzy soft dense set, fuzzy soft convergence of sequences, fuzzy soft continuous function are introduced and some basic results are investigated. To continue this work, one could study and introduce other notions.

References

[1] D. Molodtsov, Soft Set Theory- First Results, Computer Math. Appl., 37(4/5) (1999),19-31.

[2] P. K.Maji R. Biswas and A. R. Roy, Fuzzy Soft Set, J. Fuzzy Math. , 9(3)(2001), 589-602.

[3] B. Ahmad and A. Kharal ,On Fuzzy Soft Set, Hindawi Publishing Corporation, Advance in Fuzzy Systems, Article ID 586507, (2009),6 pages.

[4] B. P. Varol , A. P. Shostak and H. Aygun, Categories Related to Topology Viewed as Soft Set, Proceeding of the 17 Conference of the European Society for fuzzy logic and technology, (2011), 883-890.

[5] B. P. Varol and H. Aygun, Fuzzy Soft Topology, Hacettepe J. of Math. And Statistics, 41(3)(2012), 407-419.

[6] I. Sadeqi and F. Kia, Fuzzy normed linear space and its topological structure, Chaos Solitions and Fractals, 40 (9)(2009), 2576-2589.

[7] J. Kider, On fuzzy normed spaces, Eng. & Tech. Journal, 29(9)(2011),1790-1795.

[8] J. Kider, Completion of fuzzy normed spaces, Eng. & Tech. J. , 29(10)(2011), 559-564.

[9] J. Kider, New fuzzy normed spaces, J. Baghdad Sci., 9(2012), 2004-2012.

[1] S. Nadaban, Fuzzy continuous mapping in fuzzy normed linear spaces, I. J. Computer Communications and Control, 10 (6)(2015), 834-842.

[11] J. Kider and N. Kadhum , Properties of fuzzy norm of fuzzy bounded operators, Iraqi Journal of Science, 58(3A)(2017),1237-1281.

[12] N. Kadhum , On fuzzy norm of a fuzzy bounded operator on fuzzy normed spaces, M.Sc. Thesis 2017, University of Technology, Iraq.

[13] A. Ali, Properties of Complete Fuzzy Normed Algebra, M.Sc. Thesis 2018, University of Technology, Iraq.

[14] J. Kider and A. Ali, Properties of fuzzy absolute value on and properties finite dimensional fuzzy normed space. Iraqi Journal of Science, 59(2B)(2018), 909-916.

[15] J. Kider and M. Gheeab, Properties of a General Fuzzy Normed Space, Iraqi Journal of Science, 60(4)(2019), 847-855.

[16] J. Kider and N. Gheeab, Properties of The Space GFB(V, U), Journal of AL-Qadisiyah for computer science and mathematics, 11(1)(2019), 102-110.

[17] J. Kider and N. Kadhum , Properties of Fuzzy Compact Linear Operators on Fuzzy Normed Spaces, Baghdad Science Journal, 16(1)(2019), 104-110.

[18] M. Vasuky and A. Uma, Fuzzy soft metric space, Shanlas Publications, 2019.

[19] T. Beaulaa and C. Gunaseeli, On fuzzy soft metric spaces, Malaya J. Mat. 2(3) (2014), 197–202.

[20] E. A. Hussin, On some properties of finite dimensional fuzzy antinormed spaces and Fuzzy antiinner product spaces, M. Sc. Theses, Department of Mathematics, College of Sciences for Women, University of Baghdad, 2019.