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ABSTRACT

In this paper we inserted the notion of Δ-statistically pre-Cauchy double sequence of fuzzy numbers. Also, we establish a criterion for arbitrary double sequence of fuzzy numbers to becomes Δ-statistically pre-Cauchy.

1. Introduction

Zadeh [35] introduced the concepts of fuzzy sets and fuzzy set operations. Maltoka [20] discussed bounded and convergent sequences of fuzzy numbers and studied their some properties. After on it, sequences of fuzzy numbers have been discussed by Dutta [7,8,9], Diamond and Kloeden [15], Nanda [21], Esi [13] and many others.

A fuzzy real number X on R is a function $X : R \to I = [0,1]$ associating each $t \in R$, with its grade of membership $X(t)$.The class of all fuzzy real numbers is denoted by $R(I)$. For $0 < \alpha \leq 1$, the $\alpha-$ level set $X^{\alpha} = \{t \in R : X(t) \geq \alpha\}$, and the 0-level set $X^{0} = \{t \in R: X(t) > 0\}$, is the clouser of strong 0-cut then it is compact.

Let D denote to the set of all closed bounded intervals $B = [b_1, b_2]$.Define the relation d on D by $d(B,W) = \max\{|b_1 - w_1|, |b_2 - w_2|\}$. Clearly (D,d) is a complete metric space. (look Diamond and Kloeden [14], Nanda[21]).

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The studying of Orlicz sequence spaces have been put newly by various authors ([10], [11], [12], [25], [33], [34]).

Battor and Neamah [24] introduced a double sequence as follows: a double sequence $(X,Y) = (X_{rs},Y_{rs})$ is a double infinite matrix of elements (X_{rs},Y_{rs}), where $X = X_{rs}$ is a double infinite matrix of elements X_{rs} and $Y = Y_{rs}$ is infinite a double matrix of elements Y_{rs}, which means (X_{rs},Y_{rs}) is complex double sequences and they defined a double Orlicz function on double sequence space in the following:

$M : [0,\infty) \times [0,\infty) \to [0,\infty) \times [0,\infty)$ such that $M(X,Y) = (M_1(X), M_2(Y))$ and $(X,Y) = (X_{rs},Y_{rs})$ where $M_1 : [0,\infty) \to [0,\infty)$, $M_2 : [0,\infty) \to 0,\infty)$, and M_1, M_2 are two Orlicz functions which be continuous, non-decreasing, even, convex and satisfy the next conditions:

(i) $M_1(0) = 0$, $M_2(0) = 0 \Rightarrow M(0,0) = (M_1(0), M_2(0)) = (0,0)$

(ii) $M_1(X) > 0, M_2(Y) > 0 \Rightarrow M(X,Y) = (M_1(X), M_2(Y)) > (0,0)$, for all $X, Y > 0$

(iii) $M_1(X) \to \infty, M_2(Y) \to \infty$, as $X, Y \to \infty \Rightarrow M(X,Y) \to (\infty,\infty)$,as $(X,Y) \to (\infty,\infty)$.

***Remark* 1.1** [24] If M is a double Orlicz function, then $M_1(\lambda X) \leq \lambda M_1(X)$, $M_2(\lambda Y) \leq \lambda M_2(Y)$, for all $X \geq 0, Y \geq 0$ with $0 < \lambda < 1$, therefore $M(\lambda X, \lambda Y) = (M_1(\lambda X), M_2(\lambda Y)) \leq \lambda (M_1(X), M_2(Y)) = \lambda M(X,Y)$,for all $(X,Y) \geq (0,0)$, thus $M(\lambda X, \lambda Y) \leq \lambda M(X,Y)$, for all $(X,Y) \geq (0,0)$.

If replaced the convexity of M by $M(X+Y) \leq M(X) + M(Y)$, then it is said a modulus function (look [22]). A double Orlicz function may be bounded or unbounded, e.g. $M_1(X) = X^p, M_2(Y) = Y^p$, $0 < p \leq 1$ are unbounded, so consequently $M(X,Y) = (X^p, Y^p)$, $(0,0) < (p,p) \leq (1,1)$ is unbounded and $M_1(X) = \frac{X}{X+1}, M_2(Y) = \frac{Y}{Y+1}$ are bounded, so, $M(X,Y) = \left(\frac{X}{X+1}, \frac{Y}{Y+1}\right)$ is bounded.

A fuzzy double sequence $(X,Y) = (X_{rs},Y_{rs})$ is a double infinite matrix of elements (X_{rs},Y_{rs}) for all $r,s \in N$, where $X = X_{rs}$ is a double infinite matrix of elements X_{rs} and $Y = Y_{rs}$ is infinite a double matrix of elements Y_{rs},where $X_{rs},Y_{rs} \in R(I)$, which means (X_{rs},Y_{rs}) is a double sequences of fuzzy real numbers, where $X_{rs},Y_{rs} \in R(I)$, for each $r,s \in N$.

The earliest works on double sequences of real or complex terms is found in Bromwich [1]. Later on it further studied by Basarir and Solancan [2], Moricz [23], Tripathy and Dutta [31], Tripathy and Sarma [32] and many others. The notion of regular convergence for double sequences of real or complex terms was introduced by Hardy [18]. Fast [16] was first introduced the concept of statistical convergence and it also independently by Buck [3] and Schoenberg [28]. Further it was studied by Salat [27], Fridy [17], Cannor [4,5] and many others.

The notion of Δ-statistically pre-Cauchy double sequence of fuzzy numbers was introduced by Dutta and Reddy [29] and they established a standard for arbitrary double sequence of fuzzy numbers to become Δ-statistically pre-Cauchy.

We concept this work to introduced the following notions:

A double sequences of fuzzy numbers $(X_{kl}),(,Y_{kl})$ are said to be Δ-statistically convergent to X_0, Y_0 respectively if $\lim_{r,s\to\infty}\frac{1}{rs}\left|\left\{(p,q): d\left(\Delta X_{pq}, X\right)\geq \varepsilon,\ p\leq r, q\leq s\right\}\right| = 0$, and $\lim_{r,s\to\infty}\frac{1}{rs}\left|\left\{(p,q): d\left(\Delta Y_{pq}, Y\right)\geq \varepsilon,\ p\leq r, q\leq s\right\}\right| = 0$, respectively,

therefore, (X_{kl}, Y_{kl}) is called Δ -statistically convergent to (X_0, Y_0) if

$$\lim_{r,s\to\infty}\frac{1}{rs}\left|\left\{(p,q): \left(d\left(\Delta X_{pq}, X\right), d\left(\Delta Y_{pq}, Y\right)\right)\geq \varepsilon,\ p\leq r, q\leq s\right\}\right| = 0,$$

such that the vertical bars indicate to the number of elements in the set.

Definition 1.2 A double sequence of fuzzy number (X_{kl}, Y_{kl}) is called Δ-statistically pre-Cauchy if for all $\varepsilon > 0$ there exist $v(\varepsilon)$ and $w(\varepsilon)$ where,

$$\lim_{r,s\to\infty}\frac{1}{r^2 s^2}\left|\left\{(p,q): \left(d\left(\Delta X_{pq}, \Delta X_{vw}\right), d\left(\Delta Y_{pq}, \Delta Y_{vw}\right)\right)\geq \varepsilon,\ p\leq r, q\leq s\right\}\right| = 0.$$

In actually, the first order difference operator Δ may be represented as an infinite triangular matrix as following,

$$\Delta = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & \dots \\ 0 & 1 & -1 & 0 & 0 & \dots \\ 0 & 0 & 1 & -1 & 0 & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \end{bmatrix}$$

The fuzzy double sequence (X_{rs}, Y_{rs}) may be expressed as an infinite matrix of fuzzy numbers as following,

$$(X_{rs}, Y_{rs}) = \begin{bmatrix} X_{11}, Y_{11} & X_{12}, Y_{12} & X_{13}, Y_{13} & \dots & X_{1n}, Y_{1n} & \dots \\ X_{21}, Y_{21} & X_{22}, Y_{22} & X_{23}, Y_{23} & \dots & X_{2n}, Y_{2n} & \dots \\ X_{31}, Y_{31} & X_{32}, Y_{32} & X_{33}, Y_{33} & \dots & X_{3n}, Y_{3n} & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \end{bmatrix}$$

Then, for any fuzzy double sequence by $(X, Y) = (X_{rs}, Y_{rs})$, we have

$$\Delta(X_{rs}, Y_{rs}) = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & \dots \\ 0 & 1 & -1 & 0 & 0 & \dots \\ 0 & 0 & 1 & -1 & 0 & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \end{bmatrix}\begin{bmatrix} X_{11}, Y_{11} & X_{12}, Y_{12} & X_{13}, Y_{13} & \dots & X_{1n}, Y_{1n} & \dots \\ X_{21}, Y_{21} & X_{22}, Y_{22} & X_{23}, Y_{23} & \dots & X_{2n}, Y_{2n} & \dots \\ X_{31}, Y_{31} & X_{32}, Y_{32} & X_{33}, Y_{33} & \dots & X_{3n}, Y_{3n} & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \end{bmatrix}$$

$$= \begin{bmatrix} X_{11}-X_{21}, Y_{11}-Y_{21} & X_{12}-X_{22}, Y_{12}-Y_{22} & \dots & X_{1n}-X_{2n}, Y_{1n}-Y_{2n} & \dots \\ X_{21}-X_{31}, Y_{21}-Y_{31} & X_{22}-X_{32}, Y_{22}-Y_{32} & \dots & X_{2n}-X_{3n}, Y_{2n}-Y_{3n} & \dots \\ X_{31}-X_{41}, Y_{31}-Y_{41} & X_{32}-X_{42}, Y_{32}-Y_{42} & \dots & X_{3n}-X_{4n}, Y_{3n}-Y_{4n} & \dots \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \end{bmatrix}.$$

This way about construction on difference double sequences is useful according to discuss some properties on the spaces of such sequences.

Lindenstrauss and Tzafriri [19] took the idea of Orlicz function and defined the sequence space L_M of single sequences, later on Battor and Neamah [24] used that idea to construct a double sequence space:

$$2L_M = \left\{(X_{rs}, Y_{rs}) \in 2\omega : \sum_{r=1}^{\infty}\sum_{s=1}^{\infty}\left[\left(M_1\left(\frac{|X_{rs}|}{\rho}\right)\right) \vee \left(M_2\left(\frac{|Y_{rs}|}{\rho}\right)\right)\right] < \infty, \text{ for some } \rho > 0\right\},$$

which is a Banach space under the norm:

$$\|(X_{rs}, Y_{rs})\|_M = \inf\left\{\rho > 0 : \sum_{r=1}^{\infty}\sum_{s=1}^{\infty}\left[\left(M_1\left(\frac{|X_{rs}|}{\rho}\right)\right) \vee \left(M_2\left(\frac{|Y_{rs}|}{\rho}\right)\right)\right] \leq 1\right\},$$

which is said a double Orlicz of a double sequence space such that 2ω is a family of all $\mathbb{R}^2$ or $\mathbb{C}^2$ double sequence, that is, (X_{rs}) and (Y_{rs}) are complex or real double sequence and conclusion that the double Orlicz of a double sequence space $2L_M$ be closely related to the space $2L_p$, which is, a double Orlicz double sequence space with $M\,(X,Y) = (M_1(X), M_2(Y)) = (X^p, Y^p)$, for $(1,1) \leq (p,p) < (\infty,\infty)$ such that $M_1(X) = X^p$, for $1 \leq p < \infty$, and $M_2(Y) = Y^p$, for $1 \leq p < \infty$.

Connor, Fridy and Kline in [6] and Dutta and Reddy in [37] showed the condition of bounded sequence to becomes statistically pre-Cauchy, so we can establish the follows criterion for arbitrary double sequence of fuzzy terms to be statistically pre-Cauchy for a double Orlicz $\boldsymbol{M = (M_1, M_2)}$.

2. Main results

Theorem *2.1* Let $(X,Y) = (X_{pq}, Y_{pq})$ be a double sequence of fuzzy number and $M = (M_1, M_2)$ be a bounded double Orlicz function, then (X,Y) is Δ- statistically pre-Cauchy if and only if

$$\lim_{r,s\to\infty} \frac{1}{r^2s^2}\sum_{p,v\leq r}\sum_{q,w\leq s}\left[\left(M_1\left(\frac{d(\Delta X_{pq},\Delta X_{vw})}{\rho}\right)\right),\left(M_2\left(\frac{d(\Delta Y_{pq},\Delta Y_{vw})}{\rho}\right)\right)\right] = 0,$$

for some $\rho > 0$.

Proof : Suppose

$$\lim_{r,s\to\infty} \frac{1}{r^2s^2}\sum_{p,v\leq r}\sum_{q,w\leq s}\left[\left(M_1\left(\frac{d(\Delta X_{pq},\Delta X_{vw})}{\rho}\right)\right),\left(M_2\left(\frac{d(\Delta Y_{pq},\Delta Y_{vw})}{\rho}\right)\right)\right] = 0,$$

for some $\rho > 0$. For each $\varepsilon > 0$, $\rho > 0$ and $m, n \in N$, we get

$$\frac{1}{r^2s^2}\sum_{p,v\leq r}\sum_{q,w\leq s}\left[\left(M_1\left(\frac{d(\Delta X_{pq},\Delta X_{vw})}{\rho}\right)\right),\left(M_2\left(\frac{d(\Delta Y_{pq},\Delta Y_{vw})}{\rho}\right)\right)\right] =$$

$$\frac{1}{r^2s^2}\sum_{\substack{p,v\leq r,\, d(\Delta X_{pq},\Delta X_{vw})<\varepsilon \\ \text{and } d(\Delta Y_{pq},\Delta Y_{vw})<\varepsilon}}\sum_{q,w\leq s}\left[\left(M_1\left(\frac{d(\Delta X_{pq},\Delta X_{vw})}{\rho}\right)\right),\left(M_2\left(\frac{d(\Delta Y_{pq},\Delta Y_{vw})}{\rho}\right)\right)\right]$$

$$+\frac{1}{r^2s^2}\sum_{\substack{p,v\leq r,\, d(\Delta X_{pq},\Delta X_{vw})\geq\varepsilon \\ \text{and } d(\Delta Y_{pq},\Delta Y_{vw})\geq\varepsilon}}\sum_{q,w\leq s}\left[\left(M_1\left(\frac{d(\Delta X_{pq},\Delta X_{vw})}{\rho}\right)\right),\left(M_2\left(\frac{d(\Delta Y_{pq},\Delta Y_{vw})}{\rho}\right)\right)\right]$$

$$\geq \frac{1}{r^2s^2}\sum_{\substack{p,v\leq r, d(\Delta X_{pq},\Delta X_{vw})\geq\varepsilon \\ \text{and } d(\Delta Y_{pq},\Delta Y_{vw})\geq\varepsilon}} \sum_{q,w\leq s}\left[\left(M_1\left(\frac{d(\Delta X_{pq},\Delta X_{vw})}{\rho}\right)\right),\left(M_2\left(\frac{d(\Delta Y_{pq},\Delta Y_{vw})}{\rho}\right)\right)\right]$$

$$\geq M(\varepsilon)\left[\frac{1}{r^2s^2}\left|\left\{(p,q):\left(d(\Delta X_{pq},\Delta X_{vw}),d(\Delta Y_{pq},\Delta Y_{vw})\right)\geq\ \varepsilon,\ \ p\leq r, q\leq s\right\}\right|\right]$$

≥ 0, where $M = (M_1, M_2)$

Next, suppose that (X,Y) is Δ- statistically pre-Cauchy and that ε has been given.

Let $\varepsilon > 0$ such that $M_1(\delta) < \frac{\varepsilon}{2}$ and $M_2(\delta) < \frac{\varepsilon}{2}$ and consequently, $M(\delta) < \frac{\varepsilon}{2}$.

Since M is bounded, there exist an integer A such that $M_1(X) < \frac{A}{2}$ and $M_2(Y) < \frac{A}{2}$ for all $X, Y \geq 0$ and consequently, $M(X,Y) < \frac{A}{2}$ for all $X, Y \geq 0$. Note that, for all $n \in N$.

$$\frac{1}{r^2s^2}\sum_{p,v\leq r}\sum_{q,w\leq s}\left[\left(M_1\left(\frac{d(\Delta X_{pq},\Delta X_{vw})}{\rho}\right)\right),\left(M_2\left(\frac{d(\Delta Y_{pq},\Delta Y_{vw})}{\rho}\right)\right)\right] =$$

$$\frac{1}{r^2s^2}\ \sum_{\substack{p,v\leq r, d(\Delta X_{pq},\Delta X_{vw})<\delta \\ \text{and } d(\Delta Y_{pq},\Delta Y_{vw})<\delta}}\ \sum_{q,w\leq s}\left[\left(M_1\left(\frac{d(\Delta X_{pq},\Delta X_{vw})}{\rho}\right)\right),\left(M_2\left(\frac{d(\Delta Y_{pq},\Delta Y_{vw})}{\rho}\right)\right)\right]$$

$$+\frac{1}{r^2s^2}\sum_{\substack{p,v\leq r, d(\Delta X_{pq},\Delta X_{vw})\geq\delta \\ \text{and } d(\Delta Y_{pq},\Delta Y_{vw})\geq\delta}}\sum_{q,w\leq s}\left[\left(M_1\left(\frac{d(\Delta X_{pq},\Delta X_{vw})}{\rho}\right)\right),\left(M_2\left(\frac{d(\Delta Y_{pq},\Delta Y_{vw})}{\rho}\right)\right)\right]$$

$$\leq M(\delta) + \frac{1}{r^2s^2}\sum_{\substack{p,v\leq r, d(\Delta X_{pq},\Delta X_{vw})\geq\delta \\ \text{and } d(\Delta Y_{pq},\Delta Y_{vw})\geq\delta}}\sum_{q,w\leq s}\left[\left(M_1\left(\frac{d(\Delta X_{pq},\Delta X_{vw})}{\rho}\right)\right),\left(M_2\left(\frac{d(\Delta Y_{pq},\Delta Y_{vw})}{\rho}\right)\right)\right]$$

$$\leq \frac{\varepsilon}{2} + \frac{A}{2}\left[\frac{1}{r^2s^2}\left|\left\{(p,q):\left(d(\Delta X_{pq},\Delta X_{vw}),d(\Delta Y_{pq},\Delta Y_{vw})\right)\geq\ \delta,\ \ p\leq r, q\leq s\right\}\right|\right]$$

$$\leq \varepsilon + A\left[\frac{1}{r^2s^2}\left|\left\{(p,q):\left(d(\Delta X_{pq},\Delta X_{vw}),d(\Delta Y_{pq},\Delta Y_{vw})\right)\geq\ \delta,\ \ p\leq r, q\leq s\right\}\right|\right] \quad ...(1)$$

Since, (X,Y) is Δ- statistically pre-Cauchy, there is, N such that the right hand side of equation (1) is less than ε for each $n \in N$. Hence,

$$\lim_{r,s\to\infty}\frac{1}{r^2s^2}\sum_{p,v\leq r}\sum_{q,w\leq s}\left[\left(M_1\left(\frac{d(\Delta X_{pq},\Delta X_{vw})}{\rho}\right)\right),\left(M_2\left(\frac{d(\Delta Y_{pq},\Delta Y_{vw})}{\rho}\right)\right)\right] = 0. \blacksquare$$

Theorem 2.2 Let $(X,Y) = (X_{pq}, Y_{pq})$ be a double sequence of fuzzy number and let M be a bounded double Orlicz function, then (X,Y) is Δ- statistically convergent to (X_0, Y_0) if and only if

$$\lim_{r,s}\frac{1}{rs}\sum_{p=1}^{r}\sum_{q=1}^{s}\left[\left(M_1\left(\frac{d(\Delta X_{pq},X_0)}{\rho}\right)\right),\left(M_2\left(\frac{d(\Delta Y_{pq},Y_0)}{\rho}\right)\right)\right] = 0.$$

Proof : consider that

$$\lim_{r,s}\frac{1}{rs}\sum_{p=1}^{r}\sum_{q=1}^{s}\left[\left(M_1\left(\frac{d(\Delta X_{pq},X_0)}{\rho}\right)\right),\left(M_2\left(\frac{d(\Delta Y_{pq},Y_0)}{\rho}\right)\right)\right] = 0,$$

with a double Orlicz function $M = (M_1, M_2)$, then (X, Y) is Δ- statistically convergent to (X_0, Y_0) see[22].

Conversely, suppose that (X, Y) is Δ- statistically convergent to (X_0, Y_0). In the same manner to theorem (2.1) and using that M be a double Orlicz function we can prove that

$$\lim_{r,s} \frac{1}{rs}\sum_{p=1}^{r}\sum_{q=1}^{s}\left[\left(M_1\left(\frac{d(\Delta X_{pq},X_0)}{\rho}\right)\right),\left(M_2\left(\frac{d(\Delta Y_{pq},Y_0)}{\rho}\right)\right)\right] = 0,$$

Corollary *2.3* Let $(X, Y) = (X_{pq}, Y_{pq})$ be a double sequence of fuzzy number, then (X, Y) is Δ- statistically pre-Cauchy, if and only if

$$\lim_{r,s} \frac{1}{r^2s^2}\sum_{p=1}^{r}\sum_{q=1}^{s}\left[\left(d(\Delta X_{pq}, \Delta X_{vw}), d(\Delta Y_{pq}, \Delta Y_{vw})\right)\right] = 0.$$

Proof: Let $B_1 = \sup_{p,q} d(\Delta X_{pq}, \bar{0})$, $B_2 = \sup_{p,q} d(\Delta Y_{pq}, \bar{0})$ and define $M(X, Y) = \left(\frac{(1+2B_1)X}{1+X}, \frac{(1+2B_2)Y}{1+Y}\right)$.Then,

$$\left[\left(M_1\left(\frac{d(\Delta X_{pq},\Delta X_{vw})}{\rho}\right)\right),\left(M_2\left(\frac{d(\Delta Y_{pq},\Delta Y_{vw})}{\rho}\right)\right)\right] \le \left((1 + 2B_1)d(\Delta X_{pq}, \Delta X_{vw}),\ (1 + 2B_2)d(\Delta Y_{pq}, \Delta Y_{vw})\right)$$

and

$$\left[\left(M_1\left(\frac{d(\Delta X_{pq},\Delta X_{vw})}{\rho}\right)\right),\left(M_2\left(\frac{d(\Delta Y_{pq},\Delta Y_{vw})}{\rho}\right)\right)\right] = \left((1 + 2B_1)\frac{d(\Delta X_{pq},\Delta X_{vw})}{1+d(\Delta X_{pq},\Delta X_{vw})},\ (1 + 2B_2)\frac{d(\Delta Y_{pq},\Delta Y_{vw})}{1+d(\Delta Y_{pq},\Delta Y_{vw})}\right)$$

$$\ge \left(\frac{(1 + 2B_1)d(\Delta X_{pq}, \Delta X_{vw})}{1 + d(\Delta X_{pq}, \Delta X_{vw})}, \frac{(1 + 2B_2)d(\Delta Y_{pq}, \Delta Y_{vw})}{1 + d(\Delta Y_{pq}, \Delta Y_{vw})}\right)$$

$$\ge \left(\frac{(1 + 2B_1)d(\Delta X_{pq}, \Delta X_{vw})}{1 + (1 + 2B_1)}, \frac{(1 + 2B_2)d(\Delta Y_{pq}, \Delta Y_{vw})}{1 + (1 + 2B_2)}\right),$$

$$= \left(d(\Delta X_{pq}, \Delta X_{vw}), d(\Delta Y_{pq}, \Delta Y_{vw})\right).$$

Thus, $\lim_{r,s} \frac{1}{r^2s^2}\sum_{p=1}^{r}\sum_{q=1}^{s}\left[\left(d(X_{pq}, X_{vw}), d(Y_{pq}, Y_{vw})\right)\right] = 0$, if and only if

$\lim_{r,s} \frac{1}{r^2s^2}\sum_{p=1}^{r}\sum_{q=1}^{s}\left[\left(M_1\left(\frac{d(\Delta X_{pq},X_{vw})}{\rho}\right)\right),\left(M_2\left(\frac{d(\Delta Y_{pq},Y_{vw})}{\rho}\right)\right)\right] = 0$, and direct application of theorem (2.1) the proof completes. ■

Corollary *2.4* Let $(X, Y) = (X_{pq}, Y_{pq})$ be a double sequence of fuzzy number, then (X, Y) is Δ- statistically convergent to (X_0, Y_0), if and only if

$$\lim_{r,s} \frac{1}{rs}\sum_{p=1}^{r}\sum_{q=1}^{s}\left[\left(d(\Delta X_{pq}, X_0), d(\Delta Y_{pq}, Y_0)\right)\right] = 0.$$

Proof: Let $B_1 = \sup_{p,q} d(\Delta X_{pq}, \bar{0})$, $B_2 = \sup_{p,q} d(\Delta Y_{pq}, \bar{0})$ and define $M(X,Y) = \left(\frac{(1+B_1+X_0)X}{1+X}, \frac{(1+B_2+Y_0)Y}{1+Y}\right)$.Then in the same manner of the proof of corollary (2.3) we can get the prove. ■

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