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Anwar Khaleel Faraj
Department of Applied Sciences, University of Technology, Baghdad, Iraq., anwar_78_2004@yahoo.com

Marwa Hadi Sapur
Department of Applied Sciences, University of Technology, Baghdad, Iraq., marwahadis2016@gmail.com

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Authors Names

a. Anwar Khaleel Faraj

b. Marwa Hadi Sapur

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ABSTRACT

The main aim of this paper is to generalize Awtar's Theorem concerning derivation on Lie ideal to symmetric higher $(U,\mathcal{R})$-n-derivation by introducing many concepts have played an important role in obtaining this result such as $(U,\mathcal{R})$-n-derivation, Jordan $(U,\mathcal{R})$-n-derivation and higher$n$-derivation.

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1 .Introduction

Throughout, $\mathcal{R}$ will represent an associative ring with center $Z(\mathcal{R})$. A ring $\mathcal{R}$ is said to be n-torsion free where n is nonzero integer if $na = 0$ withh$a \in \mathcal{R}$ implies that $a = 0$. $\mathcal{R}$ is said to be prime ring if *a* $\mathcal{R}$ *b=0* implies that either $a = 0$ or $b = 0$ for all $a, b \in \mathcal{R}$, and $\mathcal{R}$ is semiprime if $a\,\mathcal{R}\,a = 0$ then $a = 0$ [8]. For all $u, v \in \mathcal{R}$, the commutator of u and v is $[u,r] = ur - ru$. An additive subgroup U/off $\mathcal{R}$ is said to be a Lie ideal of $\mathcal{R}$ if $[u,r] \in U$ for all $u \in eU$, $r \in \mathcal{R}$ [9]. A mapping $d{:}\,\mathcal{R} \to \mathcal{R}$ is a derivation of $\mathcal{R}$ if d is an additive mapping that satisfies the following $d(uv) = d(u)v + u\,d(v)$ for all $u, r \in \mathcal{R}$ and d is said to be Jordan derivation if $u = r$ and this means $d(u^2) = d(u)u + u\,d(u)$ [12]. In general every derivation is a Jordan derivation, but the converse is not true. In [4] Herstein proved that every Jordan derivation of a prime ring $\mathcal{R}$ of $char \neq 2$ is a derivation. Awtar in [2] generalized Herstein's result to Lie ideal, he proved that in a prime ring $\mathcal{R}$ of $char \neq 2$, every Jordan derivation $d_{|U}{:}\,U \to \mathcal{R}$ is a derivation, where U is a Lie ideal of $\mathcal{R}$ such that $u^2 \in U$ for all $u \in U$. A family of additive mappings $D = (d_n)_{n\in N}$ of $\mathcal{R}$ into itself where $d_0 = id_R$iis called a higher derivation (resp. Jordan higher derivation), if $d_n(uv) = \sum_{i+j=n} d_i(u)d_j(u)$ for all $u, v \in \mathcal{R}$, $n \in N$, where N is the set of natural number [6]. According to [5], Ferrero and Haetinger were extended Herstein's result to higher derivations, they proved that every Jordan higher derivation of 2-torsion free semiprime ring is a higher derivation. In [7] Haetinger extended Awtar's theorem to/higher derivation on Lie ideal. Further, Faraj, Haetinger and Majeed were expanded this result to higher $(U,\mathcal{R})$-derivation [6]. In this paper, new generalization

[a] Department of Applied Sciences, University of Technology, Baghdad, Iraq. Email: anwar_78_2004@yahoo.com

[b] Department of Applied Sciences, University of Technology, Baghdad, Iraq. Email: marwahadis2016@gmail.com

of Herstein's theorem to symmetric higher $(U,\mathcal{R})$-n-derivation will introduce as different kind from previous generalizations, where U is a Lie ideal of $\mathcal{R}$ by introducing many concepts that play role to arrive main goal. Throughout this paper d_0 is defined to be the identity mapping $id_{\mathcal{R}}:\mathcal{R}\to\mathcal{R}$.

2. Symmetric $(U,\mathcal{R})$-n-Derivation

The concept of symmetric $(U,\mathcal{R})$-n-Derivation is introduced and studied throughout this section.

We begin with following definition:

Definition (2.1) [10]:

A mapping $d:\mathcal{R}\times\ldots\times\mathcal{R}\to\mathcal{R}$ is called symmetric if the equation $d(u_1,u_2,\ldots,u_n)= d(u_{\pi(1)},u_{\pi(2)},\ldots,u_{\pi(n)})$ holds, for all $u_i\in\mathcal{R}$ and for every permutation $\{\pi(1),\pi(2),\ldots,\pi(n)\}$.

The concept of $(U,\mathcal{R})-n-$derivation as a generalization of the concept $(U,\mathcal{R})-$ derivation can be introduced as follows

Definition (2.2):

Let U be a Lie ideal of $\mathcal{R}$. An n- additive mapping $d:\mathcal{R}\times\mathcal{R}\times\ldots\times\mathcal{R}\to\mathcal{R}$ is said to be $(U,\ \mathcal{R})$-*n*-derivation of $\mathcal{R}$, if the following equations are equivalent for all $u,u_2,\ldots,u_n\in U,\ r,s\in\mathcal{R}$

$$d(u_1r+su_1,u_2,\ldots,u_n)=d(u_1,u_2\ldots,u_n)r+u_1d(r,u_2,\ldots,u_n)+d(s,u_2\ldots,u_n)u_1+sd(u_1,u_2\ldots,u_n)$$

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$$d(u_1,u_2,\ldots,u_nr+su_n)=d(u_1,u_2\ldots,u_n)r+u_nd(u_1,u_2,\ldots,r)+d(u_1,u_2,\ldots,s)u_n+s\,d(u_1,u_2,\ldots,u_n)$$

Moreover, d is said to be a symmetric$(U,\mathcal{R})$-n-derivation if the above equations are equivalent to each other. Further, if $\mathcal{R}$ is 2-torsion free and $r=s=u$, the above equations can be reduced to $d(u^2,u_2,\ldots,u_n)=d(u,u_2\ldots,u_n)u+ud(u,u_2,\ldots,u_n)$

Recall that a Lie ideal U of a ring is called a square closed Lie ideal if $u^2\in U$ for all $u\in U$ and U is called an admissible Lie ideal of $\mathcal{R}$ if U is no central closed Lie ideal [6]

Lemma (2.3):[6]

Let $\mathcal{R}$ be a prime ring of $char\neq 2$ and U be an admissible Lie ideal of $\mathcal{R}$ such that $tw^2+w^2t=0$ for all $w\in U,t\in\mathcal{R}$. Then $t=0$.

Remark (2.4):

Let $D=(d_n)_{n\in N}$ be a family of additive symmetric mappings of $\mathcal{R}\times\mathcal{R}\times\ldots\times\mathcal{R}$. We set $\phi_n(ur,u_2\ldots,u_n)$ for

$d_n(ur,u_2\ldots,u_n)-\sum_{i+j=n}d_i(u,u_2\ldots,u_n)d_j(r,u_2\ldots,u_n)$ for all $u,u_2,\ldots,u_n\in U,r\in\mathcal{R},n\in N$.

Lemma (2.5): Let U $\mathcal{R}$ be a Lie ideal of 2-torsion free ring and $d:\mathcal{R}\times\mathcal{R}\times\ldots\times\mathcal{R}\to\mathcal{R}$ be a symmetric $(U,\mathcal{R})$-n-derivation. Then

$d(uru,u_2,\ldots,u_n)=d(u,u_2,\ldots,u_n)ru+ud(r,u_2,\ldots,u_n)u+urd(u,u_2,\ldots,u_n)$ for all $u,u_2,\ldots,u_n\in U,r\in\mathcal{R}$.

Proof: Substituting $r = s = (2u)r + r(2u)$ in the Definition (2.2), then $H = d(u(2u)r + (2u)r + r(2u)(2u) + ru, u_{2,} ..,u_n)$

$=2(d(u, u_{2,}\dots, u_n)ur + d(u, u_{2,}\dots, u_n)ru + ud(u, u_{2,}\dots, u_n)r + u^2d(r, u_{2,}\dots, u_n) + u\, d(r, u_{2,}\dots, u_n)u + urd(u, u_{2,}\dots, u_n) + d(u, u_{2,}\dots, u_n)ru + u\, d(r, u_{2,}\dots, u_n)u + d(r, u_{2,}\dots, u_n)u^2 + rd(u, u_{2,}\dots, u_n)u + ur\, d(u, u_{2,}\dots, u_n) + ru\ (u, u_{2,}\dots, u_n)$

On the other hand, $H = 2d(u, u_{2,}\dots, u_n)ur$+$2ud(u, u_{2,}\dots, u_n)r$ +$2u^2d(r, u_{2,}\dots, u_n)$+$2\, d(r, u_{2,}\dots, u_n)u^2$+$2rd(u, u_{2,}\dots, u_n)u$+$2ru\, d(u, u_{2,}\dots, u_n)$+$4d(uru, u_{2,}\dots, u_n)$. Compare the both sides of H and since $\mathcal{R}$ is 2-torsion free, then we get the required result.

The following corollary is immediate from Lemma (2.9).

Corollary (2.6):

Let U $\mathcal{R}$ be a Lie ideal of 2-torsionfree ring and $d: \mathcal{R} \times \mathcal{R} \times \dots \times \mathcal{R} \to \mathcal{R}$ be a symmetric $(U, \mathcal{R})$-n-derivation of $\mathcal{R}$. Then for all $w, u, u_{2,}\dots, u_n \in U, r, s \in \mathcal{R}$. Then $d(urw + wru, u_{2,}\dots, u_n)$

$$= d(u, u_{2,}\dots, u_n)rw + ud(r, u_{2,}\dots, u_n)w + urd(w, u_{2,}\dots, u_n)$$

$$+ d(w, u_{2,}\dots, u_n)su + wd(s, u_{2,}\dots, u_n)u + wsd(u, u_{2,}\dots, u_n) .$$

Lemma (2.7):

Let $\mathcal{R}$ be a prime ring of $char(\mathcal{R}) \neq 2$, U be an admissible Lie ideal of $\mathcal{R}$ and d:$\mathcal{R} \times \mathcal{R} \times \dots \times \mathcal{R} \to \mathcal{R}$ be a symmetric mapping such that $d(u^2, u_{2,}\dots, u_n) = d(u, u_{2,}\dots, u_n)u + ud(u, u_{2,}\dots, u_n)$ for all $u, u_{2,}\dots, u_n \in U$ Then $\phi(uv, u_{2,}\dots, u_n)[u, v] =0$ for all $u, v, u_{2,}\dots, u_n \in U$.

Proof: Since U be an admissible Lie ideal of $\mathcal{R}$, then $2uv \in U$ for all $u, v \in U$. So by Definition (2.2), $d((uv)^2, u_{2,}\dots, u_n) = d(uv, u_{2,}\dots, u_n)uv + uvd(uv, u_{2,}\dots, u_n$

Replace w by $2uv$ in Corollary (2.6) to get

$H = 2d(uv(uv) + (uv)vu, u_{2,}\dots, u_n)$= $2(d(u, u_{2,}\dots, u_n)v(uv)$+ $ud(v, u_{2,}\dots, u_n)uv + uvd(uv, u_{2,}\dots, u_n)$+$d(uv, u_{2,}\dots, u_n)vu$ +$uvd(v, u_{2,}\dots, u_n)u$+$(uv)vd(u, u_{2,}\dots, u_n)$)

But $H = 2d(uv(uv) + (uv)vu, u_{2,}\dots, u_n)$= $2d((uv)^2 + uv^2u, u_{2,}\dots, u_n)$

$$2(d(uv, u_{2,}\dots, u_n)uv + uvd(uv, u_{2,}\dots, u_n) + d(u, u_{2,}\dots, u_n)v^2u$$

$$+u\{d(v, u_{2,}\dots, u_n)v + vd(v, u_{2,}\dots, u_n)\}u + uv^2d(u, u_{2,}\dots, u_n))$$

Comparing both sides of H and since $char\ (\mathcal{R}) \neq 2$, then $\phi(uv, u_{2,}\dots, u_n)[u, v] =0$ for all $u, v, u_{2,}\dots, u_n \in U$.

The proof of following lemma is similar to the proof of Lemma (2.7):

Lemma (2.8):

Let $\mathcal{R}$ be a prime ring of $char(\mathcal{R}) \neq 2$, U be an admissible Lie ideal of $\mathcal{R}$ and d:$\mathcal{R} \times \mathcal{R} \times \dots \times \mathcal{R} \to \mathcal{R}$ be a symmetric mapping such that $d(u^2, u_{2,}\dots, u_n) = d(u, u_{2,}\dots, u_n)u + ud(u, u_{2,}\dots, u_n)$ for all $u, u_{2,}\dots, u_n \in U$. Then $[u, v]\phi(uv, u_{2,}\dots, u_n) =0$ for all $u, v, u_{2,}\dots, u_n \in U$.

Lemma (2.9):

Let $\mathcal{R}$ be a prime ring of $char(\mathcal{R}) \neq 2$, U be a an admissible Lie ideal of $\mathcal{R}$ and $d: \mathcal{R} \times \mathcal{R} \times \dots \times \mathcal{R} \to \mathcal{R}$ be a symmetric mapping such that $d(u^2, u_{2,}\dots, u_n)$

$= d(u,u_2,\ldots,u_n)u + ud(u,u_2,\ldots,u_n)$ for all $u,u_2,\ldots,u_n \in U$ and for all $u \in U \in C(U) = \{x \in R: [x,U] = 0\}$. Then $d(u,u_2,\ldots,u_n) \in Z(R)$ for all $u,u_2,\ldots,u_n \in U$.

Proof: Since $U \not\subset Z(\mathcal{R})$, then [4, Lemma 2] implies that $C(U) = Z(\mathcal{R})$ which gives$u \in Z(\mathcal{R})$. In view of Definition (2.2) we have

$$d(2uv,u_2,\ldots,u_n) = d(u,u_2,\ldots,u_n)v + vd(u,u_2,\ldots,u_n) + 2ud(v,u_2,\ldots,u_n). \quad \ldots (1)$$

Put $v = vw + wv$ in equation (1), where $w \in U$, then

$$d(2u(vw + wv),u_2,\ldots,u_n) =$$

$$d(u,u_2,\ldots,u_n)(vw + wv) + (vw + wv)d(u,u_2,\ldots,u_n) + 2ud(vw + wv,u_2,\ldots,u_n) \quad \ldots (2)$$

Since $u \in Z(\mathcal{R})$ and by Corollary (2.6) we obtain

$$d(2u(vw + wv),u_2,\ldots,u_n) = 2(d(u,u_2,\ldots,u_n)vw + d(u,u_2,\ldots,u_n)vw) + 2u(d(v,u_2,\ldots,u_n)w + v\,d(w,u_2,\ldots,u_n) + d(w,u_2,\ldots,u_n)v + wd(v,u_2,\ldots,u_n)) \quad \ldots (3)$$

Comparing equations (2) and (23) in this case we have

$d(u,u_2,\ldots,u_n)(vw - wv) = (vw - wv)d(u,u_2,\ldots,u_n)$ and this means that $[d(u,u_2,\ldots,u_n),[U,U] = 0$ for all $u,u_2,\ldots,u_n \in U$. Hence, $d(u,u_2,\ldots,u_n) \in C[U,U] = C(U)$ by [4, Lemma 3]. But, as above, $C(U) = Z(\mathcal{R})$. Therefore, $d(u,u_2,\ldots,u_n) \in Z(\mathcal{R})$ for all $u,u_2,\ldots,u_n \in U$.

Lemma (2.10):

Let $\mathcal{R}$ be a prime ring of $char(\mathcal{R}) \neq 2$, U be an admissible Lie ideal of $\mathcal{R}$ and $d: \mathcal{R} \times \mathcal{R} \times \ldots \times \mathcal{R} \to \mathcal{R}$ be a symmetric mapping such that $d(u^2,u_2,\ldots,u_n) = d(u,u_2,\ldots,u_n)u + ud(u,u_2,\ldots,u_n)$ for all $u,u_2,\ldots,u_n \in U$ and $uv = vu$ for some $u \in U$. Then $\phi_1(uv,u_2,\ldots,u_n)=0$, for all $u,v,u_2,\ldots,u_n \in U$.

Proof: As an application of Corollary (2.6), we have

$$d(uvw + wvu,u_2,\ldots,u_n) = d(u,u_2,\ldots,u_n)vw + ud(v,u_2,\ldots,u_n)w + uvd(w,u_2,\ldots,u_n) + d(w,u_2,\ldots,u_n)vu + wd(v,u_2,\ldots,u_n)u + wvd(u,u_2,\ldots,u_n)$$. for all $w,u,v,u_2,\ldots,u_n \in U$. ... (1)

Since $uv = vu$ and by applying Definition (2.2), we get $d(uvw + wuv,u_2,\ldots,u_n) = d(uv,u_2,\ldots,u_n)w + uvd(w,u_2,\ldots,u_n) + d(w,u_2,\ldots,u_n)uv + wd(uv,u_2,\ldots,u_n)$ . ... (2)

Subtracting equations (1) and (2) to get

$(d(uv,u_2,\ldots,u_n) - d(u,u_2,\ldots,u_n)v - ud(v,u_2,\ldots,u_n))w + w(d(uv,u_2,\ldots,u_n) - vd(u,u_2,\ldots,u_n) - vd(u,u_2,\ldots,u_n))=0$ and this means

$\phi_1(uv,u_2,\ldots,u_n)w + w\,\phi_1(vu,u_2,\ldots,u_n) = 0$. By Remark (2.4), $\phi_1(uv,u_2,\ldots,u_n) = -\phi_1(vu,u_2,\ldots,u_n)$ for all $u,v,u_2,\ldots,u_n \in U$. Hence $[\phi_1(uv,u_2,\ldots,u_n),w] = 0$ for all $w,u,v,u_2,\ldots,u_n \in U$. By Lemma 2 and Lemma 3 of [4], we have $\phi_1(uv,u_2,\ldots,u_n) \in Z(\mathcal{R})$ for all $u,v,u_2,\ldots,u_n \in U$. Since $u^2 \in U$ for all $u \in U$ and $u^2v = v\,u^2$.

So, $d(u^2v,u_2,\ldots,u_n) - d(u^2,u_2,\ldots,u_n)v - u^2d(v,u_2,\ldots,u_n) = d(u^2v,u_2,\ldots,u_n) - d(u,u_2,\ldots,u_n)uv$

$$-ud(u,u_2,\ldots,u_n)v - u^2d(v,u_2,\ldots,u_n) \in Z(\mathcal{R}) \quad \ldots (3)$$

Since $uv = vu$ and $2uv \in U$, then $(2uv)u = u(2uv)$, so we get $d(u(2uv),u_2,\ldots,u_n) - 2d(u,u_2,\ldots,u_n)uv - 2ud(uv,u_2,\ldots,u_n) \in Z(\mathcal{R})$ and since $char\mathcal{R} \neq 2$ this means

$$d(u^2v,u_2,\ldots,u_n) - d(u,u_2,\ldots,u_n)uv - ud(uv,u_2,\ldots,u_n) \in Z(\mathcal{R}) \text{ for all } u,v,u_2,\ldots,u_n \in U. \quad \ldots (4)$$

Comparing equations (3) and (4) and $char(\mathcal{R}) \neq 2$ yields $u\phi_1(uv,u_2,\ldots,u_n) \in Z(\mathcal{R})$ for all $u,v,u_2,\ldots,u_n \in U$.

If $u\phi_1(uv, u_2, \ldots, u_n) \neq 0$, since $\mathcal{R}$ is prime and $\phi_1(uv, u_2, \ldots, u_n) \in Z(\mathcal{R})$, then we can conclude that $u \in Z(\mathcal{R})$ and in view of Lemma (2.9), $d\,(u, u_2, \ldots, u_n) \in Z(\mathcal{R})$. Hence, $2d(uv, u_2, \ldots, u_n) = d(uv + vu, u_2, \ldots, u_n)$

$= 2\big(d(u, u_2, \ldots, u_n)v + vd(u, u_2, \ldots, u_n)\big)$. Since $char \neq 2$, the last equation becomes $d(uv, u_2, \ldots, u_n) = d(u, u_2, \ldots, u_n)v + vd(u, u_2, \ldots, u_n)$ for all $u, u_2, \ldots, u_n \in U$. That is $\phi_1(uv, u_2, \ldots, u_n) = 0$ for all $u, v, u_2, \ldots, u_n \in U$ and this is a contradiction. Therefore, $\phi_1(uv, u_2, \ldots, u_n) =0$, for all $u, v, u_2, \ldots, u_n \in U$.

Theorem (2.15):

Let $\mathcal{R}$ be a prime ring of $char \neq 2$, U be an admissible Lie ideal of $\mathcal{R}$ and $d: \mathcal{R} \times \mathcal{R} \times \ldots \times \mathcal{R} \to \mathcal{R}$ be a symmetric mapping such that $d(u^2, u_{2,}, \ldots, u_n) = d(u, u_{2,}, \ldots, u_n)u + ud(u, u_2, \ldots, u_n)$ for all $u, u_2, \ldots, u_n \in U$. Then $d(uv, u_{2,}, \ldots, u_n) = d(u, u_{2,}, \ldots, u_n)v + ud(v, u_{2,}, \ldots, u_n)$ for all $u, v, u_{2,}, \ldots, u_n \in U$.

Proof: Linearizing Lemmas (2.7) and (2.8) on v, then

$\phi_1(uv, u_2, \ldots, u_n)[u, w]=-\phi_1(uw, u_2, \ldots, u_n)[u, v]$ for all $u, w, u_2, \ldots, u_n \in U$. ... (1)

Also, $[u, w]\phi_1(uv, u_2, \ldots, u_n) = -[u, v]\,\phi_1(uw, u_2, \ldots, u_n)$ for all $u, w, u_2, \ldots, u_n \in U$. ... (2)

Multiply the left hand side of equation (1) by $[u, w_1]$ and using equations (2) and (1), then

$[u, v]\phi_1(uw, u_2, \ldots, u_n)[u, w_1]= -[u, w_1]\phi_1(uw, u_2, \ldots, u_n)[u, v][u, v]$. ... (3)

Replace w_1 by $2w_1v_1$ in Equation (3) to get $[u, v]\phi_1(uw, u_2, \ldots, u_n[u, w_1]v_1 + [u, w_1]v_1\phi_1(uw, u_2, \ldots, u_n)[u, v]$
$= -[u, v]\phi_1(uw, u_2, \ldots, u_n)w_1\,[u, v_1] - w_1[u, v_1]\phi_1(uw, u_2, \ldots, u_n)[u, r]$ ... (4)

Now applying equations (1) and (2) to equation (3) , it is easily observed that
$[u, v]\phi_1(uw, u_2, \ldots, u_n)[u, w_1]$=$[u, w_1]\,\phi_1(uw, u_2, \ldots, u_n)[u, w]$, $[u, w]\phi_1(uw, u_2, \ldots, u_n)[u, w_1]$=$[u, w_1]$
$\phi\phi_1(uw, u_2, \ldots, u_n)[u, v]$

And using these in equation (4), next by using equations (1) and (2) we have $[u, w_1]\;(\phi_1(uv, u_2, \ldots, u_n)[u, w]v_1 - v_1\phi_1(uw, u_2, \ldots, u_n)[u, w])$

$= -(\;[u, r]\phi_1(uw, u_{2,}, \ldots, u_n)w_1 - w_1[u, w]\phi_1(uw, u_{2,}, \ldots, u_n)[u, r_1]$.

Or, $[u, w_1][\phi_1(uv, u_{2,}, \ldots, u_n)[u, w], v_1]$=$-[[u, v]\,\phi_1(uw, u_{2,}, \ldots, u_n), w_1][u, v_1]$. ... (5)
Putting v_1=$2v_1u_1$ in equation (5) and use it, then $[u, w_1]v_1[\phi_1\,(uv, u_{2,}, \ldots, u_n)[u, w], v_1]$=$-$
$[[u, v]\,\phi_1(uw, u_{2,}, \ldots, u_n), w_1]v_1[u, v_1]$. ... (6)

As proof of [2, Theorem] one can complete the proof to get the required result. Hence, $\phi_1(uv, u_{2,}, \ldots, u_n) = 0$ for all $u, v, u_{2,}, \ldots, u_n \in U$.

Lemma (2.16):

Let $\mathcal{R}$ be a prime ring of $char(\mathcal{R}) \neq 2$, U be an admissible Lie ideal of $\mathcal{R}$ and d:$\mathcal{R} \times \mathcal{R} \times \ldots \times \mathcal{R} \to \mathcal{R}$ be a symmetric $(U, \mathcal{R})$-n-derivation. Then $\phi_1(u^2r, u_{2,}, \ldots, u_n)$ =0 for all , $u_{2,}, \ldots, u_n \in U, r \in \mathcal{R}$.

Proof: By Theorem (2.15), $\phi_1(uv, u_2, \ldots, u_n) = 0$ for all $u, v, u_{2,}, \ldots, u_n \in U$. Then

$$0 = \phi_1(u(ur - ru), u_2\,, \ldots, u_n) = d(u^2r - uru, u_2, \ldots, u_n) - d(u, u_2\,, \ldots, u_n)(ur - ru) - d\big((ur - ru), u_2\,, \ldots, u_n\big)u$$

$$= d(u^2r, u_2, \ldots, u_n) - d\big((ur - ru), u_2\,, \ldots, u_n\big) - d(u, u_2\,, \ldots, u_n)(ur - ru) - d\big((ur - ru), u_2\,, \ldots, u_n\big)u$$

$$= d(u^2r, u_2, \ldots, u_n) - (d(u, u_2, \ldots, u_n)ur + ud(u, u_2, \ldots, u_n)r + u^2d(r, u_2, \ldots, u_n)) = \phi_1\big(u^2r, u_{2,}, \ldots, u_n\big).$$

The following result is a generalization of [2, Theorem] to $(U, \mathcal{R})$-n-derivation.

Theorem (2.17):

Let $\mathcal{R}$ be a prime ring of $char(\mathcal{R}) \neq 2$, U be a Lie ideal of $\mathcal{R}$ and d:$\mathcal{R} \times \mathcal{R} \times \ldots \times \mathcal{R} \rightarrow \mathcal{R}$ be a symmetric $(U, \mathcal{R})$-n-derivation. Then $\phi_1(ur, u_2, \ldots, u_n)$ =0 for all , $u_2, \ldots, u_n \in U, r \in \mathcal{R}$.

Proof: Since d is a symmetric $(U, \mathcal{R})$-n-derivation, then replacing r by ur in Definition (2.2) yields

$H = d(uur + uru, u_2, \ldots, u_n) = d(u, u_2, \ldots, u_n)ur + ud(ur, u_2, \ldots, u_n) + d(ur, u_2, \ldots, u_n)u + urd(u, u_2, \ldots, u_n)$

On the other hand, $H = d(u^2r, u_2, \ldots, u_n) + d(uru, u_2, \ldots, u_n)$

By Lemma (2, 16) and Lemma (2, 5), the last equation can be reduced to

$H = d(u, u_2, \ldots, u_n)ur + ud(u, u_2, \ldots, u_n)r + u^2d(r, u_2, \ldots, u_n)$

$+d(u, u_2, \ldots, u_n)ru + ud(r, u_2, \ldots, u_n)u + urd(u, u_2, \ldots, u_n)$

Comparing both sides of H impies that

$u\phi_1(ur, u_2, \ldots, u_n) + \phi_1(ur, u_2, \ldots, u_n)u$=0 for all , $u_2, \ldots, u_n \in U, r \in \mathcal{R}$. ... (1)

Substituting $u = u + w$ where $w \in U$ in the equation (1) satisfies
$u\, \phi_1(wr, u_2, \ldots, u_n) + w\phi_1(ur, u_2, \ldots, u_n) + \phi_1(ur, u_2, \ldots, u_n)w + \phi_1(wr, u_2, \ldots, u_n)u$=0. ... (2)

Put $w = w^2$ in equation (2), we get

$u\, \phi_1(w^2r, u_2, \ldots, u_n) + w^2\phi_1(ur, u_2, \ldots, u_n) + \phi_1(ur, u_2, \ldots, u_n)w^2 + \phi_1(w^2r, u_2, \ldots, u_n)u$=0.

As application of Lemma (2.16) to the last equation implies that

$w^2\phi_1(ur, u_2, \ldots, u_n) + \phi_1(ur, u_2 \ldots, u_n)w^2$=0 for all $u, w, u_2, \ldots, u_n \in U, r \in \mathcal{R}$. Now we have two cases**:**

Case 1: If $U \not\subset Z(\mathcal{R})$, by Lemma (2.3) , the last equation gives

$\phi_1(ur, u_2, \ldots, u_n) = 0$ for all $u, u_2, \ldots, u_n \in U, r \in \mathcal{R}$.

Case 2: If $U \subset Z(\mathcal{R})$ and since $char(\mathcal{R}) \neq 2$ then $w^2\phi_1$(ur,$u_2, \ldots, u_n$)=0 for all w, $uu_2, \ldots, u_n \in U, r \in \mathcal{R}$ and this implies that $0 = cw^2\phi_1(uru_2, \ldots, u_n) = w^2c\phi_1(ur, u_2, \ldots, u_n)$=0 the primness of $\mathcal{R}$ implies that either $w^2 = 0$ or $\phi_1(ur, u_2, \ldots, u_n)$=0. Since $U \neq 0$, hence $\phi_1(ur, u_2, \ldots, u_n)$=0 for all $u, u_2, \ldots, u_n \in U, r \in \mathcal{R}$.

3. Symmetric Higher.$(U, \mathcal{R})$-n-Derivation

The concept of symmetric higher $(U, \mathcal{R})$-n-Derivation is introduced and studied to extend the results of section.1.

Motivated by definition (2.2) and the definition of higher derivation, the concept of symmetric higher $(U,\mathcal{R})$-n-derivation can be defined as follows

Definition (3.1):

Let U.be a Lie ideal of a ring $\mathcal{R}$. A family of additive mappings D=$(d_n)_{n\in N}$ of $\mathcal{R} \times \mathcal{R} \times \ldots \times \mathcal{R}$ into $\mathcal{R}$ is said to be a symmetric higher $(U,\mathcal{R})$-n-derivation if for all $u, u_2, \ldots, u_n \in U, r, s \in \mathcal{R}, n \in N$, the following equations are equivalent

$d_n(u_1r + su_1, u_2, \ldots, u_n) = \sum_{i+j=n} d_i(u_1, u_2 \ldots, u_n)d_j(r, u_2 \ldots, u_n) + d_i(s, u_2, \ldots, u_n)d_j(u_1, u_2 \ldots, u_n)$

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$d_n(u_1,u_2,\ldots,u_nr + su_n) = \sum_{i+j=n} d_i(u_1,u_2\ldots,u_n)d_j(u_1,u_2\ldots,r) + d_i(u_1,u_2,\ldots,s)d_j(u_1,u_2\ldots,u_n)$

Further, if $\mathcal{R}$ is a 2-torsion free and $r = s = u_n$ for all $n \in N$ then
$d_n(u^2,u_2,\ldots,u_n) = \sum_{i+j=n} d_i(u,u_2\ldots,u_n)d_j(u,u_2\ldots,u_n)$ for all $u, u_2,\ldots,u_n \in U$. ... $(*)$

The following example explains the concept of symmetric higher $(U,\mathcal{R})$-n-derivation.

Example (3.2):

Let S be a commutative ring of characteristic.2.numbers. Consider $\mathcal{R}$=$\mathcal{M}_2(S)$.is the ring of all 2 × 2 matrices over S with addition and multiplication of matrices. It is simple matter to check that U=$\left\{\begin{pmatrix} u & v \\ v & u \end{pmatrix} \mid u,v \in S\right\}$ is a Lie ideal of $\mathcal{R}$. Define d_n: $\mathcal{R} \times \mathcal{R} \times \ldots \times \mathcal{R} \to \mathcal{R}$ as follows

$$d_n\left(\begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix},\begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix},\ldots,\begin{pmatrix} a_n & b_n \\ c_n & d_n \end{pmatrix}\right) = \begin{cases} \begin{pmatrix} 0 & -b_1b_2\ldots b_n \\ nc_1c_2\ldots c_n & 0 \end{pmatrix}, & for\ n = 1,2 \\ \quad n & \\ \quad n & \\ 0 & , n \geq 3 \end{cases}$$

For all $\begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix},\begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix},\ldots,\begin{pmatrix} a_n & b_n \\ c_n & d_n \end{pmatrix} \in \mathcal{R}, n \in N$, then d_n is a symmetric higher *(U,$\mathcal{R}$)-n*-derivation of $\mathcal{R} \times \mathcal{R} \times \ldots \times \mathcal{R}$.

Lemma (3.3):

Let $\mathcal{R}$ be a prime ring of $char(\mathcal{R}) \neq 2$, U be an admissible Lie ideal of $\mathcal{R}$ and $D = (d_n)_{n\in N}$ be a family of mappings satisfying equation $(*)$. Then
$d_n(uw + wu,u_2,\ldots,u_n)$=$\sum_{i+j=n} d_i(u,u_2\ldots,u_n)d_j(w,u_2\ldots,u_n)$ +$d_i(w,u_2\ldots,u_n)d_j(u,u_2\ldots,u_n)$ for all $u,w, u_2,\ldots,u_n \in U, n \in N$.

Proof: By hypothesis
$d_n(u^2,u_2,\ldots,u_n) = \sum_{i+j=n} d_i(u,u_2\ldots,u_n)d_j(u,u_2\ldots,u_n)$, for all $u,u_2,\ldots,u_n \in U, n \in N$. ...(1)

Putting $u = u + w$ in, equation (1) where $w \in U$ we obtain
$d_n((u + w)^2,u_2,\ldots,u_n) = \sum_{i+j=n} d_i(u,u_2\ldots,u_n)d_j(u,u_2\ldots,u_n)$ +$d_i(u,u_2\ldots,u_n)d_j(w,u_2\ldots,u_n)$
+ $d_i(w,u_2\ldots,u_n)d_j(u,u_2\ldots,u_n)$+$d_i(w,u_2\ldots,u_n)d_j(w,u_2\ldots,u_n)$ ... (2)

On the other hand, we have

$d_n((u + w)^2,u_2,\ldots,u_n) = d_n(u^2,u_2\ldots,u_n) + d_n(uw + wu,u_2\ldots,u_n) + d_n(w^2,u_2\ldots,u_n)$. ... (3)

Compare the Equations (2) and (3) yields

$d_n(uw + wu,u_2\ldots,u_n)$=$\sum_{i+j=n} d_i(u,u_2\ldots,u_n)d_j(w,u_2\ldots,u_n)$ +$d_i(w,u_2\ldots,u_n)d_j(u,u_2\ldots,u_n)$ for all $u,w, u_2,\ldots,u_n \in U$.

Lemma (3.4):

Let $\mathcal{R}$ be a 2-torsion-free ring and D=$(d_n)_{n\in N}$ be an higher $(U,\mathcal{R})$-n- derivation of $\mathcal{R}$. Then

$d_n(uru,u_2,\ldots,u_n) = \sum_{i+j+k=n} d_i(u, u_2,\ldots,u_n)d_j(r,u_2,\ldots,u_n)d_k(u,u_2,\ldots,u_n)$ for all $u, u_2,\ldots,u_n \in U, ur \in \mathcal{R}, n \in N$.

Proof: Let $r = s = (2u)r + r(2u)$ in Definition (3.1) to get $d_n(u((2u)r + r(2u)) + ((2u)r + r(2u))u,u_2,\ldots,u_n)$
$=2\sum_{i+j=n}^{n\geq 1} d_i(u,u_2,\ldots,u_n)\, d_j(ur + ru,u_2,\ldots,u_n) + d_i(ur + ru,u_2,\ldots,u_n)d_j(u,u_2\ldots,u_n)$
$=2\sum_{i+h+g=n}(d_i(u,u_2\ldots,u_n)\, d_h(u,u_2\ldots,u_n)d_g(r,u_2\ldots,u_n) +$

$d_i(u,u_2...,u_n)d_h(r,u_2...,u_n)\ d_g(u,u_2...,u_n))$+2$\sum_{p+q+j=n} d_p(u,u_2...,u_n)\ d_q(r,u_2...,u_n)d_j(u,u_2...,u_n)$+ $d_p(r,u,u_2...,u_n)\ d_q(u,u_2...,u_n)d_j(u,u_2...,u_n))$ ... (1)

On the other hand,

$d_n(u((2u)r + r(2u)) + ((2u)r + r(2u))u,u_2...,u_n)$=$d_n((2u^2)r + r(2u^2)),u_2...,u_n)$+4$d_n$(uru,$u_2...,u_n$)
=2$\sum_{a+b+j=n} d_r(u,u_2...,u_n)\ d_b(u,u_2...,u_n)d_j(r,u_2...,u_n)$+2
$\sum_{i+h+g=n}^{n\geq 1} d_i(r,u_2...,u_n)\ d_h(u,u_2...,u_n)d_g(u,u_2...,u_n)$ + 4$d_n(uru,u_2...,u_n)$. ... (2)

By comparing equations (1) and (2) and since $\mathcal{R}$ is 2-torsion-free, we get
$d_n(uru,u_2...,u_n) = \sum_{i+j+k=n} d_i(u,u_2...,u_n)\ d_j(r,u_2...,u_n)d_k(u,u_2...,u_n)$... (3)

The following corollary is a linearization of equation (3) on u.

Corollary (3.5):

Let. $\mathcal{R}$.be.a 2-torsion-free ring and $D = (d_n)_{n\in N}$ be a higher $(U,\mathcal{R})$-n- derivation of $\mathcal{R}$. Then

$d_n(urw + wru,u_2...,u_n) = \sum_{i+j+k=n} d_i(u,u_2...,u_n)\ d_j(r,u_2...,u_n)d_k(w,u_2...,u_n)$+ $d_i(w,u_2...,u_n)d_j(r,u_2...,u_n)d_k(u,u_2...,u_n)$ for all $u,w\ ,u_2,...,u_n \in U, r\in\mathcal{R}, n\in N$.

Lemma (3.6) [5, Lemma 2.2]: Let $\mathcal{R}$ be a 2-torsion free prime ring and U be an admissible Lie ideal of $\mathcal{R}$. If $\in U$, $b\in\mathcal{R}$ are such that $axb + bxa = 0$ for all $x\in\mathcal{R}$, then $axb = bxa = 0$ for all $x\in\mathcal{R}$.

Lemma (3.7) [5, Lemma 2.1]:

Let $\mathcal{R}$ be a 2-torsion free prime ring and $U \not\subset Z(\mathcal{R})$ be a Lie ideal of $\mathcal{R}$. If $a,b\in\mathcal{R}$ such that $aUb = 0$, then either $a = 0$ or $b = 0$.

Lemma (3.8):

Let $\mathcal{R}$ be a prime ring of $char(\mathcal{R}) \neq 2$, U be an admissible Lie ideal of $\mathcal{R}$ and $D = (d_n)_{n\in N}$be a symmetric higher $(U,\mathcal{R})$-n-derivation. Then $\phi_n(uw,u_2,...,u_n)$ =0 for all $u,w\ ,u_2,...,u_n \in U, n\in N$.

Proof: Since $\phi_0(uw,u_2,...,u_n) = 0$ and by Theorem (2.15), $\phi_n(uw,u_2,...,u_n) = 0$ for all $u,w\ ,u_2\ ,...,u_n \in U, r\in\mathcal{R}, n\in N$. Assume by induction on $n\in N$**,** that $\phi_m(uw,u_2,...,u_n) = 0$ for all $u,w\ ,u_2,...,u_n \in U, n,m\in N, m < n$.

Let $H = d_n(4(uwvwu + wuvuw),u_2,...,u_n)$, by Lemma (3.6) we conclude that
$H =$
$4\{d_n(uw,u_2,...,u_n)vwu + uwvd_n(wu,u_2,...,u_n) + \sum_{\mathrm{i+j+k}=n}^{\mathrm{i,k}<n} d_i(\mathrm{uw},u_2,...,u_n)\ d_j(\mathrm{v},u_2,...,u_n)\ d_k(wu,u_2,...,u_n)$
$+d_n(wu,u_2,...,u_n)vuw + wuvd_n(uw,u_2,...,u_n)\sum_{\mathrm{i+j+k}=n}^{\mathrm{i,k}<n} d_i(\mathrm{wu},u_2,...,u_n)\ d_j(\mathrm{v},u_2,...,u_n)\ d_k(uw,u_2,...,u_n)\}$

By Lemma (3.5) one can compute H in other way

$$H = 4\{uwv\sum_{s+k=n} d_s(w,u_2,...,u_n)\ d_k(u,u_2,...,u_n) + \sum_{\mathrm{i+j+k}=n} d_i(u,u_2,...,u_n)\ d_j(w,u_2,...,u_n)vwu$$

$$+\sum_{\mathrm{i+p+q+s+k}=n}^{\mathrm{i+p,s+k}<n} d_i(u,u_2,...,u_n)\ d_p(\mathrm{w},u_2,...,u_n)\quad d_q(v,u_2,...,u_n)d_s(w,u_2,...,u_n)d_k(u,u_2,...,u_n)$$

$$+wuv\sum_{s+k=n} d_s(u,u_2,...,u_n)\ d_k(w,u_2,...,u_n) + \sum_{\mathrm{i+j+k}=n} d_i(w,u_2,...,u_n)\ d_j(u,u_2,...,u_n)vuw$$
$$+\sum_{\mathrm{i+p+q+s+k}=n}^{\mathrm{i+p,s+k}<n} d_i(w,u_2,...,u_n)\ d_p(\mathrm{u},u_2,...,u_n)\quad d_q(v,u_2,...,u_n)d_s(u,u_2,...,u_n)d_k(w,u_2,...,u_n).$$

Since $\phi_m(uw, u_2, \ldots, u_n) = 0$ for all $u, w, u_2, \ldots, u_n \in U, n, m \in N, m < n$ and $char(\mathcal{R}) \neq 2$, comparing both sides of H implies that

$\phi_n(uw, u_2, \ldots, u_n)v[u, w] + [u, w]v\phi_n(uw, u_2, \ldots, u_n) = 0$

For all $u, w, u_2, \ldots, u_n \in U, n \in N$. By Lemma (3.6), $\phi_n(uw, u_2, \ldots, u_n)U[u, w] = 0$ for all $u, w, u_2, \ldots, u_n \in U, n \in N$.

In view of Lemma (3.7) and since $U \not\subset Z(\mathcal{R})$, then $\phi_n(uw, u_2, \ldots, u_n) = 0$ for all $u, w, u_2, \ldots, u_n \in U, n \in N$.

Lemma (3.9):

Let $\mathcal{R}$ be a prime ring of $char(\mathcal{R}) \neq 2$, U be an admissible Lie ideal of $\mathcal{R}$ and $D = (d_n)_{n\in N}$ be an higher $(U, \mathcal{R})$-n-derivation of $\mathcal{R}$ then $\phi_n(u^2 r, u_2 \ldots, u_n) = 0$ for all $u, u_2, \ldots, u_n \in U, r \in \mathcal{R}, n \in N$.

Proof: we have $\phi_n(uw, u_2 \ldots, u_n) = 0$ for all $u, u_2, \ldots, u_n \in U, n \in N$.

Putting $w = ur - ru$ in the last relation, where $w \in U$, we obtain $0 = \phi_n(u(ur - ru), u_2 \ldots, u_n) = d_n(u^2 r, u_2, \ldots, u_n)$-$d_n(uru, u_2, \ldots, u_n)$-$\sum_{i+j=n} d_i(u, u_2 \ldots, u_n)$

$d_j(ur - ru, u_2 \ldots, u_n)$=$d_n(u^2 r, u_2, \ldots, u_n)$-$d_n(uru, u_2, \ldots, u_n)$-
$\sum_{i+p+q=n} d_i(u, u_2 \ldots, u_n)\, d_p(u, u_2 \ldots, u_n) d_q(r, u_2 \ldots, u_n)$+$\sum_{i+p+q=n} d_i(u, u_2 \ldots, u_n)\, d_p(r, u_2 \ldots, u_n) d_q(u, u_2 \ldots, u_n)$

$=d_n(u^2 r, u_2, \ldots, u_n) - \sum_{h+q=n} (\sum_{i+p=h} d_i(u, u_2 \ldots, u_n)\, d_p(u, u_2 \ldots, u_n))\, d_q(r, u_2 \ldots, u_n) = \phi_n(u^2 r, u_2 \ldots, u_n)$.

Now, we can prove our main theorem which a generalization of [4, Theorem 2.1] and [2, Theorem].

Theorem (3.10) :

Let $\mathcal{R}$ be a prime ring of $char(\mathcal{R}) \neq 2$, U be an admissible Lie ideal of $\mathcal{R}$ and $D = (d_n)_{n\in N}$ be a symmetric higher $(U, \mathcal{R})$-n-derivation. Then $\phi_n(ur, u_2, \ldots, u_n) = 0$ for all $u, u_2, \ldots, u_n \in U, r \in \mathcal{R}, n \in N$.

Proof : Since $\phi_0(ur, u_2 \ldots, u_n) = 0$, by Theorem (2.17) $\phi_1(ur, u_2 \ldots, u_n) = 0$, then we can assume that $\phi_m(ur, u_2 \ldots, u_n) = 0$. Since D is higher $(U, \mathcal{R})$-n-derivation ,in this case we have

$d_n(uur + uru, u_2 \ldots, u_n) = \sum_{i+j=n} d_i(u, u_2 \ldots, u_n)\, d_j(ur, u_2 \ldots, u_n) + d_i(ur, u_2 \ldots, u_n) d_j(u, u_2 \ldots, u_n)$
$=ud_n(ur, u_2 \ldots, u_n) + d_n(u, u_2 \ldots, u_n)ur + \sum_{i+j=n}^{i,j<n} d_i(u, u_2 \ldots, u_n)\, d_j(ur, u_2 \ldots, u_n) + urd_n(u, u_2 \ldots, u_n) +$
$d_n(ur, u_2 \ldots, u_n)u + \sum_{i+j=n}^{i,j<n} d_i(ur, u_2 \ldots, u_n)\, d_j(u, u_2 \ldots, u_n)$
$=ud_n(ur, u_2 \ldots, u_n) + d_n(u, u_2 \ldots, u_n)ur + \sum_{i+j=n}^{i,j<n} d_i(u, u_2 \ldots, u_n) \sum_{p+q=n}^{i,j<n} d_p(u, u_2 \ldots, u_n)\, d_q(r, u_2 \ldots, u_n) + ur$
$d_n(u, u_2 \ldots, u_n) + urd_n(ur, u_2 \ldots, u_n) + d_n(ur, u_2 \ldots, u_n)u +$

$\sum_{i+j=n}^{i,j<n} \sum_{h+g=i}^{h,g<i} d_h(u, u_2 \ldots, u_n)\, d_g(r, u_2 \ldots, u_n)\, d_j(u, u_2 \ldots, u_n) = ud_n(ur, u_2 \ldots, u_n) + d_n(u, u_2 \ldots, u_n)ur +$
$\sum_{i+p+q=n}^{i,j<n} d_i(u, u_2 \ldots, u_n)\, d_p(u, u_2 \ldots, u_n) d_q(r, u_2 \ldots, u_n) + urd_n(u, u_2 \ldots, u_n) + d_n(ur, u_2 \ldots, u_n)u +$
$\sum_{h+g+j=n}^{h,g,j<n} d_h(u, u_2 \ldots, u_n)\, d_g(r, u_2 \ldots, u_n)\, d_j(u, u_2 \ldots, u_n)$ ... (1)

Also, on the other hand and by Lemma (3.4) and Lemma (3.9) we get

$d_n(uur - uru, u_2 \ldots, u_n) = \sum_{i+j=n}^{n\geq 1} d_i(u^2, u_2, \ldots, u_n)\, d_j(r, u_2, \ldots, u_n)$-
$\sum_{i+j+k=n}^{n\geq 1} d_i(u, u_2 \ldots, u_n)\, d_j(r, u_2 \ldots, u_n) d_k(u, u_2 \ldots, u_n)$=
$\sum_{i+j=n}^{n\geq 1} \sum_{g+h=j}^{n\geq 1} d_g(u, u_2 \ldots, u_n)\, d_h\, (u, u_2 \ldots, u_n)\, d_j(r, u_2 \ldots, u_n)$+

$\sum_{i+j+k=n} d_i(u,u_2...,u_n)\, d_j(r,u_2...,u_n) d_k(u,u_2...,u_n) = \sum_{g+h+j=n}^{n\geq 1} d_g(u,u_2...,u_n)\, d_h(u,u_2...,u_n)$

$d_j(r,u_2...,u_n) + \sum_{i+j+k=n} d_i(u,u_2...,u_n)\, d_j(r,u_2...,u_n) d_k(u,u_2...,u_n)$

$= d_n(u,u_2...,u_n)ur + u\sum_{h+j=n} d_h(u,u_2...,u_n)\, d_j(r,u_2...,u_n) + \sum_{g+h+j=n} d_g(u,u_2...,u_n)\, d_h(u,u_2...,u_n)$

$d_k(r,u_2...,u_n) + ur d_n(u,u_2...,u_n) + \sum_{i+j=n}^{n\geq 1} d_i(u,u_2,...,u_n)\, d_j(r,u_2,...,u_n)\, u +$

$\sum_{i+j+k=n} d_i(u,u_2,...,u_n)\, d_j(r,u_2,...,u_n) d_k(u,u_2,...,u_n)$ ... (2)

By comparing equations (1) and (2) to get

$\phi_n(ur,u_2...,u_n)u + u\phi_n(ur,u_2...,u_n) = 0$ ... (3)

A linearization Equation (3) yield $\phi_n(ur,u_2...,u_n)w + \phi_n(wr,u_2...,u_n)u + u\phi_n(wr,u_2...,u_n) +$

$\phi_n(ur,u_2...,u_n) = 0$, for all $u, u_2, ..., u_n \in U, r \in \mathcal{R}$, $n \in N$. ... (4)

Let $w = w^2$ in Equation (4), then using Lemma (3.9) we get $\phi_n(ur,u_2...,u_n) = 0$ for all $u, u_2, ... u_n \in U$, $r \in \mathcal{R}$, $n \in N$.

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