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ABSTRACT

Throughout this paper by a space we mean a supra topological space, we have studied some of properties to new set is called supra generalized-cocompact open set (𝔰-g-coc-open set)and find the relation with other sets and our concluded a new class of the function called 𝔰-g-coc-continuous, 𝔰-g-coc'-continuous . We shall provide some properties of these concepts and it will explain the relationship among them and some results on this subjects are proved. Throughout this work , and new concept have been illustrated including , 𝔰-g-coc-compact space .

MSC: 30C45, 30C50

1. Introduction

In 1983, by dropping the finite intersection condition of topological spaces, Mashhour et al.[3] came up with an idea of supra topological spaces and it deals with groups called supra open sets. They introduced the notions of (irresolute) S*-continuous maps and investigated some of their properties. Ravi et al. [2] introduced the idea of supra g-closed sets and obtained other properties of supra g-closed sets. [1] Al-shami introduced some studies on supra topology and the definition of supra open sets and their properties and presented some types of supra compact space . in [4],[5],[6] B. Meera Devi, D.K. Nathan R. Selvarathi. O. R. SAYED,] B.Meera Devi , respectively, introduced new class of functions in supra topological spaces, namely supra $\delta \hat{g}$-continuous functions. In [7] S. Al Ghour and S. Samarah introduced coc-open sets in topological spaces.

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2. Preliminaries

Definition(2.1) [3] If X belongs to μ and μ is closed under arbitrary union, a subcollection μ of 2^X is considered a supra topology on X. That μ elements is referred to as a supra open sets (ꞩ-open) of (X,μ) and its complement is referred to as supra closed sets (ꞩ-closed) .The supra closure of A (denoted by $\overline{A}^{ꞩ}$) is the intersection of all supra closed sets containing A and the supra interior of A (denoted by$A^{\circ ꞩ}$) is the union of all supra open sets contained in A.The supra relative topology μy on Y is defined as μy ={ Y⋂G : G ∊ μ }. A pair (Y, μy) is called supra subspace of (X, μ).

Definition(2.2) [2]Let (X, μ) be a supra topological space. A subset A of X is called supra g-closed (ꞩ-g-closed) if $\overline{A}^{ꞩ}$ ⊆U whenever A ⊆ U and U is supra open in (X, μ). And A is called supra g-open (ꞩ-g-open), if A^c is supra g-closed .

Remark(2.3) [2] Every ꞩ-closed set implies ꞩ-g-closed set,ꞩ-g-closed set need not imply ꞩ-closed sets, ꞩ-open set mplies ꞩ-g-open and ꞩ-g-open must not imply ꞩ-open set .

Theorem(2.4) [2] A subset of supra topological space in (X, μ) is ꞩ -g-open set if and only if B ⊆ $A^{\circ ꞩ}$ whenever B is ꞩ -closed and B ⊆ A **.**

Definition (2.5) [1] A supra topological spaces (X, μ) is called supra compact (ꞩ-compact for short) provided that every supra open cover of X has a finite subcover .

Definition (2.6) [1]. A collection {Gi : i ∊ I} of supra open sets in a supra topological spaces (X,μ) is called supra open cover of a subset E of X provided that E ⊆∪{Gi : i ∊ I}.

3. On supra generalized co-compact-open set and supra generalized co-compact -continuous functions

In this section, we introduce the definition of ꞩ-g-coc-open set, ꞩ-g-coc-closed set , ꞩ-g-coc-continuous , ꞩ-g-coc'-continuous , remarks and propositions about this concept .

Definition (3.1) A subset A of a space (X. μ) is called supra ganeralized cocompact open set (ꞩ-g-coc-open set) if for every $x \in A$ there exists ꞩ -g-open set $U \subseteq X$ and ꞩ -compact subset K such that $x \in U - K \subseteq A$, the complement of ꞩ-g-coc-open set is called ꞩ -g-coc-closed set . The family of all ꞩ-g-coc-open subsets of a space (*X*, μ) will be denoted by μ^{gk} .

Example(3.4) Let X = {a, b, c} and μ = {Ø ,X,{a, b},{a, c}}. Then μ^{gk} = p(X)

Proposition (3.5)

i- Every ꞩ-open set is ꞩ-g-coc-open set.

iii-Every ꞩ-g-open set is ꞩ-g-coc-open set.

proof

i. Let A ꞩ-open set Then A ꞩ-g-open and let K ꞩ-compact (∅).Then for all x∊A we have $x \in U - K \subseteq A$

ii. Let A ꟗ-g-open and K ꟗ-compact (∅).Then for all x∈ A we have $x \in U - K \subseteq A$

but the convers of i and ii are not true for the following example :-

Example (3.6)

Let $X = N$. $\mu = \{\emptyset . X . X - \{1\}\}$ is supra topological space. It is clear that $\{1\}$ ꟗ-g-coc-open set but not ꟗ-open and not ꟗ-g-open set .

Remark (3.7) :- Every ꟗ-g-closed is ꟗ-g-coc-closed but the converse is not true as in the example (3.6) .It is clear that $\{1\}^c$ ꟗ-g-coc-closed set but$\{1\}^c$ not ꟗ-g-closed set .

Proposition (3.8) :- The union of ꟗ-g-coc-open is ꟗ-g-coc-open set

Proof

let $\{A_\ltimes : \ltimes \in \Lambda\}$ ꟗ-g-coc-open sets . let $x \in A_\ltimes$ .Then for some $\ltimes \in \Lambda$ .Then there exists $U_\ltimes$ ꟗ-g-open set and $K_\ltimes$ ꟗ-compact such that $x \in U_\ltimes - K_\ltimes \subseteq A_\ltimes \subseteq \cup A_\ltimes$ then $x \in U_\ltimes - K_\ltimes \subseteq \cup A_\ltimes$,then $\cup A_\ltimes$ ꟗ-g-coc-open set .

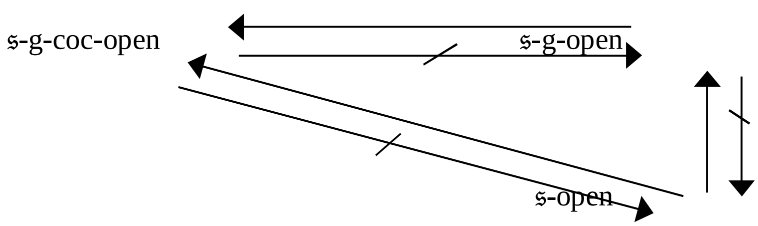
Theorem (3.9) :- μ^{gk} forms supra topology on X .

Proof.

By the definition one has directly that $\emptyset \in \mu^{gk}$.To see that $X \in \mu^{gk}$, let $x \in X$, take U =X and K = ∅. Then $x \in U - K \subseteq X$.

Let $\{U\alpha : \alpha \in \Delta\}$ be a collection of ꟗ-g-coc-open subsets of (X, μ) and $x \in \cup_{\alpha\in\Delta} U\alpha$. Then there exists $\alpha 0 \in \Delta$ such that $x \in U\alpha\circ$. Since $U\alpha\circ$ is ꟗ-g-coc-open, then there exists an ꟗ-g-open set V and ꟗ-compact subset K, such that $x \in V - K \subseteq U\alpha\circ$.Therefore, we have $x \in V - K \subseteq U\alpha\circ \subseteq \cup_{\alpha\in\Delta} U\alpha$.Hence, $\cup_{\alpha\in\Delta} U\alpha$ is ꟗ-g-coc-open.

The following diagram shows the relation between types of ꟗ-coc-open sets



Definition (3.10) Let X be a space and A ⊆ X. The intersection of all s-g-coc-closed sets of X containing A is called the s-g-coc-closure of A defined by .

$$\overline{A}^{ꟗ-g-coc} = \cap \{B : B\ ꟗ - g - coc - closed\ in\ X\ and\ A \subseteq B\}$$

Proposition (3.11) Let X be a space and A ⊆ X , then $x \in \overline{A}^{ꟗ-g-coc}$ if and only if for each ꟗ-g-coc-open in X contained point x we have $U \cap A \neq \emptyset$.

Proof :

Asume that $x \in \overline{A}^{ꟗ-g-coc}$ and let U ꟗ-g-coc-open in X , such that x∈U , and suppose U⋂A=∅ , then $A \subseteq U^c$ since U is ꟗ-g-coc-open set in X , x∈U, then U^c is ꟗ-g-coc-closed set in X , $x \notin U^c$, and $\overline{A}^{ꟗ-g-coc}$ is smallest ꟗ-g-coc-closed set contain A, then $\overline{A}^{\mathfrak{s}-g-coc} \subseteq U^c$, since $U \cap U^c = \emptyset$ $x \in U$ then $x \notin U^c$,then $x \notin \overline{A}^{\mathfrak{s}-g-coc}$, and this contradiction ,thus $\forall$ ꞩ-g-open U containing x we have $U \cap A \neq \emptyset$.

Conversely; Let U is ꞩ-g- coc-open set in X , such that $x \in U$ an $U \cap A \neq \emptyset$ .To prove $x \in \overline{A}^{\mathfrak{s}-g-coc}$, let $x \notin \overline{A}^{\mathfrak{s}-g-coc}$,then $x \in (\overline{A}^{\mathfrak{s}-g-coc})^c$,since $\overline{A}^{\mathfrak{s}-g-coc}$ is ꞩ-g-coc-closed, then $(\overline{A}^{\mathfrak{s}-g-coc})^c$ is ꞩ-g-coc-open set in X ,and $\overline{A}^{\mathfrak{s}-g-coc} \cap (\overline{A}^{\mathfrak{s}-g-coc})^c = \emptyset$,which is contradiction since $A \cap U \neq \emptyset$,$\forall$ U is ꞩ-g-coc-open set in X , such that $x \in U$.

Proposition (3.12)

Let X be a space and $A \subseteq B \subseteq X$ then

1. $\overline{A}^{\mathfrak{s}-g-coc}$ is ꞩ-g-coc-closed set
2. A is ꞩ-g-coc-closed if and only if $A = \overline{A}^{\mathfrak{s}-g-coc}$
3. $\overline{A}^{\mathfrak{s}-g-coc} = \overline{\overline{A}^{\mathfrak{s}-g-coc}}^{\mathfrak{s}-g-coc}$
4. If $A \subseteq B$ then $\overline{A}^{\mathfrak{s}-g-coc} \subseteq \overline{B}^{\mathfrak{s}-g-coc}$
5. $\overline{A}^{\mathfrak{s}-g-coc} \subseteq \overline{A}^{\mathfrak{s}}$

Proof :-

1. By Proposition (3.8) .
2. Let A be ꞩ-g-coc-closed in X . Since $A \subseteq \overline{A}^{\mathfrak{s}-g-coc}$ and $\overline{A}^{\mathfrak{s}-g-coc}$ smallest ꞩ-g-coc-closed set containing A , then $\overline{A}^{\mathfrak{s}-g-coc} \subseteq A$ then $A = \overline{A}^{\mathfrak{s}-g-coc}$

   conversely :- Let $A = \overline{A}^{\mathfrak{s}-g-coc}$. Since $\overline{A}^{\mathfrak{s}-g-coc}$ is ꞩ-g-coc-closed then A is ꞩ-g-coc-closed.
3. From (1) and (2)
4. Let $A \subseteq B$. Since $B \subseteq \overline{B}^{\mathfrak{s}-g-coc}$ then $A \subseteq \overline{B}^{\mathfrak{s}-g-coc}$ Since $\overline{B}^{\mathfrak{s}-g-coc}$ smallest ꞩ-g-coc-closed set containing A then $\overline{A}^{\mathfrak{s}-g-coc} \subseteq \overline{B}^{\mathfrak{s}-g-coc}$ (since $\overline{A}^{\mathfrak{s}-g-coc}$ smallest ꞩ-g-coc-closed set containing A).
5. Let $x \in \overline{A}^{\mathfrak{s}-g-coc}$ then for all ꞩ-g-coc-open set U ,such that $x \in U$ we have $U \cap A \neq \emptyset$ .Then for all ꞩ-open set U ,such that $x \in U$,we have $A \cap U \neq \emptyset$ by proposition (3.11). Then $x \in \overline{A}^{\mathfrak{s}}$.

Definition (3.13) Let X be a space and $A \subseteq X$. The union of all ꞩ-g-coc-open sets of X containing in A is called ꞩ-g-coc-Interior of A denoted by $A^{\circ\mathfrak{s}-g-coc}$ or ꞩ-g-coc-In (A)

$$A^{\circ\mathfrak{s}-g-coc} = \cup \{B : B \ \ \mathfrak{s}-g-coc-\text{open in } X \text{ and } B \subseteq A\}$$

Proposition (3.14) Let X be a space and $A \subseteq X$, then $x \in A^{\circ\mathfrak{s}-g-coc}$ if and only if there exists ꞩ-g-coc- open set V containing x such that $x \in V \subseteq A$.

Proof :

Let $x \in A^{\circ\mathfrak{s}-g-coc}$ then $x \in \cup V$ such that V ꞩ-g-coc-open set and $x \in V \subseteq A$.

Conversely; Let there exists V ꞩ-g-coc-open set ,such that $x \in V \subseteq A$ then $x \in \cup V$, $V \subseteq A$ and V ꞩ-g-coc-open set ,then $x \in A^{\circ\mathfrak{s}-g-coc}$.

Proposition (3.15)

Let X be a space and $A \subseteq B \subseteq X$,then .

1. $A^{\circ\mathfrak{s}-g-coc}$ is ꞩ-g-coc-open set .
2. A is ꞩ-g-coc-open if and only if $A = A^{\circ\mathfrak{s}-g-coc}$.
3. $A^{\circ\mathfrak{s}} \subseteq A^{\circ\mathfrak{s}-g-coc}$.

4. $A^{\circ\mathfrak{s}-g-coc} = (A^{\circ\mathfrak{s}-g-coc})^{\circ\mathfrak{s}-g-coc}$.
5. if $A \subseteq B$ then $A^{\circ\mathfrak{s}-g-coc} \subseteq B^{\circ\mathfrak{s}-g-coc}$.

Proof :-

1. By Proposition (3.8) .

2. let A is $\mathfrak{s}$-g-coc-open ,since $A^{\circ\mathfrak{s}-g-coc}$ largest $\mathfrak{s}$-g-coc-open containing A ,then $A^{\circ\mathfrak{s}-g-coc} \subseteq$ A and since $A \subseteq A^{\circ\mathfrak{s}-g-coc}$, then $A = A^{\circ\mathfrak{s}-g-coc}$.

Conversely; Let $A = A^{\circ\mathfrak{s}-g-coc}$,since $A^{\circ\mathfrak{s}-g-coc}$ is $\mathfrak{s}$-g-coc-open ,then A is $\mathfrak{s}$-g-coc-open .

3. Let $x \in A^{\circ\mathfrak{s}}$ then there exists U $\mathfrak{s}$-open set such that $x \in U \subseteq A$ then U $\mathfrak{s}$-g-coc-open set then U $\mathfrak{s}$-g-coc-open set such that $x \in U \subseteq A$ thus $x \in A^{\circ\mathfrak{s}-g-coc}$

4. from (1) and (2).

5. Let $x \in A^{\circ\mathfrak{s}-g-coc}$ then there exists V $\mathfrak{s}$-g-coc-open set such that $x \in V \subseteq A$ by proposition(3.19), since $A \subseteq B$ then $x \in V \subseteq B$ .Then $x \in B^{\circ\mathfrak{s}-g-coc}$ by proposition(3.14),. Thus $A^{\circ\mathfrak{s}-g-coc} \subseteq B^{\circ\mathfrak{s}-g-coc}$.

Definition (3.16)[3] Let $f: X \longrightarrow Y$ be a function of space X into space Y.Then f is called supra-irresolute ($\mathfrak{s}$-continuous) function if $f^{-1}(A)$ is $\mathfrak{s}$-open set in X for every $\mathfrak{s}$-open set A in Y

Definition (3.17) Let $f: X \longrightarrow Y$ be a function of a space X into a space Y. f is called $\mathfrak{s}$-g-coc-continuous function if $f^{-1}(A)$ is $\mathfrak{s}$-g-coc-open set in X for every $\mathfrak{s}$-open set in Y .

Proposition (3.18) Every $\mathfrak{s}$-continuous is $\mathfrak{s}$-g-coc-continuous

Proof

Let $f: X \longrightarrow Y$ be $\mathfrak{s}$-continuous function and A $\mathfrak{s}$-open set in Y. Thus $f^{-1}(A)$ is $\mathfrak{s}$-open set in X . Then $f^{-1}(A)$ is $\mathfrak{s}$-g-coc-open set in X .Then f is $\mathfrak{s}$-g-coc-continuous

But the convers not true in general for example

Example (3.19)

Let $X = \mathrm{N} . \mu = \{\emptyset. X. \{2\}\}$ supra topology on $X . Y = \{a. b. c\}$. $\acute{\mu} = \{\emptyset. Y. \{a\}\}$ supra topology on Y and $f: X \longrightarrow Y$ $defined\ by$ $f(x) = \begin{cases} a & if\ x \in \{1.3\} \\ b & if\ x \notin \{1.3\} \end{cases}$. Then f is $\mathfrak{s}$-g-coc-continuous But not $\mathfrak{s}$-continuous

Remark (3.20)

Let $f: X \longrightarrow Y$ be a function of a space X in to a space Y then

i. The constant function is $\mathfrak{s}$-g-coc-continuous
ii. If (X,μ) supra discrete topology then f $\mathfrak{s}$-g-coc-continuous
iii. If X finite set and μ any topology on X then f $\mathfrak{s}$-g-coc-continuous
iv. If (Y,μ') indiscrete topology then f $\mathfrak{s}$-g-coc-continuous

Proposition (3.21)

Let $f: X \longrightarrow Y$ be a function of a space X into a space Y then the following statements are equivalent :-

1. f $\mathfrak{s}$-g-coc-continuous function .
2. $f^{-1}(A^{\circ\mathfrak{s}}) \subseteq \left(f^{-1}(A)\right)^{\circ\mathfrak{s}-g-coc}$ for every set A of Y .
3. $f^{-1}(A)$ $\mathfrak{s}$-g-coc-closed set in X for every $\mathfrak{s}$-closed set A in X .

4. $f(\bar{A}^{\mathfrak{s}-g-coc}) \subseteq \overline{f(A)}^{\mathfrak{s}}$ for every set A of X .
5. $\overline{f^{-1}(A)}^{\mathfrak{s}-g-coc} \subseteq f^{-1}(\bar{A}^{\mathfrak{s}})$ for every set A of Y .

proof:

$1 \longrightarrow 2$

Let $A \subseteq Y$,since $A^{\circ\mathfrak{s}}$ $\mathfrak{s}$ -open set in Y .Then $f^{-1}(A^{\circ\mathfrak{s}})$ $\mathfrak{s}$ -g-coc-open set in X .Thus $f^{-1}(A^{\circ\mathfrak{s}}) = \left(f^{-1}(A^{\circ\mathfrak{s}})\right)^{\circ\mathfrak{s}-g-coc} \subseteq \left(f^{-1}(A)\right)^{\circ\mathfrak{s}-g-coc}$ .Hence $f^{-1}(A^{\circ\mathfrak{s}}) \subseteq \left(f(A)\right)^{\circ\mathfrak{s}-g-coc}$.

$2 \longrightarrow 3$

Let $A \subseteq Y$ such that A $\mathfrak{s}$-closed set in Y.Then A^c $\mathfrak{s}$-open set then $A^c = (A^c)^{\circ\mathfrak{s}}$ then $f^{-1}\left((A^c)^{\circ\mathfrak{s}}\right) \subseteq \left(f^{-1}(A^c)\right)^{\circ\mathfrak{s}-g-coc}$ .Therefore $f^{-1}(A^c) \subseteq \left(f^{-1}(A^c)\right)^{\circ\mathfrak{s}-g-coc}$ then $\left(f^{-1}(A)\right)^c \subseteq (f^{-1}(A)^c)^{\circ\mathfrak{s}-g-coc}$. Therefore$\left(f^{-1}(A)\right)^c = (f^{-1}(A)^c)^{\circ\mathfrak{s}-g-coc}$. Hence $\left(f^{-1}(A)\right)^c$ $\mathfrak{s}$-g-coc-open set in X. Hence $f^{-1}(A)$ $\mathfrak{s}$-g-coc-closed set in X.

$3 \longrightarrow 4$

Let $A \subseteq X$. Then $\overline{f(A)}^{\mathfrak{s}}$ $\mathfrak{s}$-g-coc-closed set in Y .Then by (3) we have $f^{-1}\left(\overline{f(A)}^{\mathfrak{s}}\right)$ is $\mathfrak{s}$-g-coc-closed set in X containing A ,thus $\bar{A}^{\mathfrak{s}-g-coc} \subseteq f^{-1}\left(\overline{f(A)}^{\mathfrak{s}}\right)$ (since $\bar{A}^{\mathfrak{s}-g-coc}$ intersection of all $\mathfrak{s}$-g-coc-closed sets in X containing A) . Hence $f\left(\overline{(A)}^{\mathfrak{s}-g-coc}\right) \subseteq \overline{f(A)}^{\mathfrak{s}}$

$4 \longrightarrow 5$

Let $A \subseteq Y$.Then$f^{-1}(A) \subseteq X$.Then by (4) we have $f\left(\overline{(f^{-1}(A))}^{\mathfrak{s}-g-coc}\right) \subseteq \overline{f(f^{-1}(A))}^{\mathfrak{s}}$. Hence $\overline{f^{-1}(A)}^{\mathfrak{s}-g-coc} \subseteq f^{-1}(\bar{A}^{\mathfrak{s}})$.

$5 \longrightarrow 1$

Let B $\mathfrak{s}$-open set in Y then B^c $\mathfrak{s}$-closed .Then $B^c = \overline{B^c}^{\mathfrak{s}}$. Hence $\overline{f^{-1}(B^c)}^{\mathfrak{s}-g-coc} \subseteq f^{-1}(\overline{B^c}^{\mathfrak{s}})$. Then $\overline{f^{-1}(B^c)}^{\mathfrak{s}-g-coc} \subseteq f^{-1}(B^c)$. Then $f^{-1}(B^c) = (f^{-1}(B))^c$ $\mathfrak{s}$-g-coc-closed set in X .Therefore $f^{-1}(B)$ $\mathfrak{s}$-g-coc-open set in X . Thus f $\mathfrak{s}$-g-coc-continuous function .

Remark (3.22) From proposition (3.21) we have f $\mathfrak{s}$-g-coc-continuous if and only if , the inverse image of every $\mathfrak{s}$-closed set in Y is $\mathfrak{s}$-g-coc- closed set in X .

Definition (3.23) Let $f: X \longrightarrow Y$ be a function of a space X into a space Y, then f is called $\mathfrak{s}$-g-coc irresolute ($\mathfrak{s} - g\text{–}co\acute{c}\text{–}continuous$) function if $f^{-1}(A)$ $\mathfrak{s}$-g-coc-open set in X for every $\mathfrak{s}$-g-coc-open set $in\, Y$.

Proposition (3.24) Every $\mathfrak{s} - \mathrm{g}\text{–}co\acute{c}\text{–}continuous$ function is $\mathfrak{s}$ -g-coc-continuous function

Proof:

Let $f: (X.\mu) \longrightarrow (Y.\mu')$ be $\mathfrak{s} - \mathrm{g}\text{–}co\acute{c} - continuous$ and B $\mathfrak{s}$-open set in Y. Then B is $\mathfrak{s}$-g-coc-open set . Since f $\mathfrak{s} - g\text{–}co\acute{c}\text{–}continuous$ then $f^{-1}(B)$ $\mathfrak{s}$-g-coc-open . Hence f $\mathfrak{s}$-g-coc-continuous function .

But the converse is not true in general for the following example.

Example(3.25): $f: X \longrightarrow Y$ be function defined by $f(x) = \begin{cases} 1 & if\ x \in \mathrm{Ne} \\ 2 & if\ x = \mathrm{x1} \\ 3 & if\ x \in \mathrm{No} \end{cases}$ and $X=x_1 \cup N$ ($x_1 \in R$, N nature number , $\mu =\{ \emptyset. X. A: A \subseteq N \}$. μ' supra indiscrete topology on $Y = \{1.2.3\}$, then the only s-open sets in Y are Y and $\emptyset$, then $f^{-1}(\emptyset) = \emptyset$, $f^{-1}(Y) = X$. Since Y and $\emptyset$ $\mathfrak{s}$-g-coc-open sets in Y then $f^{-1}(\emptyset). f^{-1}(Y)$ $\mathfrak{s}$-g-coc-open in X .Then f ṩ-g-coc-continuous function. But $\{2\}$ ṩ-g-coc-open in Y and $f^{-1}(\{2\}) = \{x1\}$ is not ṩ-g-coc-open set in X . $x1 \in \{x1\}$. There is no ṩ-g-open set U such that $x1 \in U$, and K ṩ-compact such that $x1 \in U - K \subseteq \{x1\}$. Then f is not ṩ-g-coc´-continuous function .

Proposition (3.26) Let $f: X \longrightarrow Y$ be ṩ-g-coc′-continuous then $f^{-1}(A)$ ṩ-g-coc-closed set in X for all A ṩ-g-coc-closed set in Y

proof

Let A is ṩ-g-coc-closed set in Y .Then A^c ṩ-g-coc-open in Y .Since f ṩ-g-coc′-continuous . Then $f^{-1}(A^c)$ is ṩ-g-coc-open in X by definition(2.10). Since $f^{-1}(A^c) = \left(f^{-1}(A)\right)^c$ .Then $\left(f^{-1}(A)\right)^c$ ṩ -g-coc-open set in X .Therefore. $f^{-1}(A)$ ṩ-g-coc-closed set in X for all A ṩ-g-coc-closed set in Y.

Remark (3.27)

i. $ṩ - g - coć -$continuous is need not to be ṩ-continuous function .

ii. ṩ-continuous is need not to be $ṩ - g - coć -$continuous function .

But the converse is not true as the follow example

Examples (3.28)

i--

Let $f: X \longrightarrow Y$. $X = \{1.2.3\}$. $\mu = \{\emptyset . X . \{3\}\}$ supra $topology\ on\ X$ and $Y = \{a.b\}$. $\mu' = \{\emptyset . Y . \{a\}\}$ $supra\ topology\ on\ Y$. $f(1) = f(3) = b$. $f(2) = a$. It is clear that f is ṩ-g-coć-continuous but not ṩ-continuous .

ii- Let $f: X \longrightarrow Y$ be function defined by $f(x) = \begin{cases} 1 & if\ x \in Ne \\ 2 & if\ x = x1 \\ 3 & if\ x \in No \end{cases}$ and X=x_1∪N (x_1∈ R, N natural number) , μ ={ $\emptyset . X . A : A \subseteq N$ } . μ' supra indiscrete topology on $Y = \{1.2.3\}$, then the only ṩ-open sets in Y are Y and $\emptyset$, then $f^{-1}(\emptyset) = \emptyset$, $f^{-1}(Y) = X$. Since Y and $\emptyset$ ṩ-open sets in Y then $f^{-1}(\emptyset) . f^{-1}(Y)$ ṩ-open in X .Then f ṩ-continuous function. But $\{2\}$ ṩ-g-coc-open in Y and $f^{-1}(\{2\}) = \{x1\}$ is not ṩ-g-coc-open set in X . $x1 \in \{x1\}$. There is no ṩ-g-open set U such that $x1 \in U$, and K ṩ-compact such that $x1 \in U - K \subseteq \{x1\}$. Then f is not ṩ-g-coc´-continuous function .

Proposition (3.29)

Let $f: X \longrightarrow Y$ be a function of space X into space Y then the following statements are equivalent .

i. $f\ is$ ṩ-g-coć-continuous .

ii. $f(\bar{A}^{ṩ-g-coc}) \subseteq \overline{f(A)}^{ṩ-g-coc}$ for every set $A \subseteq X$.

iii. $\overline{f^{-1}(B)}^{ṩ-g-coc} \subseteq f^{-1}(\bar{B}^{ṩ-g-coc})$ for every set $B \subseteq Y$.

proof:

$i \longrightarrow ii$

Let $A \subseteq X$ then $f(A) \subseteq Y$ and $\overline{f(A)}^{ṩ-g-coc}$ ṩ-g-coc-closed set in Y. Since $f\ is\ ṩ - g - coć - continuous$. Then $f^{-1}(\overline{f(A)}^{ṩ-g-coc})$ ṩ -g-coc-closed set in X by proposition(2.14). Since $f(A) \subseteq \overline{f(A)}^{ṩ-g-coc}$ then $f^{-1}(f(A)) \subseteq f^{-1}(\overline{f(A)}^{ṩ-g-coc})$ .Then $A \subseteq f^{-1}(\overline{f(A)}^{-g-coc})$ .Since $f^{-1}(\overline{f(A)}^{ṩ-g-coc})$ - ṩ g-coc-closed .Then $\bar{A}^{ṩ-g-coc} \subseteq f^{-1}(\overline{f(A)}^{ṩ-g-coc})$ thus $f(\bar{A}^{ṩ-g-coc}) \subseteq f(f^{-1}(\overline{f(A)}^{ṩ-g-coc})) \subseteq \overline{f(A)}^{ṩ-g-coc}$ .Then $f(\bar{A}^{ṩ-g-coc}) \subseteq \overline{f(A)}^{ṩ-g-coc}$

$ii \longrightarrow iii$

Let $(\bar{A}^{ṩ-coc}) \subseteq \overline{f(A)}^{ṩ-coc}$ $\forall A \subseteq X$. $B \subseteq Y$,then $f^{-1}(B) \subseteq X$, $f(\overline{f^{-1}(B)}^{ṩ-coc}) \subseteq \overline{f(f^{-1}(B))}^{ṩ-coc}$,then $f(\overline{f^{-1}(B)}^{ṩ-g-coc}) \subseteq \bar{B}^{ṩ-g-coc}$ .Hence $\overline{f^{-1}(B)}^{ṩ-g-coc} \subseteq f^{-1}(\bar{B}^{ṩ-g-coc})$

$iii \longrightarrow i$

Let B be ꞩ -g-coc-closed set in Y then B $= \bar{B}^{ꞩ-g-coc}$. Since $\overline{f^{-1}(B)}^{ꞩ-g-coc} \subseteq f^{-1}(\bar{B}^{ꞩ-g-coc})$. Then $\overline{f^{-1}(B)}^{ꞩ-g-coc} \subseteq f^{-1}(B)$ .Since $f^{-1}(B) \subseteq \overline{f^{-1}(B)}^{ꞩ-g-coc}$ .Therefore $\overline{f^{-1}(B)}^{ꞩ-g-coc} = f^{-1}(B)$. Therefore $f^{-1}(B)$ ꞩ-g-coc-closed in X . Hence is ꞩ $- g - coć - continuous$.

Proposition (3.30) Let $f: X \longrightarrow Y$ be a function of space X into space Y then f is ꞩ $-$ g$-coć-continuous$ function if and only if the inverse image of every ꞩ-g-coc-closed in Y is ꞩ-g-coc-closed set in X

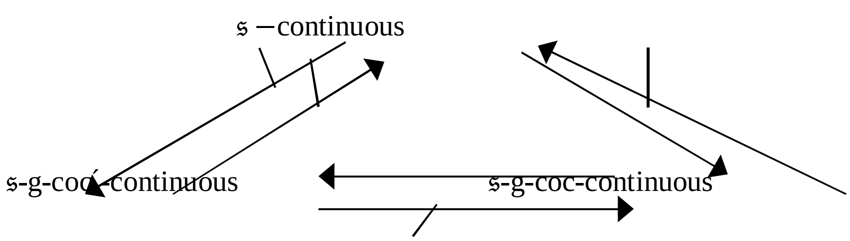
proof:

Let f be ꞩ $-$ g$-coć-continuous$, let B be ꞩ-g-coc-closed set in Y .Then B^c ꞩ-g-coc-open in Y Since f ꞩ $-$ g$-coć-continuous$, then $f^{-1}(B^c)$ ꞩ -g-coc-open in X . $f^{-1}(B^c) = f^{-1}(B - Y) = f^{-1}(Y) - f^{-1}(B) = X - f^{-1}(B) = (f^{-1}(B))^c$ .Then $(f^{-1}(B))^c$ ꞩ-g-coc-open in X. Hence $f^{-1}(B)$ ꞩ-g-coc-closed in X

Conversely:

Let M be ꞩ-g-coc-open in Y, then M^c is ꞩ-g-coc-closed in Y . Then $f^{-1}(M^c)$ ꞩ-g-coc-closed in X, since $f^{-1}(M^c) = f^{-1}(Y - M) = f^{-1}(Y) - f^{-1}(M) = X - f^{-1}(M) = (f^{-1}(M))^c$. Therefore , $f^{-1}(M^c) = (f^{-1}(M))^c$. Then $(f^{-1}(M))^c$ ꞩ-g-coc-closed in X hence $f^{-1}(M)$ ꞩ-g-coc-open set in X $t$hen f is ꞩ $-$ g $-$coć$-$continuous .

The following diagram shows the relation between types of ꞩ-coc-continuous functions



4. ꞩ-g-Coc-compact space

In this section, we introduce the concept of ꞩ-g-Coc-compact space and give some important generalization on this concept.

Definition (4.1): Let X be a supra space. A family F of subset of X is called ꞩ-g-coc-open cover of X if F covers X and F is sub family of μ^{gk}.

Definition (4.2): A space X is said to be ꞩ –g-coc-compact if every ꞩ–g-coc-open cover of X has finite sub cover.

Example (4.3):

.i. Every finite subset of a space X is an ꞩ–g-coc-compact.

ii. The indiscrete space is ꞩ-g-coc-compact space.

Remark (4.4): It is clear that every ꞩ-g-coc-compact space is ꞩ-compact but the converse is not true in general as the following example shows

Example (4.5): Let $X = \mathbb{R}$ the set of real numbers with τ is indiscrete topology, the coc-open set is $\{A: A \subseteq X\}$. Then X is $\mathfrak{s}$-compact space but not $\mathfrak{s}$—g-coc-copmpact.It is clear that, a space $(X.\mu)$ is $\mathfrak{s}$–g-coc-compact iff the space $(X.\mu^{gk})$ is $\mathfrak{s}$-compact.

Theorem (4.6):
Let $f : X \to Y$ be an onto $\mathfrak{s}$-g-coc-continuous function. If X is $\mathfrak{s}$-g-coc-compact then Y is $\mathfrak{s}$-compact.
Proof:
Let $\{V_\alpha : \alpha \in \Lambda\}$ be an $\mathfrak{s}$-open cover of Y then $\{f^{-1}(V_\alpha) : \alpha \in \Lambda\}$ is an $\mathfrak{s}$-g-coc-open cover of X, since X is $\mathfrak{s}$-g-coc-compact .Then X has finite sub cover say $\{f^{-1}(V_{\alpha i}) : i = 1.2. \dots . n\}$ and $V_{\alpha i} \in \{V_\alpha : \alpha \in \Lambda\}$. Hence $\{V_{\alpha i} : i = 1.2. \dots . n\}$ is a finite sub cover of Y. Then Y is $\mathfrak{s}$-compact.

Theorem (4.7):
Let $f : X \to Y$ be an onto $\mathfrak{s}$ –g-coc'-continuous function. If X is $\mathfrak{s}$-g-coc-compact then Y is $\mathfrak{s}$-g-coc-compact.
Proof:
Let $\{V_\alpha : \alpha \in \Lambda\}$ be an $\mathfrak{s}$-g-coc-open cover of Y then $\{f^{-1}(V_\alpha) : \alpha \in \Lambda\}$ is an $\mathfrak{s}$-g-coc-open cover of X, since X is $\mathfrak{s}$-g-coc-compact .Then X has finite sub cover say $\{f^{-1}(V_{\alpha i}) : i = 1.2. \dots . n\}$ and $V_{\alpha i} \in \{V_\alpha : \alpha \in \Lambda\}$. Hence $\{V_{\alpha i} : i = 1.2. \dots . n\}$ is a finite sub cover of Y. Then Y is $\mathfrak{s}$-g-coc-compact.

Proposition (4.8):
For any space X the following statement are equivalent:
i. X is $\mathfrak{s}$-g-coc-compact
ii. Every family of $\mathfrak{s}$-g-coc-closed sets $\{F_\alpha : \alpha \in \Lambda\}$ of X such that $\bigcap_{\alpha\in\Lambda} F_\alpha = \phi$, then there exist a finite subset $\Lambda_o \subseteq \Lambda$ such that $\bigcap_{\alpha\in\Lambda} F_\alpha = \phi$.
Proof:
(i)→(ii) Assume that X is $\mathfrak{s}$-g-coc-compact, let $\{F_\alpha : \alpha \in \Lambda\}$ be a family of $\mathfrak{s}$-g-coc-closed subset of X such that $\bigcap_{\alpha\in\Lambda} F_\alpha = \phi$. Then the family $\{X - F_\alpha : \alpha \in \Lambda\}$ is $\mathfrak{s}$-g-coc-open cover of the $\mathfrak{s}$-g-coc-compact (X,μ) there exist a finite subset Λ_o of Λ such that $X = \cup \{X - F_\alpha : \alpha \in \Lambda_o\}$ therefore $\phi = X - \cup \{X - F_\alpha : \alpha \in \Lambda_o\} = \cap\{X - (X - F_\alpha) : \alpha \in \Lambda_o\} = \cap \{X - F_\alpha : \alpha \in \Lambda_o\}$
(ii)→(i) Let $U = \{U_\alpha : \alpha \in \Lambda\}$ be an $\mathfrak{s}$-g-coc-open cover of the space (X, τ). Then $X - \{U_\alpha : \alpha \in \Lambda\}$ is a family of $\mathfrak{s}$-g-coc-closed subset of $(X,\mu)$with $\cap \{X - U_\alpha : \alpha \in \Lambda\} = \phi$ by assumption there exists a finite subset Λ_o of Λ such that$\cap \{X - U_\alpha : \alpha \in \Lambda_o\} = \phi$ so $X = X - \cap \{X - U_\alpha : \alpha \in \Lambda_o\} = \cup \{U_\alpha : \alpha \in \Lambda_o\}$. Hence X is $\mathfrak{s}$-g-coc-compact.

Definition (4.9):
A subset B of a space X is said to be $\mathfrak{s}$-g-coc-compact relative to X if for every cover of B by $\mathfrak{s}$-g-coc-open sets of X has finite sub cover of B. The sub set B is $\mathfrak{s}$-g-coc-compact iff it is $\mathfrak{s}$-g-coc-compact as a sub space.

Proposition (4.10): If X is a space such that every $\mathfrak{s}$-g-coc-open subset of X is $\mathfrak{s}$-g-coc-compact relative to X, then every subset is $\mathfrak{s}$-g-coc-compact relative to X.
Proof:
Let B be an arbitrary subset of X and let $\{U_\alpha : \alpha \in \Lambda\}$ be a cover of B by $\mathfrak{s}$-g-coc-open sets of X. Then the family $\{U_\alpha : \alpha \in \Lambda\}$ is a $\mathfrak{s}$-g-coc-open cover of the $\mathfrak{s}$-g-coc-open set $\cup \{U_\alpha : \alpha \in \Lambda\}$ Hence by hypothesis there is a finite subfamily $\{U_{\alpha i} : i = 1.2. \dots . n\}$ which covers $\cup \{U_\alpha : \alpha \in \Lambda\}$. .This the subfamily is also a cover of the set B.

Theorem (4.11): Every $\mathfrak{s}$ –g-coc-closed subset of $\mathfrak{s}$-g-coc-compact space is $\mathfrak{s}$-g-coc-compact relative.
Proof:

Let A be an ꟊ-g-coc-closed subset of X. Let $\{U_\alpha : \alpha \in \Lambda\}$ be a cover of A by ꟊ-g-coc-open subset of X. Now for each $x \in X - A$,there is a ꟊ-g-coc-open set V_x such that $V_x \cap A$ is a finite .Since X is ꟊ-g-coc-compact and the collection $\{U_\alpha : \alpha \in \Lambda\} \cup \{ V_x : x \in X - A \}$ is a ꟊ-g-coc-open cover of X , there exists a finite sub cover $\{U_{\alpha i} : i = 1. \dots . n\} \cup \{ V_{xi} : i = 1. \dots . n \}$ .Since $\cup_{i=1}^{n}(V_{xi} \cap A)$ is finite, so for each $x_j \in (V_{xi} \cap A)$, there is $U_{\alpha(x_j)} \in \{U_\alpha : \alpha \in \Lambda\}$ such that$x_j \in U_{\alpha(x_j)}$ and $j = 1. \dots . n$ .Hence $\{U_{\alpha i} : i = 1. \dots . n\} \cup \{U_{\alpha(x_j)} : j = 1. \dots . n \}$ is a finite sub cover of $\{U_\alpha : \alpha \in \Lambda\}$ and it covers A .Therefore , A is ꟊ-g-coc-compact relative to X.

Definition(4.12):

i. A space X is called CC if every ꟊ-compact set in X is ꟊ-closed.

ii. A space X is called CC' if every ꟊ-g-coc-compact set in X is ꟊ-g-coc-closed.

Theorem(4.13):

For any space(X,μ), then $(X. \mu^{gk})$ is CC.

Proof.

Let K ∈ C(X. μ^{gk}) . As μ ⊆μ^{gk} , then C(X, μ^{gk}) ⊆ C(X, μ) and hence K ∈ C(X, μ). Thus, we have X − K ∈μ^{gk}, and hence K is ꟊ-closed in the space (X, μ^{gk}).

Theorem(4.14) :

Let X be a space. Then the following statements are equivalent:

i. X is CC.

ii. $\mu = \mu^{gk}$.

Proof. i ⇒ii

Since μ ⊆μ^{gk} ,it is sufficient to see that $\mu^{gk} \subseteq \mu$, let k is ꟊ-compact then X − K ∈μ^{gk}, by i , X − K ∈ μ , then $\mu^{gk} = \mu$

ii ⇒i Let K ∈ C(X, τ). Then X − K ∈μ^{gk} , and by ii, X − K ∈ τ. Therefore, K is ꟊ-closed in X.

Remark(4.15): It is clear that every CC space is CC' space but the converse is not true in general as the following example shows:

Example(4.16): Let $X = \{1.2.3\}$, $\tau = \{X. \phi. \{1\}. \{2\}. \{1.2\}\}$, the ꟊ-coc-open sets are discrete. $\{1\}$ is ꟊ-compact set but not ꟊ-closed then X is not CC space.

Definition (4.17):

Let $f: X \to Y$ be a function of a space X into a space Y then f is called ꟊ-g-coc-compact function if $f^{-1}(A)$ is ꟊ-compact set in X for every ꟊ-coc-compact set A in Y.

Remark (4.18): Every ꟊ-g- coc-compact function is ꟊ-compact function.

Definition (4.19): Let $f: X \to Y$ be a function of a space X into a space Y then f is called ꟊ $- g - coc'$-compact function if $f^{-1}(A)$ is ꟊ-g-coc-compact set in X for every ꟊ-g-coc-compact set A in Y.

Example (4.20): Every function from a finite space into any space is ꟊ –g-coc'-compact function.

Remark (4.21): Every ꟊ –g-coc' –compact function is ꟊ –g-coc-compact function.

Remark (2.22) :

Let $f: X \rightarrow Y$ be a function for which X is CC then the following statements are equivalent:
i. f is ꞩ-continuous.
ii. f is ꞩ-g-coc-continuous.

Remark (2.23):
Let $f: X \rightarrow Y$ be a function for which X is CC then the following statements are equivalent:
i. f is ꞩ-continuous.
ii. f is ꞩ-g- coc'-continuous.

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