Al-Qadisiyah Journal of Pure Science

Manuscript 1259

Using Fibonacci sequence in Nature

Muhammad Hassan Hamid Al-Sultani

ARTICLE

Using Fibonacci Sequence in Nature

Muhammad H.H. Al-Sultani

Islamic Azad University, Isfahan, 031, Iran

Abstract

In this work, primarily focuses of the Fibonacci sequence (FS) by compute each number (N) is the total of the two preceding numbers. The quantities (N) that are associated with the Fibonacci sequence (FS) are referred to as Fibonacci numbers (FN), which are typically written as Fn. The order(S) commonly starts from 0 to 1, and presented formula for Fibonacci sequence (FS), understand Fibonacci numbers (FN) through solved various examples. Moreover introduced connection between the Fibonacci sequence (FS) and Golden Ratio (GR), relation between Fibonacci sequence (FS) and Geometric Sequence (GS) and so comparison between Lucas Sequence And (FS). Also give applied Fibonacci sequence (FS) in nature.

Keywords: Fibonacci sequence, Fibonacci number, Golden Ratio, Lucas Sequence

1. Introduction

One of the most common scientific experiments that involves the (FS) is discussed below is his experiment with rabbits [1]. Mathematics is a specialized field of science that concerns (N), shapes, numbers, and relations, (S), series. In Ref. [2] researcher is attributed with the initial description of the (S) of (N), which occurred during the (5th B.C. and 2nd or 3rd century A.D.). Since Fibonacci documented the series for the Western world, it has been recognized as significant. In The Da Vinci Code, the (FS) is integral to a significant revelation. Another take [3]. The (FS) is said for Leonardo Pisano (fame Fibonacci), an Italian mathematician. The (S) presented by Fibonacci to answer the following question: "How many duo of rabbits will be produced in a year, beginning with a single duo, if in every month each duo bears a new duo which becomes productive from the 2nd month on?" The outcome is expressed as 1, 1, 2, 3, 5, 8, 13, 21, 34,.. [4]. The Fibonacci poem's verse employs the same syllable progression (N) on each line, following the pattern of Fibonacci. (FN) Also, they can be employed to define a circle and have a beneficial effect on biologists and physicists because they are common in the natural world and are observed frequently. In Refs. [5–14] introduce patterns of branching in trees and leaves, as well as the distribution of seeds in a raspberry's berry are all indicative of the (FS). In this survey, presented role (FS) with many branches and applied its for simple examples.

2. Methodology of (FS)

In this part, presented a (FS) is the one in which every (N) offered is a sum of the previous 2 (N), where (FS) can be computed in mathematic. As each (N) in (S) is presented a idiom, that is rewrite by the idiom Fn. The n be inverted the (N) situation in the (S), initiate as zero. For instance, the (6th, 7th) term is mention to as F5, F6 respectively. The numbering of (FS) proposed as:

- F0 = 0 (implies exclusive the 1st integer)
- F1 = 1 (implies exclusive the 2nd integer)
- Fn = Fn-1 + Fn-2 (implies to all other integers)

The above equation show that each (N) of the (S) is realized by making the preceding (N). For test, to realize the 5th (N) is (F4), by the same way realize (F2, F3,…, Fn) like Employing Table 1 to locate the

Received 9 March 2024; accepted 11 April 2024.
Available online 28 September 2024
E-mail address: m20263099@gmail.com.

https://doi.org/10.29350/2411-3514.1259

Table 1. Sequence numbers preceding the target term value.

Term position	Fn value	Fibonacci number
1st	F0	0
2nd	F1	1
3rd	F2	1
4th	F3	2
5th	F4	3
6th	F5	5
7th	F6	8
8th	F7	13
9th	F8	21
10th	F9	34
11th	F10	55
12th	F11	89
13th	F12	144
14th	F13	233

sequence numbers preceding the target expersion value. For test, Now computation locate the (FN) for the expression in the 10th situation (F9):

F_9=F_(9-1)+F_(9-2)=F_8+F_7=21+13=34

The defy with a recursive formula is that it constantly relies on knowing the previous (FN) in order to compute a specific (N) in the (S). For test, we need calculate (98th and 99th) terms before the value of the 100th term. Other equations are possible, but Binet's formula is the most popular, and it provides a closed-form solution to the (FS) problem. Also many programs of applications for recursive formula code such as Java, Python or PHP. It is that soft test to find is 21 + 34 = 55 from (s) above show that as

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811,... as shown in Table 2.

From (S) above can be represented the ‘Rule’ of (FS).

So 6th expression is submitted (x6 = 8). Lastly we get the following:

x_n=x_(n-1)+x_(n-2)

Where:
x_n is idiom (N) “n”
x_(n-1) is the previous idiom (n-1)
x_(n-2) is the idiom before that (n-2)

2.1. Connection between (FS) and (GR)

The survey fundamentally focuses on the utilize of the (GR) and the (FS). The connection between them find in nature.

With the assist of the (FS) scientists, many mysteries associated with nature have been resolved. In order to sequence the numbers, they are organized in a specific manner and follow a common pattern. Four primary types of (S) exist (Arithmetic Sequence (AS), Geometric Sequence (GS) and Harmonic Sequence (HS)). (FN) can be explained as a circle with the (N)'s representing the widths of the (S)'s. The squares seem to fit together quite well, creating a circle. In this test, (5 + 8 = 13), (8 + 13 = 21), etc. The (FS) is often associated with the (GR), a proportion (roughly 1:1.6) that is common in the natural world and is employed in multiple disciplines of human activity. The (FS) and (GR) are employed to direct the design of architecture as shown in Fig. 1.

2.2. Relation between (FS) and (GS)

In this section, show that (FS) is not (GS) since $\left(\frac{F_1}{F_0}, \frac{F_2}{F_1}, \frac{F_2}{F_1}\right)$ are (not defined, $\frac{1}{1}$, $\frac{2}{1}$) have different values. Instead of being dissuaded, we should calculate several ratios of successive numbers. (FN) are expressed in Table 3 as:

As n turn out larger, the (FS) increasingly like a (GS) with a collective share of around 1.6. Since the upcoming integer values of the (FS) seem to have a significant impact, let's not concern ourselves with the primary values of the (S). Instead, we aim for a soft outcome (GS) $x_n = ar^n$ that implies the Fibonacci recurrence $x_{n+2} = x_{n+1} + x_n$. That is, we want to consider values of r for whichar$^{n+2} = ar^{n+1} + ar^n$,

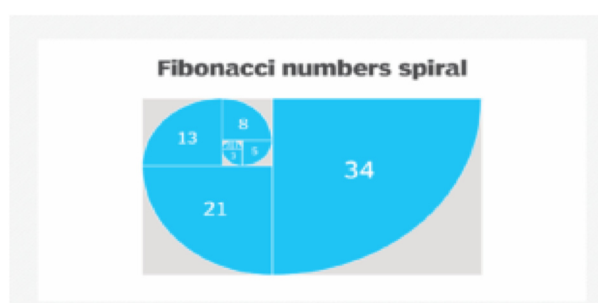

Fig. 1. Fibonacci numbers spiral.

Table 2. Rule of (FS).

n=	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	…
xn=	0	1	1	2	3	5	8	13	21	34	55	89	144	233	377	…

Table 3. Ratios of successive (FN).

n	0	1	2	3	4	5	6	7	8
F_n	0	1	1	2	3	5	8	13	21
F_{n+1}/F_n	–	1.0	2.0	1.5	1.667	1.600	1.625	1.615	1.615

a constant $r \neq 0$, which $r^{n+2} = r^{n+1} + r^n$, $r^2 = r + 1$ is the quadratic formula, we obtain there are two roots r of the quadratic equation: $p = \frac{1+\sqrt{5}}{2}$ and $q = \frac{1-\sqrt{5}}{2}$. $p = 1.618$, We believe we've identified the magnitude we sought in the numerical table above. For any two arbitrary numbers a and b, the geometric sequences ap^n and bq^n satisfy the Fibonacci law. Additionally, since $ap^{n+2} = ap^{n+1} + ap^n$ and $bq^{n+2} = bq^{n+1} + bq^n$, we can add these two equations to see that

$$\left(ap^{n+2} + bq^{n+2}\right) = \left(ap^{n+1} + bq^{n+1}\right) + \left(ap^n + bq^n\right).$$

That is, for any values of a and b, $x_n = ap^n + bq^n$ solves the Fibonacci recurrence.

After that get the several result as:

Theorem (1). The Fibonacci recursion formula $x_{n+2} = x_{n+1} + x_n$, is solved by $x_n = ap^n + bq^n$ where $p = \frac{1+\sqrt{5}}{2}$, $q = \frac{1-\sqrt{5}}{2}$, and a and b are arbitrary constants.The constant $p = \frac{1+\sqrt{5}}{2} = 1.6180\ldots$ is the celebrated (GR). It was recognized (but not named) in ancient Greek mathematics, because it resolved the problem: determine the point C on a line segment. $\overline{AB}$ so that AB/AC = AC/BC. The second solution $q = \frac{1-\sqrt{5}}{2} = -0.618\ldots$, of $r^2 - r - 1 = 0$ can be write as quadratic polynomial.

$$r^2 - r - 1 = (r-p)(r-q) = r^2 - (p+q)r + pq$$

Then the coefficients presented as $p + q = 1$ and $pq = -1$. It's show the relationships without requiring the explicit formulas for p and q that involve $\sqrt{5}$.

2.3. Comparison between Lucas Sequence And (FS)

When take is $a = 1, b = 1$ for $x_n = ap^n + bq^n$ in the theorem above get:

$L_n = p^n + q^n = \left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{1-\sqrt{5}}{2}\right)^n$. For instance,

$L_0 = p^0 + q^0 = 1 + 1 = 2$ and $L_0 = p + q = 1$, and we found that no $\sqrt{5}$ expression appears. Moreover, because $L_{n+2} = L_{n+1} + L_n$ and $L_0 = 2$, $L_1 = 1$, the expression of the sequence are $\forall Z^+$. We have rediscovered the (FS): 2, 1, 3, 4, 7, 11, 18, 29,… dedicated to Edouard Lucas (1842−1909). Despite this, the (S) is less commonly recognized. The (FS), the Lucas sequence is also characterized by several properties that benefit. To get the (FN), we must assess the potential for choosing the constants a and b to produce the desired result $F_n = ap^n + bq^n$. Because $F_0 = 0$ and $ap^0 + bq^0 = a + b$, if $tb = -a$ .Also, since $F_1 = 1$ and

$ap^1 + bq^1 = a(p - q) = a\left(\frac{1+\sqrt{5}}{2} - \frac{1-\sqrt{5}}{2}\right) = a\sqrt{5}$, we must take $= \frac{1}{\sqrt{5}}$. Thus, the nth (FN) is given by the explicit formula

$$F_n = \frac{p^n - q^n}{\sqrt{5}} = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$$

This formula is referred to as Binet's formula for the (FN), after Jacques Binet (1786−1865) who found it in 1843. However, the formula's discovery was first made in 1718 by Abraham DeMouivre (1667−1754). Since Binet's formula implies that F0 = 0, we can now understand why Jason began with 0 in his list of (FN). The formula also demonstrates why the (FS) despite not being geometric, is increasingly similar to one. Since $q = \frac{1-\sqrt{5}}{2} \approx -0.618$ we see that $|q^n|$ is increasingly small as n becomes large. Then, we have the approximate equality $F_n \approx \frac{1}{\sqrt{5}}p^n$. This explains why the (FS) is nearly a (GS), as we noticed in the table of values compute earlier. If we let {x} denote the "round to the integer nearest" to x function, it is easy to check that $F_n = \left\{\frac{p^n}{\sqrt{5}}\right\}$ for all $n \geq 0$. Similarly, the Lucas numbers can be written as $L_n = \{p^n\}$ when $n \geq 1$.

3. Applications

In the part, implied formula $x_n = x_{n-1} + x_{n-2}$ for some examples of Fibonacci Numbers as:

Example (1). calculate the addition of the 1st-10th (FN).

Sol. The list of (FN) is introduced like: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34.

$$\sum = 0 + 1 + 1 + 2 + 3 + 5 + 8 + 13 + 21 + 34 = 88.$$

Example (2). Write the value of the (12th, 13th) (FN). The 9th and 10th terms in the sequence are 21 and 34.

Sol. we require the (12th, 13th) (FN) as: Firstly present 11th term from (9th, 10th) term as $21 + 34 = 55$, 12th term as $34 + 55 = 89$, 13th term as $55 + 89 = 144$.

Example (3). Let that $F_{14} = 377$, compute the next (FN).

Sol. from $F_{n+1} = F_n \times (GR)$. Here, $(GR) = 1.61803398875$. Thus, $F_{15} = F_{14} \times 1.615 = 377 \times 1.61803398875 \approx 610$.

3.1. Applied (FN) in nature

3.1.1. Shells

Shells are probably the most celebrated test of the (S) because the lines are very distinct and apparent, and because the math is small and personal as shown in Fig. 2.

3.1.2. Trees

Initially, tree's appear as if they are part of a larger structure that analyses the growth of the part from the tree and the other? No, because you're typical and have a more favorable test result (FS).

3.1.3. Flower pistils

The portion of the flower that is in the middle of the petals (the pistil) has a greater degree of similarity to the (FS) than other parts of nature. The pattern that results from the repeated composition of the sequence is aesthetically pleasing and intricate as shown in Fig. 3.

3.1.4. Flower petals

All varieties of flowers follow this pattern, but roses are the most preferred type of flower for use as a test of (FS). I enjoy it because the petals aren't uniformly distributed, and the espiral is more apparent and lucid as shown in Fig. 4.

3.1.5. Leaves

The veins on the leaves are progressing toward a more and more outward-oriented pattern as the (FS) increases as shown in Fig. 5.

3.1.6. Storms

Specifically, hurricanes and tornadoes are frequented by many systems of storm. I think this isn't very attractive, but it's more intriguing as shown in Fig. 6.

3.1.7. Human body

The human body has several ways of expressing the (FS) proportions, including your face, your ear, and your hands, as well as other ways. You're now mathematically brilliant as shown in Fig. 7.

Fig. 2. (FS) of shells.

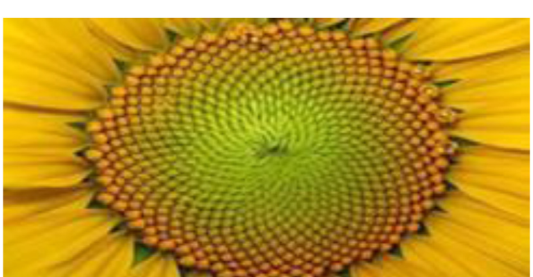

Fig. 3. (FS) of flower pistils.

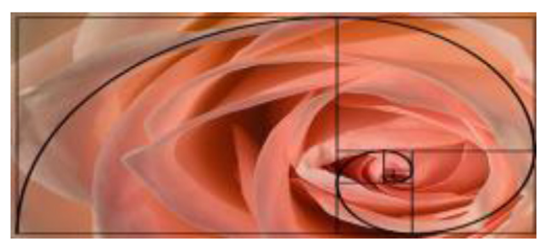

Fig. 4. Fibonacci sequence of flower petals.

Fig. 5. (FS) of leaves.

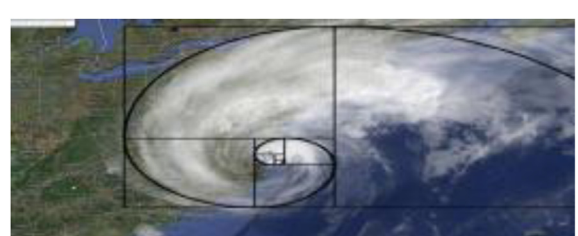

Fig. 6. (FC) of storms.

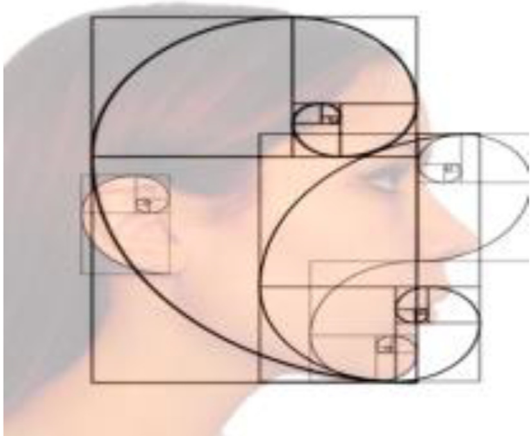

Fig. 7. (FC) of human body.

4. Conclusion

In this paper, understanding the (FN) are Nature's numbering system, (FS), the (1st, 2nd) idiom is (0, 1) respectively and The (FN) have numerous practical uses in computer science, music, financial transactions, and other fields. (FN) are present in nature in various patterns and forms. As a result, the number of (FS) will change depending on their position in the series. To calculate the general formula: $x_n = x_{n-1} + x_{n-2}$. Also study the difference (Fs) and Lucas through The creation of both Fibonacci and Lucas numbers is identical (the first number is derived from the previous two, every number is added to them). The sole discrepancy is that (FN) begins from 0 to 1, whereas Lucas' numbers begin from 2 to 1, and show that connection between the (FS) and (GR), relation between (FS) and (GS).

Funding

There is no funding for this article.

References

[1] Knott, Ron, Fibonacci's Rabbits, University of surrey faculty of engineering and physical sciences.
[2] The On-Line Encyclopedia of Integer Sequences. www.research.att.com/~njas/sequences/index.html.
[3] Weisstein, Eric. MathWorld—A Wolfram Web Resource http://mathworld.wolfram.com/.
[4] Hom EJ. What is the Fibonacci sequence? LifeScience; 2013. http://www.livescience.com/37470-fibonacci-sequence.html.
[5] Agarwal P, Agarwal N, Saxena R. 'Data encryption through Fibonacci sequence and unicode characters. MIT Int J Comput Sci Inf Technol 2015;5(2):79−82.
[6] Bortner Cashous W, Peterson Allan C. The history and applications of Fibonacci numbers. UCARE Res. Prod. 2016;42. http://digitalcommons.unl.edu/ucareresearch/42.
[7] Knott, Dr. Ron (25 September 2016), The Fibonacci Numbers and Golden section in Nature − 1, University of Surrey, retrieved 27 November 2018.
[8] Garg M, Garg P, Vohra RK. Advanced Fibonacci sequence with golden ratio. Int J Sci Eng Res 2014;5(6):388−91.
[9] Nikhat P. Fibonacci in nature. http://www.fibonacciinnature. [Accessed 15 January 2017].
[10] Sarma S, University Gauhati, Bhuyan RK. Fibonacci number, golden ratio and their connection to different flo-ras. Int J Math Trends Technol 2018;61(2):95−9.
[11] Hisert GA. The use of cragmont Fibonacci matrices in analyzing and cataloging identities with powers of Fibonacci and Lucas numbers. JP J Algebra, Number Theory Appl 2019; 41(1):95−119.
[12] Sah PK, Raj AM, Sah AK. Fibonacci sequence with golden ratio and its application. Int J Math Trends Technol 2020; 66(3):28−32.
[13] Křížek M, Somer L, Šolcová A. Fibonacci and Lucas numbers. In: From great discoveries in number theory to applications. Cham: Springer International Publishing; 2021. p. 151−81.
[14] Ambrish KP, Shriya K, Alok KV. Applications of Fibonacci sequences and golden ratio. J Infor Electr Electron Eng 2023; 4(1):1−11. S. No. 001.